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Use of Queuing Model in One Day International Cricket World Cup

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Abstract:

In this chapter we assay to apply queuing model in ICC world cup cricket matches. In particular one day international cricket game in which always two batsmen open the innings. The cricket match which is going to played on a 22 yard pitch is take as single server and the number one and number two batsmen in a playing team is taken as a customer and it is also assumed that third number batsmen who will wait for their turn is considered as a waiting customer. In this paper we will obtain the utilization factor of single server after considering the batting of two batsmen of a pitch. We can drive an arrival rate and service rate from observation, we will also determine probabilities of all three possible result i.e. tie, no result, win or loss. As a result, we conclude that the result of the cricket match is either a win or tie or no result with respective probabilities.

Keywords: Cricket, Two batsmen playing on a ground, Queuing theory, One day international game, and world cup matches two batsmen on a ground, M/M/1 queuing model, Queue, game of cricket.

History of ODI cricket:-A one day international is a form of limited over cricket which is played between two teams with international status in which each team faces a limited numbers of over , fifty. One day international matches are also called limited over international. The first ODI was played on 5 January 1971 between Australia and England at the Melbourne cricket ground which has 40 eight – all over per side. in this game Australia won the game y 5 wickets.

In the late 1970 kerry packer established the rival world series cricket competition ad it brings many of the features of one day international cricket that are common place, including coloured uniforms, matches played at night under flood lights with a white balls and dark sight screen , multiple camera angles. In the main the laws of cricket apply. In the early day of cricket the number of over was generally 60 per side, but now it has been uniformly fixed at 50 over. The international cricket council determines which teams have ODI status. The nations are listed below with the date of each nations ODI debut shown in brackets.

1.5 January 1971 England

2.5 January 1971 Australia

3. 11 February 1973 Pakistan

4. 5 September 1973 West indices

5. 13 July 1974, India

6. 7 June 1975 Sri lank

7 10 November 1991 South Africa

Since 2005, the ICC has granted temporary ODI status to six other teams. There are two major ODI tournaments which feature most or all permanent ODI teams and often also associate members.

1. Cricket world cup played every four years since 1975

2. ICC champion's trophy played every two years since 1998

HISTORY OF CRICKET WORLD CUP:

The first cricket world cup was played during 1975 in England .The first three matches were also recognized as prudential cup with the sponsorship of prudential plc; it is a pecuniary services company.

The cricket matches consisted of 60 over per players and it was played with established white uniform and white red balls. There were Matches held only during day and the event is held after four years.

Till the 1992 cricket world cup only 8 teams participated in the cricket tournaments.

Later on, the numbers of teams were certainly increased and in cricket world cup 2007, 16 teams took part in the world cup.

In 1975 , England , New Zealand , India , east Africa, Australia , West indices, Pakistan and sri lanka took participation and during 1979 canada were replaced by east Africa . In 1983, Zimbabwe made an entry and Canada was out of game. In the 1987 world, same team took part in the tournament.

In 1992 South Africa made an entry in the group and pertaining years 9 team took part in the cricket tournament.

In 1996, the number of teams even increased more up to 12 with the participation of the fresh group UAE, Netherlands and Kenya.

Bangladesh and Scotland were replaced by UAE and Netherlands during 1999 cricket world cup

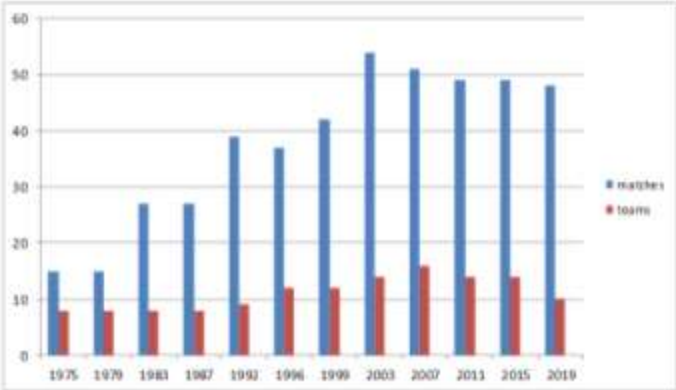
In 2003 world cup there were 14 team played in tournaments. There were 54 matches played in whole tournament, In 2007 world cup there were 16team played in tournament. There were 51 matches played in whole tournament.

In 2011 world cup there were 14team played in tournament. There were 49 matches played I whole tournament.

In 2015 world cup there were 14team played in tournament. There were 49 matches played in whole tournament.

In 2019 world there were 10 team played in tournament. There were 48matches played in whole tournament.

Here is the graph which shows number of team and number of math in world cup



England successively hosted the first three matches and during 1987 match become the first world cup to be hosted outside England .

All the cricket world cup matches played have also contributed more records in the cricket world cup history

In Matches of cricket world cup, two batsmen start the inning and third batsman is waiting for his turn. The pitch in which two player takes batting is considered as server i.e. single server. The third batsman who is waiting for his turn is considered as a waiting customer. By observing the innings of two batsmen, we can obtain the rate of arriving and rate of servicing of customers.

Assumptions:

While starting the 1st inning, the waiting customer has already padded up.

Each team has exactly 11 number of player

Each batsman complete his innings i.e. no retired hurt in inning takes place.

There is no obstructing like bad light, raining in the match.

The queuing system is assumed to be in a steady state.

M/M/1 ODI Model:

As in the single server Queuing model which has a limited number of customers in system, an ODI model can be considered as single server model which has maximum 10 number of customers given by N in each innings which are served by single server i.e. pitch of ground.

The arrival rate of the customers is denoted by λ and the service rate is given by μ.

When arrival of customers reaches its maximum limit i.e. N=10, no extra arrival of customers is allowed in the system.

So, we have $\lambda_n = \begin{pmatrix} \lambda, n=0,1,2,3,\ldots\ldots N-1 \\ 0, n=N, N+1, \ldots\ldots\ldots \end{pmatrix}$

$$\mu_n = \mu, n = 0,1,2, \ldots \ldots \ldots$$

The probability of n customers in the system given by P_n=

$$\begin{cases} (\frac{\lambda}{\mu})^n P_0 \;, n \le N \\ 0 \;, n > N \end{cases}$$

Where P_0 denote the probability of no customers in the system and it is given by properties of probability of happening of any event is equal to one

So, $\sum_{n=0}^{N} P_n = 1$

$$P_0 + P_1 + P_2 + \cdots \ldots . + P_N = 1$$

$$P_0 + \left(\frac{\lambda}{\mu}\right) P_0 + (\frac{\lambda}{\mu})^2 P_0 + \cdots \ldots \ldots \ldots \ldots \ldots (\frac{\lambda}{\mu})^N P_0 = 1$$

$$P_0 \left[1 + \frac{\lambda}{\mu} + \left(\frac{\lambda}{\mu}\right)^2 + \cdots \ldots \ldots . \left(\frac{\lambda}{\mu}\right)^N\right] = 1$$

$$P_0 \left[\frac{1 - \left(\frac{\lambda}{\mu}\right)^{N+1}}{1 - \frac{\lambda}{\mu}}\right] = 1$$

$$P_0 = \frac{1 - \frac{\lambda}{\mu}}{1 - (\frac{\lambda}{\mu})^{N+1}}$$

$$\text{So, } P_0 = \begin{cases} \frac{1 - \frac{\lambda}{\mu}}{(1 - \frac{\lambda}{\mu})^{N+1}} \; if \; \frac{\lambda}{\mu} \\ \frac{1}{N+1}, \; \frac{\lambda}{\mu} = 1 \end{cases}$$

Hence, the probability of n customer in the system (P_n) is given by

$$P_n = \begin{cases} \frac{\left(1 - \frac{\lambda}{\mu}\right)(\frac{\lambda}{\mu})^n}{1 - (\frac{\lambda}{\mu})^{N+1}}, \frac{\lambda}{\mu} \neq 1 \\ \frac{1}{N+1}, \qquad \frac{\lambda}{\mu} = 1 \end{cases}$$

In M/M/1 ODI model, we considered that arrival rate is same as service rate so, ratio of arrival and service rate i.e. $\frac{\lambda}{\mu}$ is equal to 1

The expected number of customer in the system is given by $L_s = \sum_{n=0}^{N} nP_n$

$$P_0 + P_1 + 2P_2 + \cdots \ldots \ldots \ldots \ldots . NP_N$$

$$P_0+\left(\frac{\lambda}{\mu}\right)P_0+2\left(\frac{\lambda}{\mu}\right)^2P_0+\cdots\ldots\ldots\ldots+N\left(\frac{\lambda}{\mu}\right)^NP_0$$

$$P_0[1+\frac{\lambda}{\mu}+2\left(\frac{\lambda}{\mu}\right)^2+\cdots\ldots\ldots\ldots+N\left(\frac{\lambda}{\mu}\right)^N$$

As, $\frac{\lambda}{\mu}=1$ so,

$$L_s=\frac{1}{N+1}\{1+2+3+\cdots..+N\}$$

$$\frac{1}{N+1}\left[\frac{N(N+1)}{2}\right]=\frac{N}{2}$$

So, as in ODI maximum number of customer is N=10 so, the expected number of customer in system is $L_s=\frac{10}{2}=5$ which gives that 5 wickets is falling in each innings.

The expected number of customer in queue is given by L_q and is obtain by well known Little's formula given by

$$L_s=L_q+\frac{\lambda_{eff}}{\mu}$$

$$\frac{N}{2}=L_q+\frac{\lambda_{eff}}{\mu}$$

λ_{eff} is obtained by relationship $\lambda=\lambda_{eff}+\lambda_{lost}$

Where λ_{lost} is the rate of customers lost in the system which takes place if number of customers in the system is already N.

So, the proportion that customers that will not enter the system is given by P_N

So, $\lambda_{lost}=\lambda\, P_N$

Hence,

$$\lambda_{eff}=\lambda-\lambda P_N=\lambda[1-P_N]=\lambda\left[1-\frac{1}{11}\right]=\frac{10\lambda}{11}=\frac{10\mu}{11}$$

So, $L_q=\frac{N}{2}-\frac{10}{11}=5-\frac{10}{11}=4.09$

Which shows that the expected number of customers (batsmen) are waiting in the queue is equal to 4.09 i.e. 4 batsmen are waiting in the queue in each innings.

The average number of busy server is given by difference between expected number of customers in system and the expected number of customers in queue

$$c=L_s\text{-}L_q=\frac{\lambda_{eff}}{\mu}=\frac{10}{11}=0.9090$$

This means that 1 server remain busy

So, we can say that the utilization factor of system is equal to one.

Calculations of probability:

A coin is tossed to decide the team who will bat first

Let A_1 be the event that the team but in 1st innings and A_2 be the event that the team bat in 2nd innings.

$$P(A_1)=P(A_2)=\frac{1}{2}$$

Let B_1 be event that team having 1st innings win and B_2 be the event team having 2nd innings win.

So, $P(B_2)=P(A_2)P(n\le 9\ in\ 2nd\ innings)=\frac{1}{2}\left(\frac{1}{11}+\frac{1}{11}+\cdots..+\frac{1}{11}\right)10\ times$

$$=\frac{1}{2}\left(\frac{10}{11}\right)=\frac{10}{22}$$

The probability of team having 1st innings win is

$$P(B_1)=P(A_1)+P(A_2)P(n=10\ in\ the\ 2nd\ innings)=\frac{1}{2}+\frac{1}{22}=\frac{12}{22}$$

This shows that out of ODI matches in world cup 12 matches are won by team batting first and 10 matches are won by team batting second. This can be verified from the data of the ICC world cup 2019. The actual record of the world cup 2019 is as under in the table:

Match number	Ist inning	2nd inning
1	W	L
2	L	W
3	L	W
4	L	W
5	W	L
6	W	L
7	W	L
8	L	W
9	L	W
10	W	L
11	NR	NR
12	W	L
13	L	W
14	W	L
15	NR	NR
16	NR	NR
17	W	L
18	NR	NR
19	W	L
20	L	W
21	L	W
22	W	L
23	L	W
24	W	L

25	L	W
26	W	L
27	W	L
28	W	L
29	W	L
30	W	L
31	W	L
32	W	L
33	L	W
34	W	L
35	L	W
36	L	W
37	W	L
38	W	L
39	W	L
40	W	L
41	W	L
42	W	L
43	W	L
44	L	W
45	W	L
46	W	L
47	L	W

The above Table shows that during ICC world cup 2019 out of 47 matches 28 matches have been won by team batting first which give 5% chance of winning those team which have batting first and 15 matches have been won by team batting second which give 0.31% chance of winning those team which have batting second. From this we observe that theoretical probabilities match with actual probabilities.

Probability of winning an ODI match by any one of the team is

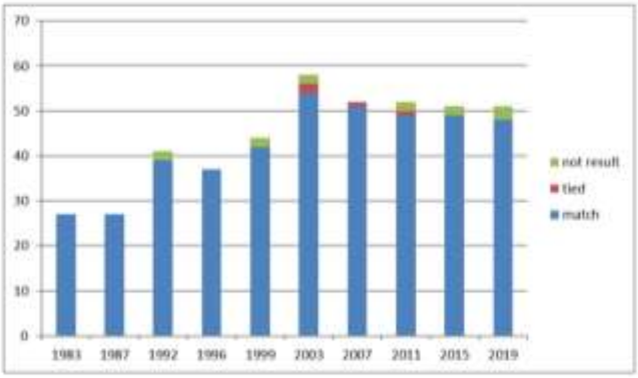
$$P(B_1) + P(B_2) = \frac{12}{22} + \frac{9}{12} = \frac{21}{22} = 0.954$$

From the law of probability, total probability is equal to one so, probability of tied matches is

1-0.954=0.046

The graph given below shows number of tied and not result matches in various world cups and we observe that chance of tied or not result matches is actually 4% which is same as from theoretical aspects.

Total match played	Number of wins	Number of tied or no result	Probability of tied or no result match
423	406	17	17/423=0.04



Probability of winning of team who will bat first or bat second is given by

$$\sum_{i=1}^{n} x_i p(x_i) = \frac{1}{2}\left(\frac{12}{22}\right) + \frac{1}{2}\left(\frac{9}{22}\right) = \frac{12}{44} + \frac{9}{44} = \frac{21}{44} = 0.47$$

Actual 2019 world cup record:

Team	Matches	Won	Loss	Tied	No result	T+NR	Win%
England	11	7	3	1	0	1	63.64
India	9	7	2	0	0	0	77.78
Australia	10	7	3	0	0	0	70
New Zealand	10	6	3	1	0	1	60
Pakistan	8	5	3	0	0	0	62.5
South Africa	9	3	5	0	1	1	33.33
Sri Lanka	7	3	4	0	0	0	42.86
Bangladesh	8	3	5	0	0	0	37.5
West Indies	9	2	6	0	1	1	22.22
Afghanistan	9	0	9	0	0	0	0
Total	**90**	**43**	**43**	**2**	**2**	**4**	**47.80**

So, from the table, we observed that actual probability match with calculated probability. Thus we can verify our result.

Conclusion: As a result, we conclude that the expected percentage of team winning batting first or second is 47.7%

Almost 95.5% ODI result in a win by any one of the two team.

References

[1] Taha H.A; Operations research –An introduction.

[2] Cooper RB (1972). Introduction to Queuing Theory. McMilan: New York

[3] Tijms HC (1986). Stochastic Modeling and Analysis.

[4] Steenson WJ (1996). Production/Opertions Management.

[5] Rust K., "Using Little's Law to Estimate Cycle Time and Cost", Proceedings of the 2008 Winter Simulation Conference, IEEE Press, Dec. 2008, doi:10.1109/WSC.2008.4736323.

[6] Little J.D.C., "A Proof for the Queuing Formula: ", Operations Research, vol. 9(3), 1961, pp. 383-387, doi:10.2307/167570.

[7] Worthington D and Wall A (1999). Using the discrete time modeling approach to evaluate the time-dependent behavior of queuing systems. J Opl Res Soc 50: 777-888.