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Analysis of Heat and Mass Transfer Effects on MHD Stagnation Flow Along a Vertical Stretching Sheet

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Abstract -**This research investigates the problem of heat and mass transfer near a stagnation-point of an incompressible and viscous fluid flow past a linearly stretching vertical sheet with the effects of magnetohydrodynamics (MHD) and mass transfer of chemical reaction. The two-dimensional governing partial differential equations are converted into a system of nonlinear ordinary equations by using similarity transformation. The resulting transformed equations are solved numerically using bvp4c method. The validity of bvp4c is guaranteed by comparing numerical solutions with existing solutions. The effects of various physical parameter on the fluid flow of heat and mass transfer such as Prandtl number, magnetic parameter, buoyancy parameter, and Schmidt number are being discussed. It is found that local Nusselt number and local Sherwood number significantly increases when the magnetic parameter increases but decreases in local skin friction coefficient, which means that the magnetic parameter accelerates the fluid velocity. However, a contradict behavior happens in fluid temperature and fluid concentration, which both have decreased trend s in the boundary layer. The influence of Schmidt number can significantly enhance the Sherwood number which leads to increase the rate of mass transfer but decreases the heat transfer rate The analysis of the influence of various physical parameters is discussed.**

Index Terms –Heat and Mass Transfer, chemical reaction, MHD, vertical stretching sheet, stagnation flow, bvp4c method.

INTRODUCTION

In 1904, Ludwig Prandtl revealed a theoretical approach that could be used on viscous flows in issues of great practical importance [1]. Ludwig Prandtl is the prior of fluid mechanics introduced the boundary layer theory.

Heat transfer is the energy transfer that takes place between material bodies as a result of a temperature difference until they reach an equilibrium state. In general, there are three types of heat transfer which are conduction, convection, and thermal radiation. The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange will take place under certain specified conditions [2].

Mass transfer is the movement of a component from one phase to another due to concentration difference between the phases. While heat transfer has three different types of processes, mass transfer encompasses diffusion and convection currents set up due to flows, laminar or turbulent, temperature differences, and concentration differences, which are complementary to one another, across phase boundaries [3]. Mass transfer is important as it involves some of the most typical chemical engineering problems such as design and operation of chemical process equipment specifically carrying out chemical reactions, diffusion of chemical contamination in rivers and ocean from natural or artificial sources, preparation of reactants, and separation of chemicals in distillation procedure [4].

Magneto-fluid-dynamics, often known as hydro-magnetics, is a branch of physics that studies the dynamics of electricity conducting fluids such plasmas, liquid metals, and salt water. Magnetohydrodynamics (MHD) is a combination words of magneto (magnetic field), hydro (liquid), and dynamics (movement of particles) [5]. Magnetic fields are being used by many natural and forced flows in various process such as heating, pumping, stirring, and increasing liquid metals in the metallurgical industries [6]. Historically, the term of magnetohydrodynamics (MHD) has been used to describe the relative of fluid flow and magnetic fields, where the fluid must be an electrical conductor [7]. The magnetic field induced the current flows in dynamic fluid and creates force on the fluid. This force is called Lorentz force and it is used to control the flow of metals.

Stagnation point flow is a point in a flow field where the local velocity of the fluid is zero. Stagnation flow is about the fluid motion near the stagnation point. The fluid pressure, heat transfer, and the rate of mass are highest in the stagnation area [8]. There are various of real-world technical applications such as solar central receivers, emergency core cooling systems, blood flow problems, flow over tips of rockets, aircraft and submarines, glass production, and crude oil purification that stagnation point flow induced by a variable heated stretching surface [9].

MHD should be understood well as it had great connection and influences in real life physical mechanism. When a conductor moves into a magnetic field, electric current is induced in the conductor and creates its own magnetic field, which is called as Lenz's Law. Since the induced magnetic field tend to eradicate the original and external supported field, the lines of magnetic field will be excluded from the conductor. So, when magnetic field causes the conductor to move it out of the field, the induced field amplifies the applied field. As a result of this process, the force lines occur to be dragged accompanied by the conductor.

In the world of fluid mechanics, the conductor is the fluid with complex motions, and to understand its dynamical, there is a Lorentz force that acts on the fluid and changes its motion when currents are induced by a motion of a conducting through magnetic field that we need to know. In MHD, the motion modifies the field and vice versa. This makes the theory highly nonlinear [10]. Therefore, since MHD is nonlinear and can modify the motion, the involvement of MHD in boundary layer offers a unique means to have a greater control over the flow of the fluid as it involves the connection between fluid flow and magnetic fields [7].

Studies over the past three decades have provided important of combination heat and mass transfer problems in boundary layer flow across a quiescent liquid on a continuously moving surface have received significant interest [11]. This attention is due to various usage of chemical reaction effects in species diffusion such as take out the diffusing solute in the region that have been using in oil recovery, or in pure the water [12], fibrous insulation, oxidation, and synthesis materials [13].

In chemical processes, chemical reaction rates are dependent on the rate of mixing. Mixing the chemicals is a problem that is very popular in fluid mechanics. Improving rate of reaction can be directly understood by understanding the underlying fluid mechanics. Fluid mechanics is important in chemical engineering because most of the substances that are handled are in the form of a fluid, where it could be in gas, liquid or more complex to calculate, condensing steam. This is why that modelling chemical reactions along with flow is a very popular and challenging problem. As with heat transfer, the effectiveness of mass transfer between phases is also important of the fluid flow. MHD flow under different physical conditions has been investigated by [14-18].

Therefore, the purpose of the present study is to extend the work by Zaimi and Ishak [14] by including the MHD in the momentum equation and mass transfer effects to the stagnation-point flow towards a stretching vertical sheet. The magnetic field parameter is being added in the momentum equation and the concentration equation with molecular diffusivity of chemically species will be involved in the governing equations.

MATHEMATICAL FORMULATION

In this study we consider a steady two-dimensional stagnation point flow of a viscous and incompressible fluid towards a linearly stretching vertical sheet. The aim of the research is to examine the MHD effect of heat and mass transfer on stagnation point flow towards a stretching sheet.

The stagnation flow placed in the plane $y = 0$ and the flow is being limited to $y > 0$. A coordinate system is chosen in such a way that x-axis is in stretching direction, and y-axis is perpendicular to it. The u and v are the velocity components in the x and y directions. This study also takes the slip model that introduced by Navier in 1823 [15] written as $u = L\left(\partial u / \partial y\right)$, where $\partial u / \partial y$ is the fluid velocity on the surface of a solid body is proportional to the shear rate of on the surface. This study also considers the fluid temperature as T and fluid concentration as C. It is assumed that the temperature of the surface denoted as $T_w(x) = T_\infty + bx$ and concentration of the surface as $C_w(x) = C_\infty + bx$, where T_∞ and C_∞ is the ambient temperature and concentration, respectively. The governing equations are Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \textbf{(1)}$$

chemical contamination in rivers and ocean from natural or artificial sources, preparation of reactants, and separation of chemicals in distillation procedure

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+v\frac{\partial^2 u}{\partial y^2}+g\beta_T\left(T-T_\infty\right) \tag{2}$$

$$-\frac{\sigma B^2}{\rho}\left(u-U\right)+g\beta_C\left(C-C_\infty\right)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Concentration equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

To simplify these problems, similarity transformation is being used in such of the following from Zaimi and Ishak [14]

$$\eta=\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}y,\quad \psi=\left(\nu xU\right)^{\frac{1}{2}}f\left(\eta\right),$$
$$\theta\left(\eta\right)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi\left(\eta\right)=\frac{C-C_\infty}{C_w-C_\infty}, \tag{5}$$

where η is the independent similarity variable, f is the dimensionless stream function, θ is the dimensionless temperature, and ϕ is the dimensionless concentration with respect to η, ψ is the stream variable which u and v is defined as

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}, \tag{6}$$

with the boundary conditions

$$u=cx+L(\partial u/\partial y),\ v=0,\ T=T_w \text{ and } C=C_w \text{ at } y=0$$
$$u\rightarrow U(x)=ax,\ T\rightarrow T_\infty,\ C\rightarrow C_\infty \text{ as } y\rightarrow\infty, \tag{7}$$

By using the similarity transformation in (5), the nonlinear differential equations obtained are

$$f'''(\eta)+f(\eta)f''(\eta)+1-f'^2(\eta)+\lambda_t\theta(\eta)+\lambda_c\phi(\eta)$$
$$+M\left(1-f'(\eta)\right)=0, \tag{8}$$

$$\theta''(\eta)+\Pr\left(f(\eta)\theta'(\eta)-f'(\eta)\theta(\eta)\right)=0 \tag{9}$$

$$\phi''(\eta)+Sc\left(f(\eta)\phi'(\eta)-f'(\eta)\phi(\eta)\right)=0 \tag{10}$$

The new boundary conditions are

$$f'(0)=\varepsilon+\delta f''(0),\ f(0)=0,\ \theta(0)=1,$$
$$\text{and } \phi(0)\rightarrow 1, \text{ at } \eta=0$$
$$f'(\eta)\rightarrow 1,\ \theta(\eta)\rightarrow 0,\ \phi(\eta)\rightarrow 0 \text{ as } \eta\rightarrow\infty, \tag{11}$$

where $\lambda_t=g\beta_t b/a^2$ and $\lambda_c=g\beta_c b/a^2$ are the buoyancy parameters and concentration buoyancy parameter, respectively, $M=\sigma B^2/a\rho$ is the magnetic parameter, $Pr=v/\alpha$ is the Prandtl number, and $Sc=v/D$ is the Schmidt number.

In this study, the physical quantities involved are local skin friction coefficient, C_f, local Nusselt number, Nu_x, and local Sherwood number, Sh_x are written

$$C_f=\frac{\tau_w}{\rho U^2},\ Nu_x=\frac{xq_w}{k\left(T_w-T_\infty\right)},\ Sh_x=\frac{xj_w}{D\left(C_w-C_\infty\right)} \tag{12}$$

where τ_w denoted as the surface shear stress, q_w denoted as the surface of heat flux, and j_w denoted as the surface of mass flux, which are given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ \tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},\ j_w=-D\left(\frac{\partial C}{\partial y}\right)_{y=0} \tag{13}$$

Using (5), the local skin-friction coefficient, local Nusselt number, and local Sherwood number are

$$f''(0)=C_f x\sqrt{\frac{a}{\nu}},\ -\theta'(0)=\frac{Nu_x}{x\sqrt{\frac{a}{\nu}}},\ -\phi'(0)=\frac{Sh_x}{x\sqrt{\frac{a}{\nu}}} \tag{14}$$

where $\mathrm{Re}_x=Ux/\nu$ is the local Reynolds number.

RESULTS AND DISCUSSION

The governing equations (1) – (4) and boundary conditions (7) are reduced to nonlinear ordinary (similarity) differential equations (8) – (10) together with the new boundary conditions (11) and then are solved numerically by using bvp4c solver in MATLAB scheme. In MATLAB code, the η_∞ is set up η = 3, which it is sufficient to achieve the solutions for the considered physical parameters.

The numerical solutions have achieved the values of skin friction coefficient, rate of heat transfer and rate of species concentration on the surface, alongside the velocity, temperature, and concentration profiles are illustrated graphically which display the behavior of the fluid flow in the boundary layer. The impact of several physical parameters, namely magnetic parameter M, Schmidt number Sc, Prandtl number Pr, buoyancy parameter λ, velocity slip parameter δ, and stretching parameter ε is observed. For numerical results, $M = 1$, $Sc = 3$, $Pr = 1$, $\lambda = 1$, $\delta = 1$, and $\varepsilon = 3$ are kept constant except the varied parameters as shown in figures and tables.

Table 1 and Table 2 shows the comparison data of $f''(0)$ and $-\theta(0)$ with the previous studies of Ishak et al. [16], and Zaimi and Ishak [14]. The velocity, temperature, and concentration profile for different value of magnetic field parameter, M are shown in Figure 1, Figure 2, and Figure 3, respectively.

Figure 1 spots that with increasing the magnetic parameter into the problem will raise the velocity field. This fluid flow accelerates is due to the presence of a transverse magnetic field in a fluid flow that causes a drag force known as Lorentz. This Lorentz force improves the body force; hence it speeds up the movement of the fluid flow as well as increases the momentum boundary layer.

TABLE I

Comparison of the values of $f''(0)$with different values of Pr, by setting $\varepsilon = c/a = 1$, $\lambda_t = 1$, $\lambda_c = 1$, $\delta = 0$, $M = 0$, and $Sc = 0$.

Pr	Ishak et al. [16]	Zaimi and Ishak [14]	Present
0.72	0.3645	0.36449	0.36446418
6.80	0.1804	0.18041	0.18041526
10.00		0.15563	0.15563839
20.00	0.1175	0.11750	0.11750010
30.00		0.09889	0.09889336
40.00	0.0873	0.08724	0.08724247
50.00		0.07903	0.07903702
60.00	0.0729	0.07284	0.07284223
70.00		0.06794	0.06794448
80.00	0.0640	0.06394	0.06394241
90.00		0.06059	0.06059020
100.00	0.0578	0.05772	0.05772760

TABLE 2

Comparison of the values of $-\theta'(0)$ with different values of Pr, by setting $\varepsilon = c/a = 1$, $\lambda_t = 1$, $\lambda_c = 1$, $\delta = 0$, $M = 0$, and $Sc = 0$.

Pr	Ishak et al. [16]	Zaimi and Ishak [14]	Present
0.72	1.0931	1.09310	1.09314645
6.80	3.2902	3.28957	3.28957359
10.00		3.98240	3.98240060
20.00	5.6230	5.62013	5.62013140
30.00		6.87771	6.87771563
40.00	7.9463	7.93830	7.93830563
50.00		8.87292	8.87292137
60.00	9.7327	9.71801	9.71801323
70.00		10.49524	10.49524595
80.00	11.2413	11.21874	11.21874102
90.00		11.89831	11.89831038
100.00	12.5726	12.54109	12.54109988

Figure 2 and Figure 3 display the temperature and concentration profiles of the behavior of the magnetic parameter. Figure 2 shows a decreasing on the dimensionless temperature profile. Physically, applying the magnetic field causes a decrease in the thermal boundary layer thickness and consequently cool down the fluid temperature. Figure 3 depicts decreases on the concentration profile as increasing the magnetic parameter. This implies that the magnetic force acts to reduce the diffusion of the solute in the boundary layer.

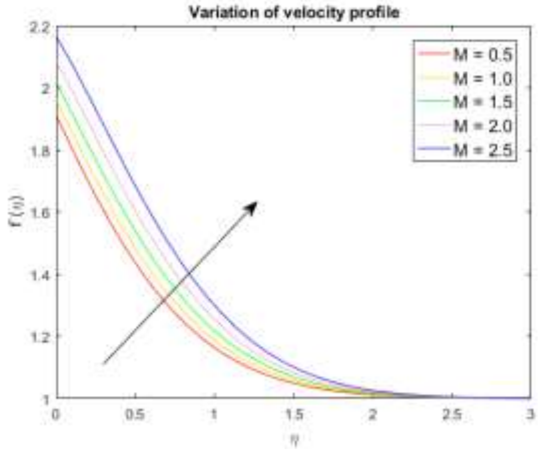

FIGURE 1

Velocity Profile of the Effects of Magnetic Parameter, M when $Sc = 3$, $Pr = 1$, $\lambda_T = 1$, $\lambda_C = 1$, $\delta = 1$, and $\varepsilon = 3$.

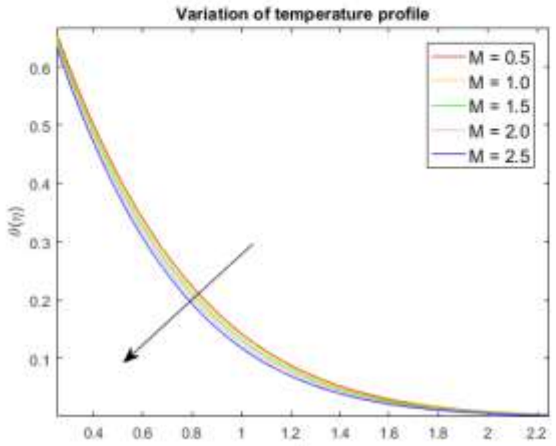

FIGURE 2

Temperature Profile of the Effects of Magnetic Parameter, M when $Sc = 3$, $Pr = 1$, $\lambda_T = 1$, $\lambda_C = 1$, $\delta = 1$, and $e = 3$.

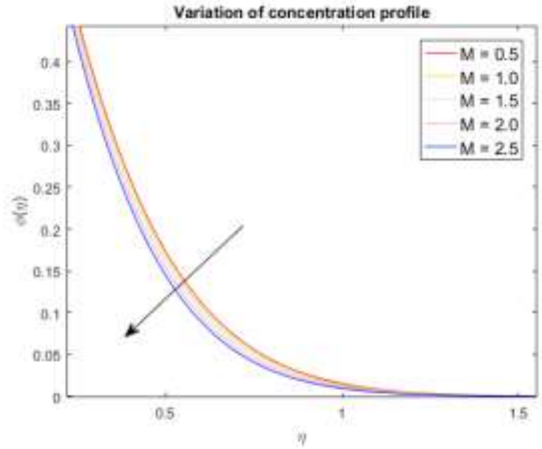

FIGURE 3

Concentration Profile of the Effects of Magnetic Parameter, M when $Sc = 3$, $Pr = 1$, $\lambda_T = 1$, $\lambda_C = 1$, $\delta = 1$, and $e = 3$.

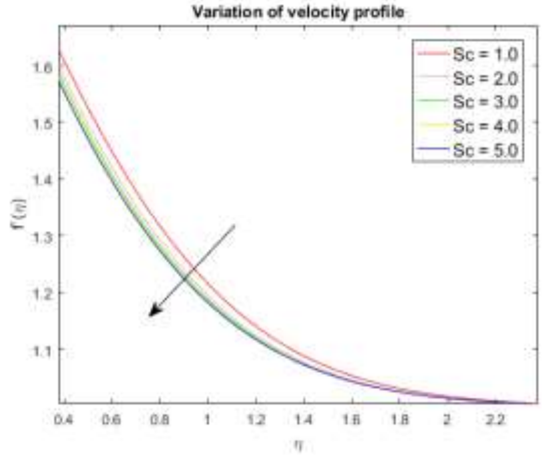

FIGURE 4

Velocity Profile of the Effects of Schmidt Number, Sc when $M = 1$, $Pr = 1$, $\lambda_T = 1$, $\lambda_C = 1$, $\delta = 1$, and $\varepsilon = 3$.

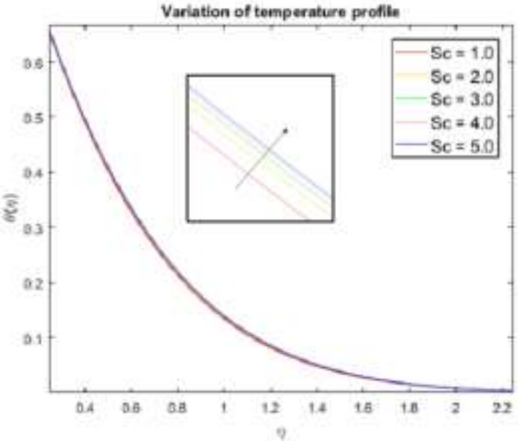

FIGURE 5
Temperature Profile of the Effects of Schmidt Number, *Sc* when M = 1, Pr = 1, λ_T = 1, λ_C = 1, δ = 1, and ε = 3.

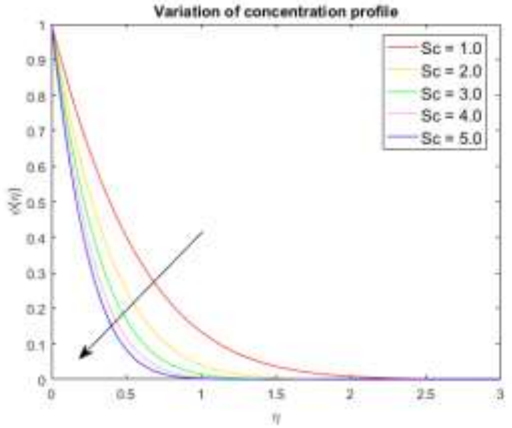

FIGURE 6
Concentration Profile of the Effects of Schmidt Number, *Sc* when M = 1, Pr = 1, λ_T = 1, λ_C = 1, δ = 1, and ε = 3.

The effects on the Schmidt number, *Sc* on the velocity, temperature, and concentration profiles are presented in Figure 4, Figure 5, and Figure 6, respectively.

Figure 4 shows behaviour of the fluid velocity decelerates as increasing the *Sc*. The effect of increasing the Schmidt number is to reduce the momentum boundary layer, hence this leads to thinning of the diffusion layer.

Figure 5 demonstrates the temperature distribution with increasing the Schmidt number. From this curve, it shows that fluid temperature slightly increases as rising the values of *Sc*.

The impact of the Schmidt number on the species concentration is displayed in Figure 6 shows the greater value of *Sc* leads to the decrease the concentration of boundary layer region. It is true due to the fact that the molecular diffusivity decreases when rising the Schmidt number. Hence, use smaller value of *Sc* for higher species concentration and use larger values of *Sc* for lower the species concentration.

CONCLUSION

In general, the magnetic parameter is related with the Lorentz force that caused by the electromagnetism of the magnetic field. As there is a connection of the fluid flow and magnetic field, the involvement of the MHD is being considered to control the flow of the fluid. Besides, the influence of the mass transfer by molecular diffusivity caused the chemical potential that have tendency to chemically react which involved species concentration gradient.

Therefore, this present research is the modification from Zaimi and Ishak [14] paper, by adding the term of magnetohydrodynamics (MHD) that signified as magnetic parameter into the momentum equations and considering the effects of mass transfer into the boundary value problems.

The analysis of the influence of various physical parameters such as magnetic parameter M, Schmidt number Sc, Prandtl number Pr, buoyancy parameter λ_t, concentration buoyancy parameter λ_c, velocity slip parameter δ, and stretching parameter ε on the momentum, heat and mass transfer characteristics is acknowledged.

The findings from this study are summarized as: An increasing in M, increases the fluid velocity as well as drops the fluid temperature and concentration of the fluid towards the boundary layer region. It implies thickening the velocity boundary layer thickness and thinning the thermal and concentration boundary layer thickness. The increase in Sc decelerates the fluid flow and the concentration, while enhances the temperature distribution. These phenomena leads to decrease the velocity and concentration boundary layer thicknesses but increases the thermal boundary layer thickness. The increment of *Pr* significantly increase the rate of heat transfer on the surface because of when fluid with a higher Prandtl number has a lower thermal diffusivity.

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