Graphs are Lucky and Proper

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Abstract:

Let $f: V(G) \rightarrow \mathbb{N}$ positive integers are used to label the nodes of a graph G. The Lucky is the graph that has a sum of adjacent nodes that are not equal $S(u) \neq S(v)$ where $S(u)$ and $S(v)$ denotes the sum of adjacent node. It was related with proper which means labeling of the adjacent u and v are not same $f(u) \neq f(v)$ where $f(u)$ and $f(v)$ denotes the adjacent labeling. If vertex is isolate, then the sum will be represented as zero $S(u) = 0$. Here we showed a least possible lucky number that can be labeled the star graph S_n, fish graph F and cross graph C, So these graphs are called as lucky graph.. Also we showed a least possible proper lucky number that can be labeled the star graph, fish graph and cross graph, so these called these graphs as proper lucky graph

. **Keywords:** Fish graph, cross graph, lucky, proper

Introduction

The Lucky is the graph which has sum of adjacent nodes are not equal [3]. It is dented as $\eta(G)$. If the graphs sum of adjacent nodes are not equal as well as its adjacent labels are equal then it is said to be Proper lucky [4]. It is dented as $\eta_P(G)$. f and s denoted label and its sum respectively.

Sandhya and Aswathy computed the root square for star graph, fish graph and cross graph. Maria Chitra and Murugan computed the star related graphs are Lucky Edge [5]. Kaneria, Meghpara, Meera and Jariya, computed a some star related graphs are Graceful Labeling [2]. Hameeda and Annamma derived some star related graphs are Prime Labeling [1]. Maheshwari, Balaji and Sudhakar derived star graphs are Cordial Labeling [8]. Dhanalakshmi and Parvathi dervived star related graphs are Mean Square Cordinal Labeling [6].

Preliminaries

Star Graph

A star graph contains n + 1 vertices and n edges. It is denoted by S_n. $V(S_n) = \{u_1, u_2, \ldots, u_n\} \cup \{v\}$ and $E(S_n) = \{vu_i\}$ where n ∈ ℕ and $1 \leq i \leq n$ refer fig 3 [5].
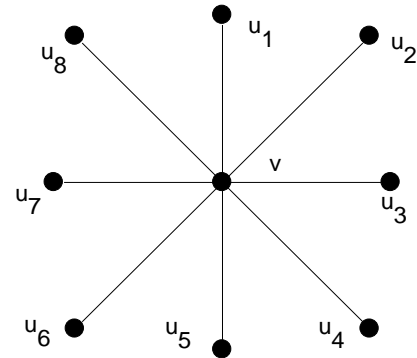

Figure 1: Star graph S_8

Fish Graph

A fish graph contains 6 vertices and 7 edges. It is denoted as F. $V(F) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ $E(F) = \{u_n u_{n+1}\} \cup \{u_2 u_3\} \cup \{u_{n+1} u_3\}$ where $n \in \mathbb{N}$ and $n < 6$ refer fig 2 [7].
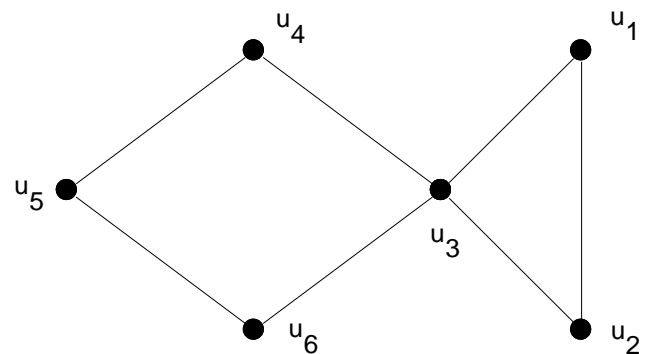

Figure 2: Fish graph

Cross Graph

A cross graph contains 6 vertices and 5 edges. It is denoted by C. $V(C) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ $E(C) = \{u_n u_{n+1}\} \cup \{u_2 u_{n+1}\} \cup \{u_{n+2} u_2\}$ where $n \in \mathbb{N}$ and $n < 5$ refer fig 3 [7].
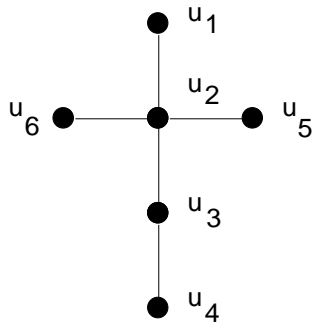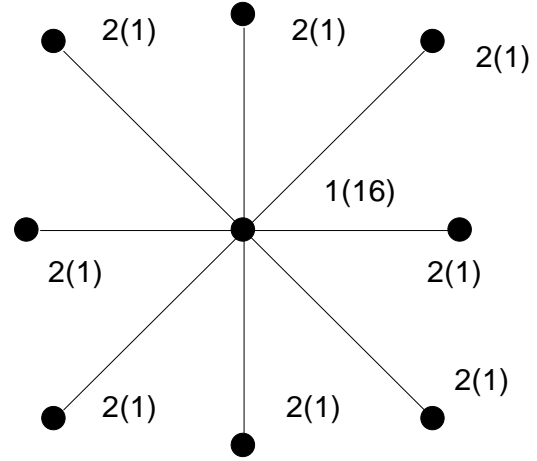
Figure 3: Cross graph

Main Results

Theorem 1

A star graph is lucky graph with $\eta(S_n) = 1$.

Proof

Let $V(S_n) \to \{1\}$ be defined by

$$f(v) = f(u_i) = 1, 1 \le i \le n$$
$$s(v) = n$$
$$s(u_i) = 1, 1 \le i \le n$$

Therefore, star graph admits lucky with $\eta(S_n) = 1$ as shown in fig 4.

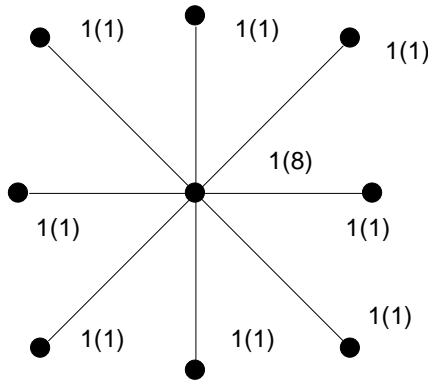**Figure 4:** Lucky star graph S_8

Theorem 2

A star graph is proper lucky with $\eta_p(S_n) = 2$.

Proof

Let $V(S_n) \to \{1\}$ be defined by

$$f(v) = 1$$
$$f(u_i) = 2, 1 \le i \le n$$
$$s(v) = 2n$$
$$s(u_i) = 1, 1 \le i \le n$$

Therefore, star graph admits proper lucky with $\eta_p(S_n) = 2$ as shown in fig 5.

Figure 5: Proper Lucky star graph S_8

Theorem 3

A fish graph is lucky graph with $\eta(F) = 2$.

Proof

Let $V(F) \to \{1,2\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i \ne 1,5 \\ 2 & i = 1,5 \end{cases}$$
$$s(u_i) = \begin{cases} 2 & i = 1,5 \\ 3 & i = 2,4,6 \\ 5 & i = 3 \end{cases}$$

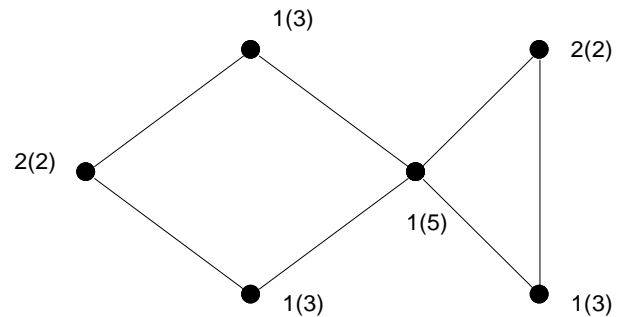Therefore, fish graph admits lucky with $\eta(F) = 2$ as shown in fig 6.

Figure 6: Lucky fish graph

Theorem 4

A fish graph is proper lucky with $\eta_p(F) = 3$.

Proof

Let $V(F) \to \{1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i = 3,5 \\ 2 & i = 1,4,6 \\ 3 & i = 2 \end{cases}$$
$$s(u_i) = \begin{cases} 2 & i = 4,6 \\ 3 & i = 2 \\ 4 & i = 1,5 \\ 9 & i = 3 \end{cases}$$

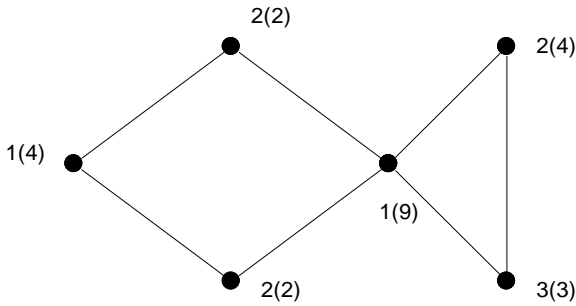Therefore, fish graph admits proper lucky with $\eta_p(F) = 3$ as shown in fig 7.
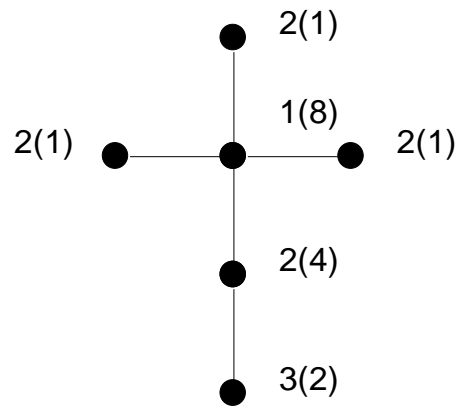
Figure 7: Proper Lucky fish graph

Theorem 5

A cross graph is lucky with $\eta(C) = 1$

Proof

Let $V(C) \rightarrow \{1\}$ be defined by

$$f(u) = 1$$

$$s(u_i) = \begin{cases} 1 & i = 1, 4, 5, 6 \\ 2 & i = 3 \\ 4 & i = 2 \end{cases}$$

Therefore, cross graph admits lucky with $\eta(C) = 1$ as shown in fig 8.
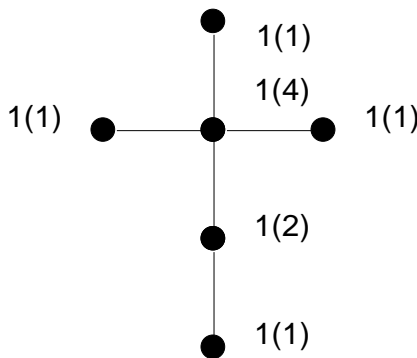

Figure 8: Lucky cross graph

Theorem 6

A cross graph is proper lucky with $\eta_p(C) = 3$.

Proof

Let $V(C) \rightarrow \{1,2,3\}$ be defined by

$$f(u_i) = \begin{cases} 1 & i = 2 \\ 2 & i = 1, 3, 5, 6 \\ 3 & i = 4 \end{cases}$$

$$s(u_i) = \begin{cases} 1 & i = 1, 5, 6 \\ 2 & i = 4 \\ 4 & i = 3 \\ 8 & i = 2 \end{cases}$$

Therefore, cross graph admits proper lucky with $\eta_p(C) = 3$ as shown in fig 9.

Figure 9: Proper lucky cross graph

Conclusion

We can conclude that star graph, fish graph and cross are lucky graph so we can call as lucky star graph, lucky fish graph and lucky cross graph. Its lucky number is as same as its minimum degree i.e., $\eta(s_n) = \delta(s_n), \eta(F) = \delta(F), \eta(C) = \delta(C)$. Also it is observed that fish graph and cross graph is proper lucky so we can call as proper lucky star graph, proper lucky fish graph and proper lucky cross graph. The fish graph and cross graph's proper lucky number is one less than its maximum degree i.e., $\eta_p(F) = \Delta(F) - 1, \eta_p(C) = \Delta(C) - 1$ and for star graph as same as its maximum degree respectively i.e., $\eta_p(s_n) = n$.

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