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Significance of l-m homomorphism and congruence relations on fuzzy modular l-filters

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Abstract: This work investigates the concept of congruence relations for the group of all fuzzy modular l-filters (fml-filters). Consider F_f the set of all fuzzy modular l-filters defined on the commutative l-m group. This study aims to introduce the research of fuzzy congruence relations on family of fuzzy modular l-filter, as well as key features for the relationship and its congruence connections. After that, we concentrated into the idea of l-m homomorphism on fuzzy modular l-filters.

Keywords: fml-filters, congruence relations, l-m homomorphism, family of fuzzy modular l-filters.

1. Introduction

One of the most essential areas of fuzzy inference system is fuzzy set theory. L.A. Zadeh[14] investigated these at the outset in a hazy sense. Rosenfeld [12] introduced the theory of fuzzy groups by applying it to group theory. Das [3] described fuzzy subgroups as level subgroups of their level subgroups. Das introduced level subgroups as fuzzy subgroups. Since then, scholars in a variety of mathematical areas have been attempting to apply their findings to a broader framework of the fuzzy setting. Gu [17] created, notion of fuzzy groups with operators, which was then extended by S. Subramanian and R. Natarajan [16] to m-fuzzy groups with operators.

Since then, researchers in a variety of mathematical disciplines have been attempting to apply their findings to a wider framework of the fuzzy environment. In addition, several authors have contributed to the field of fuzzy lattice theory.

2. Preliminaries

Definition 2.1:

A commutative l-m group is a void set G if,

i.$(G,\bullet)$ $is\ commutative\ \ group$

ii.$(G,\vee,\wedge)$ $is\ a\ \ lattice$

iii.$a(mx\vee my) = (amx)\vee(amy)$

iv.$a(mx\wedge my) = (amx)\wedge(amy),\ \forall a,b,x,y\in G, m\in M.$

Definition 2.2:

A modular l-filters is a void subset F of G if,

i.F is modular sub lattice

ii.$0 < mx < a\ \ and\ mx\in F \Rightarrow a\in F.$

3. l-m homomorphism on fuzzy modular l-filter

Theorem 3.1:

Let ϕ: $G_1 \rightarrow G_2$ be the l- m homomorphism. Let (G_1, μ_1) and (G_1, μ_2) be the fml-filter of G_1 and G_2. Then the pre image of ϕ defined by $\phi^{-1}(\mu_2)[(mu) = \mu_2[\phi(mu)$ is a fml-filter of G_1.

Proof:

Let the l- m homomorphism is defined by ϕ: $G_1 \rightarrow G_2$.

For any $mu_1, mu_2 \in G$

i.$\phi^{-1}(\mu_2)[(mu)(mv)] = \mu_2(\phi((mu)(mv)))$

$\geq [\mu_2(\phi(mu), \mu_2(\phi(mv)]$

$\geq [(\phi^{-1}(\mu_2)[(mu)], \phi^{-1}(\mu_2)[(mv)]$

ii.$\phi^{-1}(\mu_2)[(mu)\vee(mv)] = \mu_2(\phi((mu)\vee(mv)))$

$\geq [\mu_2(\phi(mu), \mu_2(\phi(mv)]$

$\geq [(\phi^{-1}(\mu_2)[(mu)], \phi^{-1}(\mu_2)[(mv)]$

iii.Likewise, $\phi^{-1}(\mu_2)[(mu)\wedge(mv)] \geq [(\phi^{-1}(\mu_2)[(mu)], \phi^{-1}(\mu_2)[(mv)]$

iv.$\phi^{-1}(\mu_2)[(mu\vee mv)\wedge\mu(mu\vee mv)] = \mu_2(\phi(mu), [\mu_2(\phi(mv)\wedge\mu_2(\phi(mu\vee mw))]$

$\geq [\mu_2(\phi(mu), [\mu_2(\phi(mv)\wedge\mu_2(\phi(mu\vee mv))])$

$\geq [(\phi^{-1}(\mu_2)[(mu)], \phi^{-1}(\mu_2)[(mv)\wedge\phi^{-1}(\mu_2)((mu\vee mw))]]$

⇒ Thus the pre image of ϕ is a fml-filter of G_1.

4. Congruence relations on set of FML-F

Definition 4.1:

Let μ is equivalence relation on G denoting the fuzzy equivalence relation. Using $\mu: U \rightarrow [0,1]$. if there's a fuzzy congruence on G,

i. $\mu(m(ru), m(sv)) \geq \mu(mr, ms) \wedge \mu(mu, mv)$
ii. $\mu(mr \wedge mu, ms \wedge mv) \geq \mu(mr, ms) \wedge \mu(mu, mv)$
iii. $\mu(mr \vee mu, ms \vee mv) \geq \mu(mr, ms) \wedge \mu(mu, mv)$
iv. $\mu(mr \vee mu, ms \vee mv \wedge \mu(mr \vee mu, mt \vee mw)$
$\geq \mu(mr \vee mu) \vee [\mu(ms \vee mv) \wedge \mu(mr \vee mu, mt \vee mw)], \forall u, v, r, s \in G, m \in M, M \subseteq G.$

Example 4.2: Assume $G = \{0, mu_f, mv_f, mw_f, 1\}$. Let $\mu: U \rightarrow [0,1]$ in G.

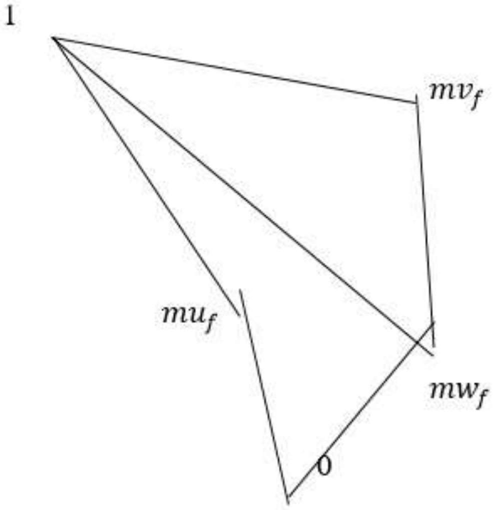

Fig. 1. Hasse diagram for the given data

Table 1. Infimum values

$\wedge$	μ(0)	$\mu(mu_f)$	$\mu(mv_f)$	$\mu(mw_f)$	μ(1)
μ(0)	μ(0)	$\mu(mu_f)$	μ(0)	μ(0)	μ(0)
$\mu(mu_f)$	μ(0)	$\mu(mu_f)$	μ(0)	μ(0)	μ(0)
$\mu(mv_f)$	μ(0)	μ(0)	$\mu(mv_f)$	μ(0)	μ(0)
$\mu(mw_f)$	μ(0)	μ(0)	μ(0)	$\mu(mw_f)$	μ(0)
μ(1)	μ(0)	μ(0)	μ(0)	μ(0)	μ(1)

The Figure 1 depicts a poset with an assumed upward inclination in graphical form. $\mu: U \rightarrow [0,1]$ satisfies all the axioms of congruence relations on set of FML-F.

Theorem 4.3:

Let F_f be the set of all FML-F on G. The binary relation θ_F, defined on F such that $\mu_1 \equiv \mu_2\ (\theta_F)$ if and only if $\theta \vee \mu_1 = \theta \vee \mu_2$ is a congruence relation for $\theta, \mu_1, \mu_2 \in F_f$ and $\theta \subseteq \mu_1, \theta \subseteq \mu_2$.

Proof

Let $\mu_1, \mu_2 \in F_f$

The binary relation θ_F defined on F_f such that $\mu_1 \equiv \mu_2\ (\theta_{F_f})$ if and only if $\theta \vee \mu_1 = \theta \vee \mu_2$ is reflexive, symmetric and transitive.

To prove the substitution property,

Assume that $\mu_1 \equiv \mu_2\ (\theta_{F_f})$ and $\mu_3 \equiv \mu_4\ (\theta_{F_f})$
$\Rightarrow \theta \vee \mu_1 = \theta \vee \mu_1$ and $\theta \vee \mu_3 = \theta \vee \mu_4$
$\Rightarrow \theta \vee (\mu_1 \vee \mu_3) = (\theta \vee \mu_1) \vee \mu_3$
$\Rightarrow \quad = (\theta \vee \mu_2) \vee \mu_3$
$\Rightarrow \quad = \theta \vee (\mu_2 \vee \mu_3)$
$\Rightarrow \quad = \theta \vee (\mu_3 \vee \mu_2)$
$\Rightarrow \quad = (\theta \vee \mu_3) \vee \mu_2$
$\Rightarrow \quad = (\theta \vee \mu_4) \vee \mu_2$
$\Rightarrow \quad = \theta \vee (\mu_4 \vee \mu_2)$
$\Rightarrow \quad = \theta \vee (\mu_2 \vee \mu_4)$
$\Rightarrow \mu_1 \vee \mu_3 = \mu_2 \vee \mu_4 (\theta_{F_f})$
$\Rightarrow \theta_{F_f}$ is a congruence relation.

Theorem 4.4:

Consider $\mu_1 \equiv \mu_2\ (\theta_{F_f})$. Then $\mu_1 \vee \mu_2 \equiv \mu_1 \cup \mu_2 (\theta_{F_f})$.

Proof

Given that, $\mu_1 \equiv \mu_2 (\theta_{F_f})$
$\Rightarrow (\theta \vee \mu_1) = (\theta \vee \mu_2)$
$Now, (\mu_1 \vee \mu_2)(mu) = (\mu_1 \cup \mu_2)(mu)$
$\Rightarrow \mu_1 \vee \mu_2 \geq \mu_1 \cup \mu_2.$

$Since\ \theta \subseteq \mu_1\ and\ \theta \subseteq \mu_2,$
$(\theta \vee (\mu_1 \vee \mu_2)) \geq (\theta \vee (\mu_1 \cup \mu_2))$ (1)
$Let\ mu = r \vee s$
$\Rightarrow mu \geq r\ and\ mu \geq s$

$Since\ \mu_1\ is\ a\ FML-F$
$\Rightarrow \mu_1(r) \geq \mu_1(mu)$

$Since\ \mu_2\ is\ a\ FML-F$
$\Rightarrow \mu_2(r) \geq \mu_2(mu)$
$\Rightarrow max\{\mu_1(mu), \mu_2(mu)\}$
$\Rightarrow (\mu_1 \cup \mu_2)(mu) \geq \mu_1 \vee \mu_2$
$Since\ \theta \subseteq \mu_1\ and\ \theta \subseteq \mu_2,$
$(\theta \vee (\mu_1 \cup \mu_2))(mu) \geq (\theta \vee (\mu_1 \cup \mu_2))$ (2)

$From\ (1)\ and\ (2)$

$\Rightarrow \mu_1 \vee \mu_2 \equiv \mu_1 \cup \mu_2 (\theta_{F_f}).$

5. Fuzzy Congruence Relation on the Family of FML-Fs.

Theorem 5.1:

Let $\mu_1, \mu_2 \in F_f$ and θ_{F_f} be the congruence relation on F_f. If $\mu_1 \equiv \mu_2 (\theta_{F_f})\ for\ \theta \subseteq \mu_1, \theta \subseteq \mu_2$ then there exists a fuzzy congruence θ on θ_p for p∈[0,1] such that,

$\bar{\theta}(\boldsymbol{\mu}(mu_f), \boldsymbol{\mu}(mv_f)) = \begin{cases} \theta(mu_f) \vee \theta(mv_f) & \text{if } mu_f \neq mv_f \\ 1 & \text{if } mu_f = mv_f \end{cases}$

Proof

Assume that $\mu_1 \equiv \mu_2 \ (\theta_{F_f})$

$\Rightarrow$ There exists $\theta \in F_f$ such that $\theta \vee \mu_1 = \theta \vee \mu_2$, for $\theta \in F_f$

Let $\theta_p = \{mu_f \in G / \theta(mu_f) = p\}$

Let $max(\theta(mu_f, mw_f), \bar{\theta}(mw_f, mv_f)) = p$

$Now\ \bar{\theta}(mw_f, mu_f) = 1$

$\Rightarrow \bar{\theta}\ fuzzy\ reflexive.$

$\bar{\theta}(mw_f, mv_f) = \theta(mu_f) \vee \theta(mv_f)$

$= \theta(mv_f) \vee \theta(mu_f)$

$= \bar{\theta}(mv_f, mu_f)$

$\Rightarrow \bar{\theta}$ is fuzzy symmetric.

$Now,$

$\bar{\theta}(mu_f, mv_f) = \theta(mu_f) \vee \theta(mv_f)$

$\leq p$

$(\bar{\theta} \bullet \bar{\theta})\ (mu_f, mv_f) = inf\ \{max[\bar{\theta}(mu_f, mw_f), \bar{\theta}(mw, mv_f)] = p$

$\leq \bar{\theta}(mu_f, mv_f)$

$\Rightarrow (\bar{\theta} \bullet \bar{\theta}) \subseteq \bar{\theta}$

$\Rightarrow \bar{\theta}$ is fuzzy transitive.

Since θ is a fuzzy modular l-filter

$= \bar{\theta}(r, s) \vee \bar{\theta}(mu_f, mv_f)$

$\Rightarrow \bar{\theta}(r(mu_f), s(mv_f)) \leq \bar{\theta}(r, s) \vee \bar{\theta}(mu_f, mv_f)$

Similarly,

$\bar{\theta}(r \vee (mu_f), s \vee (mv_f)) \leq \bar{\theta}(r, s) \vee \bar{\theta}(mu_f, mv_f)$

$and\, \bar{\theta}(r \wedge (mu_f), s \wedge (mv_f)) \leq \bar{\theta}(r, s) \vee \bar{\theta}(mu_f, mv_f)$

Hence $\bar{\theta}$ is fuzzy congruence.

6. Conclusion

The idea of congruence relations for the group of all fuzzy modular l-filters is investigated in this paper. The goal of this research is to introduce fuzzy congruence relations research on the family of fuzzy modular l-filters, as well as essential elements of the relationship and its congruence connections. We then focused on the notion of l-m homomorphism on fuzzy modular l-filters.

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