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Mean Sum Labeled Independent Domination

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Abstract: Let $G(V,E)$ be a simple (p,q) graph. A function f^* is called a mean sum labeled independent dominating function if $f:E(G)\to\{1,2,3\}$ such that the induced map f^* defined by

$$f^*(v_i)=\begin{cases}0 & if \ \sum\frac{f(e_j)}{d(v_i)}\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i and the set $\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set.

Keywords: labeled domination, icosahedron, dodecahedron

Introduction: Domination and Labeling are two concepts in graph theory. We make an attempt to combine them both. In this paper we introduce a new type of graph domination called mean sum labeled independent domination. We label the vertices of the graph G by imposing some conditions on the label of the edges thereby determining the minimal independent dominating set.

Theorems

Definition: A dominating set D is an independent dominating set [2] if no two vertices in D are adjacent. The independent domination $i(G)$ of a graph G is the minimum cardinality of an independent dominating set.

Theorem: Path graph $P_m, m>3$ satisfies mean sum labeled independent domination.

Proof: Let $v_1,v_2,\ldots,v_m$ be the vertices and $e_1,e_2,\ldots,e_{m-1}$ be the edges of P_m.

Case i) When $m=3x, x>1$.

Define $f:E(G)\to\{1,2,3\}$ by $f(e_2)=1$.

$$f(e_i)=\begin{cases}2 & if \ i=1, m-1\\ 3 & if \ i=3k, k=1,2,\ldots,\frac{m-3}{3}\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum\frac{f(e_j)}{d(v_i)}\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i and the set $D=\left\{v_{3i-1}, 1\le i\le\frac{m}{3}\right\}$ is the minimal independent dominating set.

Case ii) When $m=3x+1$.

Define $f:E(G)\to\{1,2,3\}$ by $f(e_2)=1$.

$$f(e_i)=\begin{cases}2 & if \ i=1\\ 3 & if \ i=3k, k=1,2,\ldots,\frac{m-1}{3}\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum\frac{f(e_j)}{d(v_i)}\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. $D=\left\{v_{3i-1}, 1\le i\le\frac{m-1}{3}\right\}\cup\{v_m\}$ is the minimal independent dominating set.

Case iii) $m=3x+2$.

Define $f:E(G)\to\{1,2,3\}$ by $f(e_2)=1$

$$f(e_i)=\begin{cases}2 & if \ i=1\\ 3 & if \ i=3k, k=1,2,\ldots,\frac{m-2}{3}\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum\frac{f(e_j)}{d(v_i)}\equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. $D=\left\{v_{3i-1}, 1\le i\le\frac{m+1}{3}\right\}$ is the minimal independent dominating set. In the above three cases the set D satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is a minimal mean sum labeled independent dominating set

Theorem: Cycle graph C_m satisfies mean sum labeled Independent domination.

Proof: Let $v_1, v_2, \ldots, v_m$ be the vertices and $e_1, e_2, \ldots, e_m$ be the edges of C_m.

Case i) When $m = 3x, x > 1$.

Define $f : E(G) \to \{1,2,3\}$ by $f(e_2) = 1$

$$f(e_i) = \begin{cases} 2 & if \ \ i = 1, m-1, m \\ 3 & if \ \ i = 3k, k = 1,2,\ldots,\frac{m-3}{3} \end{cases}$$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i. $D = \left\{ v_{3i-1}, 1 \le i \le \frac{m}{3} \right\}$ is the minimal independent dominating set.

Case ii) $m = 3x + 1$.

Define $f : E(G) \to \{1,2,3\}$

by $f(e_2) = 1$ $f(e_i) = \begin{cases} 2 & if \ \ i = 1, m \\ 3 & if \ \ i = 3k, k = 1,2,\ldots,\frac{m-1}{3} \end{cases}$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i.

$D = \left\{ v_{3i-1}, 1 \le i \le \frac{m-1}{3} \right\} \cup \{v_m\}$ is the minimal independent dominating set.

Case iii) When $m = 3x + 2$.

Define $f : E(G) \to \{1,2,3\}$ by $f(e_2) = 1$

$$f(e_i) = \begin{cases} 2 & if \ \ i = 1, m-1, m \\ 3 & if \ \ i = 3k, k = 1,2,\ldots,\frac{m-2}{3} \end{cases}$$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i. In the above three cases the set D satisfies $\{v_i / f^*(v_i) = 1\}$. Therefore D is the minimal mean sum labeled independent dominating set.

Theorem: Comb graph P_m^+ admits mean sum labeled independent domination.

Proof: Let $\{v_i u_i\}_{i=1}^m$ be the vertex set and $\{e_i, f_i\}$ be the edge set of P_m^+ where $e_i = v_i v_{i+1}, 1 \le i \le m-1, f_i = v_i u_i, 1 \le i \le m$.

Case i) when m is even.

Define $f : E(G) \to \{1,2,3\}$ by $f(f_i) = 3 \, \forall i$

$$f(e_i) = \begin{cases} 1 & if \ \ i = 2k-1, k = 1,2,\ldots,\frac{m}{2} \\ 2 & otherwise \end{cases}$$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i.

$D = \{u_1, u_2, \ldots, u_m\}$ is a independent dominating set.

Case ii) when m is odd.

Define $f : E(G) \to \{1,2,3\}$ by

$$f(e_i) = \begin{cases} 1 & if \ \ i = m-1, 2k-1, k = 1,2,\ldots,\frac{m-1}{2} \\ 2 & otherwise \end{cases}$$

$$f(f_i) = \begin{cases} 3 & if \ \ i = 2k-1, k = 1,2,\ldots,\frac{m+1}{2} \\ 2 & otherwise \end{cases}$$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i. $D = \{u_1, v_2, u_3, v_4, \ldots, v_{m-1}, u_m\}$ is the minimal independent dominating set. In both the cases the set D satisfies $\{v_i / f^*(v_i) = 1\}$. Therefore D is a minimal mean sum labeled independent dominating set.

Theorem: Crown graph C_m^+, m even concedes mean sum labeled independent domination.

Proof: Let $\{v_i u_i\}_{i=1}^m$ be the vertex set and $\{e_i, f_i\}$ be the edge set of C_m^+ where

$e_i = v_i v_{i+1}, 1 \le i \le m-1, e_m = v_m v_1,$

$f_i = v_i u_i, 1 \le i \le m$

Define $f : E(G) \to \{1,2,3\}$ by $f(f_i) = 3 \, \forall i$

$$f(e_i) = \begin{cases} 1 & if \ \ i = 2k-1, k = 1,2,\ldots,\frac{m}{2} \\ 2 & otherwise \end{cases}$$

The induced vertex labeling are

$$f^*(v_i) = \begin{cases} 0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2) \\ 1 & else \end{cases}$$

where e_j is an edge incident with v_i. The set that satisfies

$\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set D where $D=\{u_1,u_2,\ldots,u_m\}$.

Theorem: Wheel graph W_m concedes mean sum labeled Independent domination.

Proof: Let $v_1,v_2,\ldots,v_m$ be the vertices where v is the apex vertex and e_i, f_i be the edges where $e_i = v_i v_{i+1}; 1 \le i \le m-1, e_m = v_m v_1$, $f_i = v v_i;\ 1 \le i \le m.$

Case i) when m is odd.

Define $f: E(G) \to \{1,2,3\}$ by

$$f(e_i)=\begin{cases}3 & if\ \ i=2k-1, k=1,2,\ldots,\frac{m-1}{2}\\ 2 & otherwise\end{cases}$$

$$f(f_i)=\begin{cases}1 & if\ \ i=2k-1, k=1,2,\ldots,\frac{m}{2}\\ 2 & otherwise\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if\ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i.

Case ii) when m is even.

Define $f: E(G) \to \{1,2,3\}$ by $f(f_i)=1 \forall i$

$$f(e_i)=\begin{cases}3 & if\ \ i=2k-1, k=1,2,\ldots,\frac{m}{2}\\ 2 & otherwise\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if\ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. In both the cases the set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set D where $D=\{v\}$.

Theorem: Shell graph S_m satisfies mean sum labeled independent domination.

Proof: Let $\{v, v_i\}_{i=1}^m$ be the vertex set and $\{e_i, f_i\}$ be the edge set of the shell graph S_m. $e_i = v_i v_{i+1}; 1 \le i \le m-1,\ f_i = v v_i;\ 1 \le i \le m.$

Case i) When m is odd. Define $f: E(G) \to \{1,2,3\}$ by

$$f(e_i)=\begin{cases}2 & if\ \ i=2k, k=1,2,\ldots,\frac{m-1}{2}\\ 3 & otherwise\end{cases}$$

$$f(f_i)=\begin{cases}2 & if\ \ i=m\\ 1 & otherwise\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if\ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i.

Case ii) when m is even.

Define $f: E(G) \to \{1,2,3\}$ by $f(f_i)=1 \forall i$

$$f(e_i)=\begin{cases}2 & if\ \ i=2k, k=1,2,\ldots,\frac{m-2}{2}\\ 3 & otherwise\end{cases}$$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if\ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. In both the cases the set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set D where $D=\{v\}$.

Theorem: Helm graph H_m concedes mean sum labeled Independent domination.

Proof: Let $\{v, v_i, u_i\}_{i=1}^m$ be the vertex set where v is the apex vertex and $\{e_i\}_{i=1}^{m-1}, \{f_i, g_i\}_{i=1}^m$ be the edge set where $e_i = v_i v_{i+1}; 1 \le i \le m-1, e_m = v_m v_1$, $f_i = v v_i;\ 1 \le i \le m, g_i = v_i u_i; 1 \le i \le m$ of the helm graph where $\{u_i\}_{i=1}^m$ be the pendant vertices adjacent to $\{v_i\}_{i=1}^m$ respectively to obtain H_m.

Define $f: E(G) \to \{1,2,3\}$ by

$f(e_i)=2,\ f(f_i)=1,\ f(g_i)=3; i=1,2,..,m$

The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if\ \sum \frac{f(e_j)}{d(v_i)} \equiv 0 (\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. $D=\{v, u_1, u_2, \ldots, u_m\}$ is the minimal independent dominating set. Also this set satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is the minimal mean sum labled independent dominating set.

Mean sum labeled Independent domination for some special graphs

Theorem: Tetrahedron concedes mean sum labeled independent domination.

Proof: Let G be a Tetrahedron [1] graph. Let u, u_1, u_2, u_3 be the vertices of the tetrahedron graph. Let e_i, f_i be the edges where $e_i = u u_i; i=1,2,3,\ f_i = u_i u_{i+1}; i=1,2,\ f_3 = u_3 u_1$

Define $f: E(G) \rightarrow \{1,2,3\}$ by
$f(f_2)=1,\ f(e_1)=f(f_1)=f(f_3)=2,$
$f(e_2)=f(e_3)=3.$
The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i . $D=\{v\}$ is the miniaml independent dominating set. Also this set satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is the minimal mean sum labeled independent dominating set.

Theorem: Octahedron admits mean sum labeled independent domination

Proof: Let $u_1, u_2, u_3, v_1, v_2, v_3$ be the vertices and e_i, f_i, g_i be the edges of octahedron [1] where $e_i = v_i v_{i+1}; i=1,2, e_3 = v_3 v_1,\ f_i = u_i u_{i+1}; i=1,2,\ f_3 = u_3 u_1,$ $g_1 = u_i v_i, i=1,2, g_3 = u_2 v_1, g_4 = u_2 v_3, g_5 = u_3 v_2, g_6 = u_3 v_3$

Define $f: E(G) \rightarrow \{1,2,3\}$ by
$f(e_2)=f(e_3)=f(f_1)=f(f_3)=1,$
$f(e_1)=f(f_2)=f(g_1)=f(g_2)=f(g_4)=f(g_6)=2,$
$f(g_3)=f(g_5)=3.$
The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. $D=\{u_1, v_3\}$ is a independent dominating set. Also this set satisfies $\{v_i / f^*(v_i)=1\}$. Therefore D is the minimal mean sum labeled independent dominating set.

Theorem: Icosahedron admits mean square labeled Independent domination.

Proof: Let $\{u_1, u_2, u_3, v_1, v_2, v_3, w_1, w_2, w_3, x_1, x_2, x_3\}$ be the vertex set and $\{e_i, f_i, g_i, h_i, j_i, l_i, m_i, n_i, o_i, p_i, q_i, r_i\}$ be the edge set of icosahedron [1] graph where $e_i = u_i u_{i+1}; i=1,2, e_3 = u_3 u_1,\ f_i = v_i w_{i+1}; i=1,2,\ f_3 = v_3 w_1,$ $g_i = w_i v_i; i=1,2,3, h_i = x_i x_{i+1}; i=1,2, h_3 = x_3 x_1,\ j_i = x_i v_i;$ $i=1,2,3, l_i = u_i w_i; i=1,2,3, m_i = x_i w_i; i=1,2, n_i = x_2 w_{i+1};$ $i=1,2, o_i = x_3 w_1; i=1,3,\ p_i = v_i u_i; i=1,2,\ q_i = v_2 u_{i+1};$ $i=1,2, r_i = v_3 u_i; i=1,3.$

Define $f: E(G) \rightarrow \{1,2,3\}$ by
$f(f_1)=f(g_1)=1,\ f(f_3)=f(g_2)=3$
$f(h_i)=f(h_3)=f(j_i)=f(l_i)=f(m_i)=f(n_i)=2$
$f(o_i)=f(p_i)=2=f(q_i)=2$
The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set D where $D=\{v_1, v_2, v_3\}$.

Theorem: Dodecahedron concedes mean sum labeled independent domination.

Proof: Let G be a Dodecahedron graph [1]. Let $\left\{\begin{matrix} u_1, u_2, u_3, u_4, u_5, v_1, v_2, v_3, v_4, v_5, \\ w_1, w_2, w_3, w_4, w_5, x_1, x_2, x_3, x_4, x_5 \end{matrix}\right\}$ be the vertex set of G . Let $\{e_i, f_i, g_i, h_i, j_i, k_i\}$ be the edge set of G where $e_i = u_i u_{i+1}; i=1,2,3,4, e_5 = u_5 u_1,\ f_i = u_i v_i;$ $i=1,2,3,4,5,\ g_i = v_i w_i; i=1,2,3,4,5,\ h_i = w_i v_{i+1};$ $i=1,2,3,4, h_5 = w_5 v_1,\ j_i = w_i x_i; i=1,2,3,4,5,$ $k_i = x_i x_{i+1}; i=1,2,3,4,\ k_5 = x_5 x_1.$

Define $f: E(G) \rightarrow \{1,2,3\}$ by

$f(e_4)=f(h_5)=f(j_i)=1; i=1,2,3,4,$
$f(e_i)=2; i=1,2,3,\ f(f_i)=2, i=2,3,4,5,\ f(k_i)=2, i=2,4,5$
$f(j_5)=f(g_i)=2; i=1,2,3,4,5,\ f(h_i)=2; i=1,2,3,4,$
$f(f_1)=f(e_5)=f(k_1)=f(k_3)=3.$
The induced vertex labeling are

$$f^*(v_i)=\begin{cases}0 & if \ \sum \frac{f(e_j)}{d(v_i)} \equiv 0(\text{mod } 2)\\ 1 & else\end{cases}$$

where e_j is an edge incident with v_i. The set that satisfies $\{v_i / f^*(v_i)=1\}$ is a minimal mean sum labeled independent dominating set D where $D=\{u_1, u_4, w_1, w_2, w_3, w_4, w_5\}$.

Conclusion: In this article we define mean sum labeled Independent domination and shown that some graphs admits it.

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