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Inventory model with type-2 fuzzy demand and fuzzy holding cost for faulty products

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Abstract: *An EOQ degrading model with stock-dependent demand is built in this paper. This approach does not allow for shortages. The model was created for both crisp and fuzzy models, where the discount grows as the number of flaws in the lots for defective quality products increases. It is believed to be a fuzzy variable since demand and holding cost cannot be forecast exactly because they are dependent on several unpredictable and perturbing market actions. This study developed a defuzzification method for type-2 fuzzy variables related to interval approximation in this regard. The outcomes have been mathematically and visually confirmed. The crisp model is validated for optimality by a sensitivity analysis.*

Keywords: *deterioration, EOQ, stock dependent demand, type-2 fuzzy number, Interval approximation*

1. Introduction

Today, all businesses have a significant issue in terms of inventory management, administration, and control. In general, in company, appropriate inventory management meets consumer demand and market behaviour. Customer demand, which is one of the most essential market behaviours, is strongly tied to the effective and efficient utilisation of inventory. Assuming various types of consumer demand, such as constant and variable, as well as diverse inventory management situations, numerous models have been established and various strategies have been used to optimise profit and decrease cost. In the competition market, we constructed a mathematical model based on several business-like scenarios, such as demand reliant on stock, continual degradation, and proportionate discount. This sort of model is commonly used in industries such as jewellery, food, clothing, garments, medicine, cosmetics,handbags and so on. Deterioration of the product, whether in the form of decay or expiration, is a significant topic that we cannot overlook in this study. In the inventory system, deterioration has a large influence. We can utilise proportional discounts for faulty products in the proposed approach.

"As the complexity of the system grows, our capacity to make accurate and yet meaningful claims about its behaviour diminishes until a barrier is crossed beyond which accuracy and importance (or relevance) become practically mutually incompatible features," wrote L. Zadeh (1973). The characteristics feature and true scenario in the real world are frequently ambiguous. Due to a lack of knowledge, the future condition of the systems is absolutely uncertain. This ill-known facts and lack of knowledge result in an inaccurate border of the demand, and it may be represented as a fuzzy variable. There has been a lot of discussion on fuzzy inventory systems that take into account fuzzy and fuzzy random requests Shekarian (2017). However, it is not always viable to define the demand quantity using a set membership grade. Assume that ten specialists have been tasked with forecasting the demand for motor components in the future year. Due to the aforementioned causes, each expert opinion is hazy, and it is provided as a hazy set with a membership grade. Consider that three experts believe the grade of membership of 100 units is 0.3, four experts believe it is 0.4, two experts believe it is 0.2, and two experts believe it is 0.2. Here uncertainty in grade of membership is also inherited. This ambiguity can be described using a fuzzy set in the domain [0, 1]. As a result, ambiguity in demand as well as membership grade has been noted. The use of type-2 fuzzy variables can be used to simulate this sort of demand. The goal of this research is to create an inventory model that takes into account type-2 fuzzy demand and type-2 fuzzy holding cost. A defuzzification approach for type-2 fuzzy variables based on interval approximation has been presented in this procedure. These uncertainties are handled properly utilising fuzzy theory. For example, in the start of the inventory system, demand rate uncertainty is quite high because customers take longer to complete the procedure. Customer hesitancy makes it less likely to complete the process in a timely manner, and it has a direct impact on demand and customer satisfaction. Customer satisfaction, on the other hand, was determined by the cycle time. When the length of the cycle is long, the uncertainty is large, and vice versa. Uncertainty has been removed, allowing the system to accomplish dependability and acceptance, as well as significant customer achievement, optimise profit, and reduce system costs.

2. Literature survey

Harris (1913) was the first to establish the EOQ model, which assumed a constant rate of demand. The model was then expanded by Resh et al. (1976), who took into account linearly growing demand. Furthermore, Kim et al. (1995) provided a model that computed optimum profit, optimal order size, and unit wise retail pricing based on a price-dependent demand pattern. Giri et al. (1996) created a model for degrading products that took into account stock-dependent demand. Following that, Mandal and Maiti (1997) used the profit maximisation concept to estimate the optimal order amount and built a shortage model for damageable commodities based on stock dependent demand. Under the stock-based selling rate Chang (2006) developed a partially backlogged EOQ model for perishable products that is time sensitive. Panda (2010) proposed an EOQ model with stock-dependent demand and the assumption that each lot received has a known percentage of faulty products. Khan and Jaggi (2011) highlighted and examined the implications of defective quality products in lot sizing policy. Khanna(2017) established a shortage and totally backlogged model using tread-credit programmes with price-dependent demand. Indrajitsingha(2018) created a fuzzy EOQ model for degrading products, with the first interval demand depending on stock and the second interval demand assumed to be constant. Karmakar(2018) investigated a basic EOQ model under a foggy fuzzy demand rate. For the cultivation of things of mediocre quality Economic order quantity model was established by Sebatjane (2019). Pervin(2019) developed a multi-item degrading two-echelon inventory model with price- and stock-dependent demand based on a two-level supply chain strategy. Shaikh(2019) created a partially backlogged model that takes into account stock-dependent demand patterns and a price discount facility for degrading commodities. De (2019) proposed a model using proportional discount for faulty products and a clouded demand rate. Our study's key dimension is depicted in the comparison table.

Table-1

Observation about Published work and present work

Author	**Types of demand**	**Deteri-oration**	**Imperfect Items**	**Fuzzy**
Harris(1915)	Constant	no	no	no
Resh(1976)	Linearly increasing	no	no	no
Kim(1995)	Price	no	no	no
Giri(1996)	Stock	yes	no	no
Chang,Goyal & Teng (2006)	Stock	no	no	no
Panda(2010)	Stock	no	yes	no
Khan,Jaber & Bonney, (2011)	Constant	no	yes	no
Jaggi & Mittal(2011)	Constant	yes	yes	no
Sarkar & Sarkar (2013)	Stock	yes	no	no
Khanna,Gautam & Jaggi (2017)	Price	yes	yes	no
Indrajitsingha, Samanta & Misra (2018)	Stock	yes	no	yes
Karmakar,De& Goswami(2018)	constant	no	no	yes
Ravi Shankar kumar(2018)	Constant	no	yes	yes
Sebatjane & Adetunji (2019)	constant	no	yes	no
Pervin, Roy & Weber (2019)	Stock	yes	no	no
Shaikh,Khan,Panda,& Konstantaras, I.(2019)	Stock	yes	no	no
De & Mahata (2019).	Constant	no	yes	yes
Handa, N (2020)	Stock	yes	no	no
Rahaman, M(2021)	Price	no	no	no
This paper	Stock	yes	yes	yes

3. Research gap and our contribution

We constructed our current model based on Ravi Shankar Kumar's (2018) concept. The influence of deterioration and the fluctuating demand of the inventory condition were not taken into account in their model. We adjusted Ravi Shankar Kumar's (2018) model based on the following assumption:

(i) Assuming constant deteriorating effect of the inventory system.

(ii) Assuming stock-dependent demand pattern.

4. Preliminaries

Following definitions and rules are used to developed type-2 fuzzy model.

IT2TrFS ("Interval type -2 trapezoidal fuzzy set")

Let $\tilde{D}$, defined as $\tilde{D} = \left[\tilde{D}^L, \tilde{D}^U\right] = \left[\left(d_{1,}^L, d_2^L, d_3^L, d_4^L; h_D^L\right), \left(d_{1,}^U, d_2^U, d_3^U, d_4^U; h_D^U\right)\right]$,where $d_i^L \le d_{i+1}^L, d_i^U \le d_{i+1}^U, i = 1, 2, 3, d_1^U \le d_1^L$, $d_4^L \le d_4^U$ and $0 < h_D^L \le h_D^U \le 1$, be the IT2TrF number $\tilde{D}^L$ and $\tilde{D}^U$ are defined by lower membership function $\mu_{\tilde{D}}^L$ and upper membership function $\mu_{\tilde{D}}^U$ as follows:

$$\mu_{\tilde{D}}^L = \begin{cases} h_D^L\left(\dfrac{x - d_{1,}^L}{d_2^L - d_{1,}^L}\right), & d_{1,}^L \le x \le d_2^L \\ h_D^L, & d_{2,}^L \le x \le d_3^L \\ h_D^L\left(\dfrac{d_{4,}^L - x}{d_4^L - d_{3,}^L}\right), & d_{3,}^L \le x \le d_4^L \\ 0, & Otherwise \end{cases}$$

And

$$\mu_{\tilde{D}}^{U}=\begin{cases} h_D^U\left(\dfrac{x-d_{1,}^U}{d_2^U-d_{1,}^U}\right), & d_{1,}^U\le x\le d_2^U\\ h_D^U, & d_{2,}^U\le x\le d_3^U\\ h_D^U\left(\dfrac{d_{4,}^U-x}{d_4^U-d_{3,}^U}\right), & d_{3,}^U\le x\le d_4^U\\ 0, & Otherwise\end{cases}$$

Alpha cut of IT2TrFS

If

$$\tilde{\tilde{D}}=\left[\tilde{D}^L,\tilde{D}^U\right]=\left[\left(d_{1,}^L,d_2^L,d_3^L,d_4^L;h_D^L\right),\left(d_{1,}^U,d_2^U,d_3^U,d_4^U;h_D^U\right)\right]$$

is an IT2TrFS, then alpha cut of $\tilde{\tilde{D}}$, where $0\le\alpha\le1$, is defined as follows:

$$D_\alpha=\left[D_\alpha^L,D_\alpha^U\right]$$

$$=\begin{cases}\left[\left[{}^lD_\alpha^L,{}^rD_\alpha^L\right],\left[{}^lD_\alpha^U,{}^rD_\alpha^U\right]\right], 0\le\alpha\le h_D^l\\ \left[{}^lD_\alpha^U,{}^rD_\alpha^U\right], & h_D^l\le\alpha\le h_D^U\end{cases}$$

Where ${}^lD_\alpha^L=d_1^L+\left(d_2^L-d_1^L\right)\alpha/h_D^L$,

${}^rD_\alpha^L=d_4^L-\left(d_4^L-d_3^L\right)\alpha/h_D^L$,

${}^lD_\alpha^U=d_1^U+\left(d_2^U-d_1^U\right)\alpha/h_D^U$,and

${}^rD_\alpha^U=d_4^U-\left(d_4^U-d_3^U\right)\alpha/h_D^U$.

Interval approximation of IT2TrFS

Definition The interval approximation of **IT2TrFS** $\tilde{\tilde{D}}$ is defined as follows:

$$\left[D_\alpha^-,D_\alpha^+\right]=\left[\frac{1}{2}\int_0^1\left({}^lD_\alpha^L+{}^iD_\alpha^U\right)d\alpha,\frac{1}{2}\int_0^1\left({}^rD_\alpha^L+{}^rD_\alpha^U\right)d\alpha\right]$$

$$\left[\frac{1}{4}\left[\left(d_1^L+d_2^L\right)h_A^L+\left(d_1^U+d_2^U\right)h_D^U\right],\left[\frac{1}{4}\left[\left(d_3^L+d_4^L\right)h_D^L+\left(d_3^U+d_4^U\right)h_D^U\right]\right]\right]$$

Definition Mean (de-fuzzified to a scalar) value of the interval approximation of IT2TrFS $\tilde{\tilde{D}}$ is defined as follow:

$$M\left(\tilde{\tilde{D}}\right)=\frac{D_\alpha^-+D_\alpha^+}{2}=\frac{1}{4}\left[\left(d_1^L+d_2^L+d_3^L+d_4^L\right)h_D^L+\left(d_1^U+d_2^U+d_3^U+d_4^U\right)h_D^U\right]$$

If we take

$d_1^L=d_2^L=d_3^L=d_4^L=d_1^U=d_2^U=d_3^U=d_4^U=d$, then

$M\left(\tilde{\tilde{D}}\right)=\dfrac{d(h_D^L+h_D^U)}{2}$. Again if we take $h_D^L=h_D^U=1$, then $M\left(\tilde{\tilde{D}}\right)=d$, i.e., if no fuzziness occurs, the defuzzified value of IT2TrFS correlates to the crisp number.

5. Notations and Assumptions

5.1 Notations

The following notations are used

Q_S : Order size for each cycle.

C_V : Variable cost.

K_C : Fixed ordering cost.

H_C : Cost associated holding of items

P_D : The defective % in Q_S .

S_P : Unit wise selling price for good quality items.

S_R : Rate of Screening

S_C : One unit items screening cost.

T : Cycle time.

C_{TR} : Total revenue

C_{TC} : Total cost.

C_{TP} : Total profit cycle wise.

U_{TP} : Total profit unit wise

5.2 Assumptions

The following assumptions are made in developing the models.

1. Stock dependent demand is denoted by $D_R=a+bI(t)$, where $a>0$ be the initial demand, $0\le b\le1$ is stock sensitivity parameter and $I(t)$ is the instantaneous level of inventory at time t.
2. The constant rate of deterioration is θ , where $0<\theta<<1$.
3. Instantaneous delivery and no shortage.
4. Fixed selling price for non-defective items.
5. Rate of Screening is greater than rate of demand.
6. Each size lot comprises a small number of faulty products, which are sold in a single batch at a proportional discount.
7. During screening period t, perfect quality of items $\left(1-P_D\right)Q_S$ must be greater than or equal to D_Rt , that is , $1-P_D\ge\dfrac{D_R}{S_R}$.
8. The time horizon of the inventory system is infinite.

6. Mathematical models

Crisp model

Based on the above assumption the crisp model is starts with initial lot size Q_S unit at time t=0 with defective and deteriorating item. After 100% screening all the items the defective and good quality items separated. The defective items collect a single batch and sold as proportionate discounted price but good quality item sold as fixed selling price. Behaviour of the inventory level as discussed as ,where cycle length T, screening time t and the number of defective and good quality items drawn from the inventory are $P_D Q_S$ and $(1-P_D)Q_S$. The cycle starts with the initial lot size Q_S at time t=0, due to the combined effect of deterioration and customer demand the inventory level decreases during the time [0, t_1], at time t = t_1 the inventory level $I(t_1)$ becomes $(1-P_D)Q_S - D_R t_1$ and time t=T inventory level becomes zero. To avoid shortage within screening time t_1, the defective percentage is restricted $P_D \leq 1-\frac{D_R}{S_R}$, where $t_1 = \frac{Q_S}{S_R}$. Instantaneous inventory level over the period [0, T] as described by following differential equations:

$$\frac{dI(t)}{dt} + \theta I(t) = -D_R \quad , \quad 0 \leq t \leq t_1 \tag{6.1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D_R \quad , \quad t_1 \leq t \leq T \tag{6.2}$$

where $0 < \theta << 1$ and $D_R = a + bI(t)$.

Using boundary condition t=0, $I(t) = Q_S$, t=t_1, $I(t_1) = (1-P_D)Q_S - D_R t_1$ and t=T $I(t) = 0$ in equ.(6.1) and equ.(6.2) get as follows:

$$I(t) = Q_S e^{-(\theta+b)t} + \frac{a}{\theta+b}\left(e^{-(\theta+b)t} - 1\right), \quad 0 \leq t \leq t_1 \tag{6.3}$$

$$I(t) = ((1-P_D)Q_S - D_R t_1)e^{(\theta+b)(t_1-t)} + \frac{a}{\theta+b}\left(e^{-(\theta+b)t} - 1\right), \quad t_1 \leq t \leq T \tag{6.4}$$

And
$$T = t_1 - \frac{1}{\theta+b} ln\left(\frac{\frac{a}{\theta+b}}{(1-P_D)Q_S - D_R t_1 + \frac{a}{\theta+b}}\right) \tag{6.5}$$

The cycle wise total cost is:

C_{TC} = Ordering cost + variable cost +Screening cost+ holding cost

$$= K_C + C_V Q_S + S_C Q_S + H_C\left[\frac{Q_S^2(1-P_D)}{D_R}\right] \tag{6.6}$$

Total revenue during time period (0, T):

C_{TR} =sum of total sale with good quality and imperfect quality items

$$\frac{2S_P(1-P_D)Q_S + (Q_S P_D + 1)\begin{bmatrix} K_C + C_V Q_S + S_C Q_S + \\ H_C\left\{\frac{(Q_S^2(1-P_D))}{D_R}\right\}\end{bmatrix}}{2Q_S + Q_S P_D + 1} \tag{6.7}$$

The cycle wise total profit:

$$C_{TP}(Q_S) = C_{TR}(Q_S) - C_{TC}(Q_S)$$

$$\frac{2S_P Q_S^2 - 2Q_S\left[K_C + C_V Q_S + S_C Q_S + H_C\left(\frac{Q_S^2(1-P_D)}{D_R}\right)\right]}{2Q_S + Q_S P_D + 1} \tag{6.8}$$

The Unit wise total profit :

$$U_{TP}(Q_S) = \frac{C_{TP}(Q_S)}{T} = \frac{2D_R(S_P Q_S - K_C - C_V Q_S - S_C Q_S)}{(1-P_D)(2Q_S + Q_S P_D + 1)} - \frac{2H_C Q_S^2}{(2Q_S + Q_S P_D + 1)} \tag{6.9}$$

Differentiate U_{TP} with respect to Q_S two times are given as follows:

$$\frac{dU_{TP}(Q_S)}{dQ_S} = \frac{1}{(2Q_S + Q_S P_D + 1)^2}\left[\frac{2D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{(1-P_D)} - \{2H_C Q_S(2 + 2Q_S + Q_S P_D)\}\right] \tag{6.10}$$

and $\frac{d^2U_{TP}(Q_S)}{dQ_S^2} < 0$

The negative value of $\frac{d^2U_{TP}(Q_S)}{dQ_S^2}$ shows profit function is concave in nature and setting $\frac{dU_{TP}(Q_S)}{dQ_S} = 0$ and solving we get optimal order size that represent the maximum annual profit. After some basic manipulation we get

$$(Q_S)_{max} = \sqrt{\frac{D_R(S_P - C_V - S_C + 2K_C + K_C P_D)}{H_C(2+P_D)(1-P_D)}} \tag{6.11}$$

When $P_D = 0$, $S_P - C_V - S_C = 2K_C$ then $(Q_S)_{max}$ reduce to the traditional EOQ formula.

$$(Q_S)_{max} = \sqrt{\frac{2K_C D_R}{H_C}} \tag{6.12}$$

Fuzzy Model

As stated in the beginning, demand forecasting is based on a variety of sources/activities that are unexpected, troublesome, and unreliable. As a consequence, inherent ambiguity and inaccuracy will not dictate an appropriate interpretation of demand. We may effectively represent it in terms of a fuzzy variable. Furthermore, if there are numerous experts and each expert's view is a fuzzy set, it changes from expert to expert. In this case, the degree of membership of a particular demand value becomes a fuzzy interval set [0, 1]. That is, there might be fuzziness in the degree of membership meaning itself. Assume that stock-dependent demand and holding cost are IT2TrF numbers, with the following membership grades:

Now let us consider the **IT2TrF** number $\tilde{\tilde{D}}_R$ and $\tilde{\tilde{H}}_C$ are given as :

$\tilde{D}_R = \left[\tilde{D}_R^L, \tilde{D}_R^U\right] = \left[\left(D_{R1}^L, D_{R2}^L, D_{R3}^L, D_{R4}^L; h_D^L\right), \left(D_{R1}^U, D_{R2}^U, D_{R3}^U, D_{R4}^U; h_D^U\right)\right]$

,

where $D_{Ri}^L \le D_{R(i+1)}^L$ $D_{Ri}^U \le D_{R(i+1)}^U$ $.i = 1,2,3$, $D_{R1}^U \le D_{R1}^L$, $D_{R4}^L \le D_{R4}^U$ and $0 \langle h_{D_R}^L \le h_{D_R}^U \le 1$.

$\tilde{H}_C = \left[\tilde{H}_C^L, \tilde{H}_C^U\right] = \left[\left(H_{C1}^L, H_{C2}^L, H_{C3}^L, H_{C4}^L; h_H^L\right), \left(H_{C1}^U, H_{C2}^U, H_{C3}^U, H_{C4}^U; h_H^U\right)\right]$

,

where $H_{Ci}^L \le H_{C(i+1)}^L$ $H_{Ci}^U \le H_{C(i+1)}^U$ $.i = 1,2,3$, $H_{C1}^U \le H_{C1}^L$, $H_{C4}^L \le H_{C4}^U$ and $0 \langle h_{H_C}^L \le h_{H_C}^U \le 1$.

$\tilde{D}_R^L$ and $\tilde{D}_R^U$ are defined by lower membership function $\mu_{\tilde{D}_R}^L$ and upper membership function $\mu_{\tilde{D}_R}^U$ as follows:

$$\mu_{\tilde{D}_R}^L = \begin{cases} h_{D_R}^L\left(\dfrac{x - D_{R1}^L}{D_{R2}^L - D_{R1}^L}\right), & D_{R1}^L \le x \le D_{R2}^L \\ h_{D_R}^L & , D_{R2}^L \le x \le D_{R3}^L \\ h_{D_R}^L\left(\dfrac{D_{R4}^L - x}{D_{R4}^L - D_{R3}^L}\right), & D_{R3}^L \le x \le D_{R4}^L \\ 0 & .\ otherwise \end{cases}$$

And

$$\mu_{\tilde{D}_R}^U = \begin{cases} h_{D_R}^U\left(\dfrac{x - D_{R1}^U}{D_{R2}^U - D_{R1}^U}\right), & D_{R1}^U \le x \le D_{R2}^U \\ h_{D_R}^U & , D_{R2}^U \le x \le D_{R3}^U \\ h_{D_R}^U\left(\dfrac{D_{R4}^U - x}{D_{R4}^U - D_{R3}^U}\right), & D_{R3}^U \le x \le D_{R4}^U \\ 0 & .\ otherwise \end{cases}$$

$\tilde{H}_C^L$ and $\tilde{H}_C^U$ are defined by lower membership function $\mu_{\tilde{H}_C}^L$ and upper membership function $\mu_{\tilde{H}_C}^U$ as follows:

$$\mu_{\tilde{H}_C}^L = \begin{cases} h_{H_C}^L\left(\dfrac{x - H_{C1}^L}{H_{C2}^L - H_{C1}^L}\right), & H_{C1}^L \le x \le H_{C2}^L \\ h_{H_C}^L & , H_{C2}^L \le x \le H_{C3}^L \\ h_{H_C}^L\left(\dfrac{H_{C4}^L - x}{H_{C4}^L - H_{C3}^L}\right), & H_{C3}^L \le x \le H_{C4}^L \\ 0 & .\ otherwise \end{cases}$$

And

$$\mu_{\tilde{H}_C}^U = \begin{cases} h_{H_C}^U\left(\dfrac{x - H_{C1}^U}{H_{C2}^U - H_{C1}^U}\right), & H_{C1}^U \le x \le H_{C2}^U \\ h_{H_C}^U & , H_{C2}^U \le x \le H_{C3}^U \\ h_{H_C}^U\left(\dfrac{H_{C4}^U - x}{H_{C4}^L - H_{C3}^L}\right), & H_{C3}^U \le x \le H_{C4}^U \\ 0 & .\ otherwise \end{cases}$$

Now, the fuzzy total profit is as follows:

$$Max\tilde{U}_{TP} = \left(\frac{AQ_S}{BQ_S + 1} - \frac{2K_C A'}{BQ_S + 1}\right)\tilde{D}_R - \frac{2\tilde{H}_C Q_S^2}{BQ_S + 1}$$

(6.13)

where, $A = \dfrac{2S_P - 2C_V - 2S_C}{1 - P_D}$, $B = 2 + P_D$, $A' = \dfrac{1}{1 - P_D}$

Subject to $\tilde{Q}_S = \dfrac{\tilde{D}_R T}{1 - P_D}$

Arithmetic operations defined by Chen(2011) are used here to calculate the fuzzy total profit per unit time.Hence the IT2TrFS fuzzy objective function $\tilde{U}_{TP}$ can be expressed as

$$\tilde{U}_{TP} = \left[U_{TP_i}^L, U_{TP_i}^U\right] = \left[\left(U_{TP_1}^L, U_{TP_2}^L, U_{TP_3}^L, U_{TP_4}^L; h_{D_R}^L\right), \left(U_{TP_1}^U, U_{TP_2}^U, U_{TP_3}^U, U_{TP_4}^U; h_{D_R}^U\right)\right]$$

(6.14)

Where

$$U_{TPi}^L = \left(\frac{AQ_S}{BQ_S + 1} - \frac{2K_C A'}{BQ_S + 1}\right)D_{Ri}^L - \frac{2H_{C(5-i)}^L \left(Q_S\right)^2}{BQ_S + 1}, i = 1,2,3,4$$

(6.15)

And

$$U_{TPi}^U = \left(\frac{AQ_S}{BQ_S + 1} - \frac{2K_C A'}{BQ_S + 1}\right)D_{Ri}^U - \frac{2H_{C(5-i)}^U \left(Q_S\right)^2}{BQ_S + 1}, i = 1,2,3,4$$

(6.16)

The de-fuzzified by mean value of the interval approximation of IT2TrFS profit function $\tilde{U}_{TP}$ given as and order quantities are respectively given by

$$\begin{cases} M\left(\tilde{U}_{TP\left(h_{D_R}^L, h_{D_R}^U\right)}\right) = \frac{1}{8}\left(\left(D_{R1}^L + D_{R2}^L + D_{R3}^L + D_{R4}^L\right)h_{D_R}^L + \left(D_{R1}^U + D_{R2}^U + D_{R3}^U + D_{R4}^U\right)h_{D_R}^U\right)\left(\frac{AQ_S}{BQ_S + 1} - \frac{2K_C A'}{BQ_S + 1}\right) \\ -\frac{1}{8}\left(\left(H_{C4}^L + H_{C3}^L + H_{C2}^L + H_{C1}^L\right)h_{H_C}^L + \left(H_{C4}^U + H_{C3}^U + H_{C2}^U + H_{C1}^U\right)h_{H_C}^U\right)\left(\frac{2Q_S^2}{BQ_S + 1}\right) \\ M\left(\tilde{Q}_S\right) = \frac{1}{8}\left(\left(D_{R1}^L + D_{R2}^L + D_{R3}^L + D_{R4}^L\right)h_{D_R}^L + \left(D_{R1}^U + D_{R2}^U + D_{R3}^U + D_{R4}^U\right)h_{D_R}^U\right)\frac{T}{1 - P_D} \end{cases}$$

(6.17)

$$\begin{cases} M\left(\tilde{U}_{TP\left(h_{D_R}^L, h_{D_R}^U\right)}\right) = \left(\left(D_R^L\right)m + \left(D_R^U\right)n\right)\left(\frac{AQ_S}{BQ_S + 1} - \frac{2K_C A'}{BQ_S + 1}\right) - \left(\left(H_C^L\right)m + \left(H_C^U\right)n\right)\frac{2Q_S^2}{BQ_S + 1} \\ M\left(\tilde{Q}_S\right) = \left(\left(D_R^L\right)m + \left(D_R^U\right)n\right)\left(\frac{T}{1 - P_D}\right) \end{cases}$$

(6.18)

Where $D_R^L = \frac{1}{8}\left(D_{R1}^L + D_{R2}^L + D_{R3}^L + D_{R4}^L\right)$,

$$D_R^U = \frac{1}{8}\left(D_{R1}^U + D_{R2}^U + D_{R3}^U + D_{R4}^U\right)$$

$$H_C^L = \frac{1}{8}\left(H_{C4}^L + H_{C3}^L + H_{C2}^L + H_{R1}^L\right) ,$$

$$H_R^U = \frac{1}{8}\left(H_{C4}^U + H_{C3}^U + H_{C2}^U + H_{C1}^U\right)$$

$$h_D^L = m \;\; and \;\; h_D^U = m$$

Theorem-1

$M(\tilde{\tilde{U}}_{TP(m,n)})$ is a concave function of Q_S that represents the unit wise total profit. In comparison, the optimum strategy is

$$(Q_S)_{\max}^* = \sqrt{\frac{\left(\left(D_R^L\right)m + \left(D_R^U\right)n\right)\left(S_P\text{-}C_V\text{-}S_C + 2K_C + K_C P_D\right)}{\left(\left(H_C^L\right)m + \left(H_C^U\right)n\right)\left(2 + P_D\right)\left(1\text{-}P_D\right)}}$$

.

Proof: $M(\tilde{\tilde{U}}_{TP(m,n)})$ w.r.t Q_S are has a 1st and 2nd derivatives are

$$\frac{dM\tilde{\tilde{U}}_{TP}(Q_S)}{dQ_S} = \frac{1}{(2Q_s + Q_s P_D + 1)^2}\left[\begin{array}{c}\frac{2\left(\left(D_R^L\right)m + \left(D_R^U\right)n\right)\left(S_P\text{-}C_V\text{-}S_C + 2K_C + K_C P_D\right)}{(1\text{-}P_D)} \\ \text{-}\left\{2\left(\left(H_C^L\right)m + \left(H_C^U\right)n\right)Q_s\left(2 + 2Q_s + Q_s P_D\right)\right\}\end{array}\right]$$

$$\frac{d^2 M(\tilde{\tilde{U}}_{TP(m,n)})}{dQ^2} < 0$$

Now optimal order quantity can be obtained from the relation $\frac{dM\tilde{\tilde{U}}_{TP}(Q_S)}{dQ_s} = 0$ which implies

$$(Q_S)_{\max}^* = \sqrt{\frac{\left(\left(D_R^L\right)m + \left(D_R^U\right)n\right)\left(S_P\text{-}C_V\text{-}S_C + 2K_C + K_C P_D\right)}{\left(\left(H_C^L\right)m + \left(H_C^U\right)n\right)\left(2 + P_D\right)\left(1\text{-}P_D\right)}}$$

And $\frac{d^2 M(\tilde{\tilde{U}}_{TP(m,n)})}{dQ^2} < 0$ shows that $M(\tilde{\tilde{U}}_{TP(m,n)})$ is a concave function.

7. Numerical and graphical analysis of models

7.1 Numerical **result**: A firm that makes handbags creates and sells handbags to a shop. The store is aware that certain handbags are partially damaged as a result of inefficient manufacturing methods and/or transportation. As a result, he inspects all of the handbags before selling them, and faulty handbags are sold at a proportionate discount. The predicted yearly demand and holding cost are an IT2TrFS, based on prior years' demand patterns and experts' opinions. The chart also includes cost parameters and other pertinent information.'

Table 2.Cost and other parameters

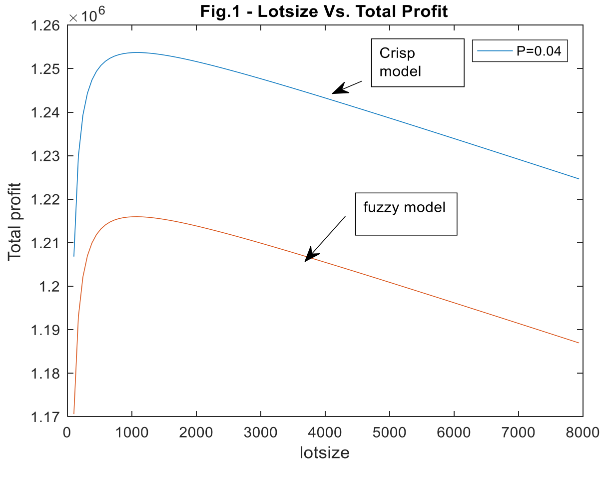
Notation	Co-efficient
$\tilde{\tilde{a}}$	((45000,50000,55000,60000;0.8),(40000,50000,60000,70000;1))
$\tilde{\tilde{H}}_C$	((4.5,5,5.5,6;0.9),(4,5,6,7;0.95))
S_P	\$50/unit
S_C	\$0.25/unit
C_V	\$25/unit
K_C	\$100/cycle
θ	0.02
P_D	0.04
b	0.02
$I(t)$	1000

The maximize profit U_{TP} and optimal order size $(Q_S)_{max}$ of EOQ model is 1,253,699.263/year and 1080.97units, screening time =0.0061 and expected cycle length 0.0268 year but for the fuzzy model 1215971.876/year and 1067.31units are the total profit and optimal order size.

7.2 Graphical analysis of model

1. The concave form of figure 1 indicates that the profit function should be maximised.
2. We find that the optimal order size of both models is the same, but the optimal profit of the crisp model is higher than the profit of the fuzzy model.
3. Figures 2,3,4 demonstrate the proportion of defective goods vs total profit per unit time. The fuzzy model outperforms the crisp model, as seen in Figure 2. In the range of 0.8 to 1.5, we can see a gap when the percentage of faulty goods is one, after which the profit of the models decreases in the other direction. Figure 4 shows that profit is once again on the upswing.

Fig.1 - Lotsize Vs. Total Profit



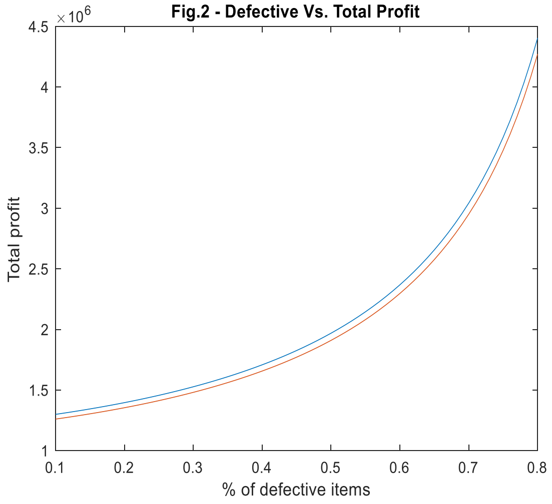


Fig.2 - Defective Vs. Total Profit

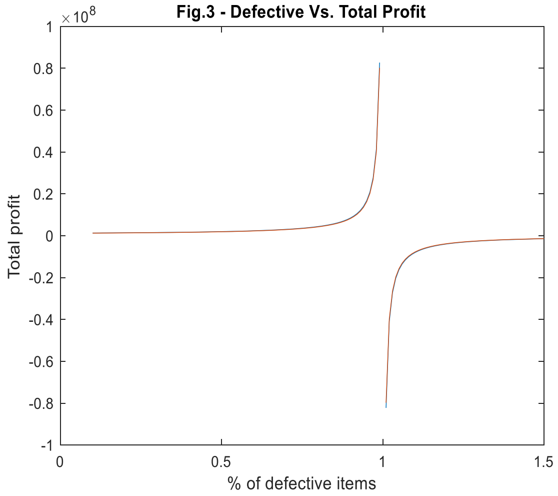


Fig.3 - Defective Vs. Total Profit

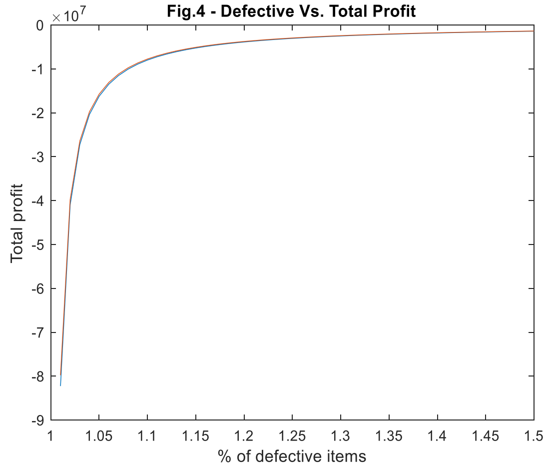


Fig.4 - Defective Vs. Total Profit

8. Sensitivity Analysis

From the above example the sensitivity analysis of the model have been done by changing -10% to +10% the value of one parameter and other parameter remain fixed values. Following Table show that influence of different parameters on screening time, cycle length, optimal order size and total profit of the system.

Table-3

Parameters	Percentage change	Screening time	T	$(Q_S)_{max}$	$TP_U(Q_S)$
a	+10	0.0064	0.0262	1133.71	1379560.874
	+5	0.0063	0.0265	1107.66	1316626.991
	-5	0.0060	0.0272	1053.61	1190778.159
	-10	0.0058	0.0276	1025.52	1127864.208
b	+10	0.0061	0.0268	1080.99	1253749.603
	+5	0.0061	0.0268	1080.98	1253724.433
	-5	0.0061	0.0268	1080.96	1253674.093
	-10	0.0061	0.0268	1080.95	1253648.923
K_C	+10	0.0064	0.0280	1128.14	1253236.795
	+5	0.0063	0.0274	1104.81	1253465.561
	-5	0.0060	0.0262	1056.60	1253938.237
	-10	0.0058	0.0256	1031.65	1254182.855
S_P	+10	0.0062	0.0271	1092.72	1508996.648
	+5	0.0062	0.0270	1086.86	1381347.800
	-5	0.0061	0.0267	1075.05	1126051.043
	-10	0.0061	0.0265	1069.09	998403.144
C_V	+10	0.0061	0.0267	1075.05	1126051.043
	+5	0.0061	0.0268	1078.02	1189875.113
	-5	0.0061	0.0269	1083.92	1317523.492
	-10	0.0062	0.0270	1086.86	1381347.800
P_D	+10	0.0061	0.0268	1083.14	1256554.873
	+5	0.0061	0.0268	1082.06	1255148.112
	-5	0.0061	0.0268	1079.93	1252359.300
	-10	0.0061	0.02	1078.8	1250977.

			69	6	159
s_C	+10	0.0061	0.0268	1080.91	1252422.779
	+5	0.0061	0.0268	1080.94	1253061.021
	-5	0.0061	0.0268	1081.00	1254337.505
	-10	0.0061	0.0268	1081.03	1254975.747
H_C	+10	0.0058	0.0256	1030.67	1253182.476
	+5	0.0060	0.0262	1054.92	1253437.789
	-5	0.0063	0.0275	1109.05	1253967.366
	-10	0.0065	0.0283	1139.45	1254242.628

(i) In positive and negative effect of screening time is depended on parameters a,K_C and holding cost but there is no effect in screening time with the change of parameters like b,S_P, P_D, C_V andS_C.

(ii) In positive way the cycle length dependent on the value of K_C, S_P but in negative way it depend on H_C ,C_V and initial demand. In other hand with the change of b,S_C *and* P_D there is no effect in cycle time.

(iii) The order quantity is increased change of the parameters a,K_C , P_D , S_P and decreased change of parameters H_C ,C_V but constant with changes of the rest parameters.

(iv) We can see from the table above that initial demand, holding costs, and selling prices of high-quality products are all very profit-sensitive**.**

8. Conclusion

Traditional EOQ models can assist us in determining how much to purchase in order to manage inventories. Any business that deals with physical goods must keep track of its inventory in order to improve and avoid shortages. Many times, the lots contain defective items, resulting in a reduction in the model's effectiveness. In this study, we assume a constant rate of deterioration for decaying products with stock-dependent demand and a proportional discount for faulty items. The type-2 fuzzy demand and type-2 fuzzy holding cost are considered in this research to construct a more realistic EOQ model for commodities with unsatisfactory quality. We all know that demand is one of the most important factors in any business, and it is frequently predicted by previous data records and the anticipation of future scenarios that include many unknown events such as an incomplete data set, ambiguous scenarios, and the imprecise nature of demand, among others. As a result, a more efficient approach, namely, interval type-2 fuzzy demand rate, is discussed here to capture such sorts of uncertainty. In addition, the research looked at interval type-2 fuzzy holding cost and provided a method for defuzzing it into a crisp amount. The issue is optimised to provide the optimal answer using the mean valued interval approximation approach and an IT2T fuzzy number and defuzzification methodology. Furthermore, the numerical findings from crisp and fuzzy surroundings were compared. Finally, the modes are justified visually and in terms of sensitivity demonstration. We see a few management implications, such as:

- Initial demand, selling price, and carrying cost on the seller all have a positive or negative influence on the seller's ultimate profit. As a result, it is recommended that the management pursue a suitable marketing approach in order to retain client demand.
- The marketer's overall profit is negatively impacted by variable costs for the second time. The manager guarantees the supplier or manufacturer that the unit variable cost will be reduced for larger orders.
- The overall procedure is determined by the perfect order size and cycle duration.

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