

Distinct polar code construction for 5G Radio network

B. Yamini pushpa (Ph.D), GITAM University, Dr Sunitha panda, Assistant Professor, GITAM University

Abstract: In 5G communication networks, polar codes with different sizes can be used in practical applications. By using shortening and puncturing techniques polarization speed gets increased with a substantial loss and decoding complexity increases. Hence we are introducing a new technique calls distinct polar code construction for variable sizes of transmitted data.

The best channel coding technique which achieves channel capacity for infinite code length is Polar codes; these are also the forward error correction codes. In 5G communication networks, polar codes with different sizes can be used in practical applications. For that Puncturing and Shortening techniques have been used to express block lengths of different sizes other than 2^n . For the construction of polar codes with arbitrary lengths these Puncturing technique and Shortening techniques use mother polar codes with some disadvantages. In terms of polarization speed there is substantial loss and decoding complexity increases while using these shortening and puncturing techniques. Because of loss in the transmission of data there is a poor performance of error rate. These are not suitable for practical implementation, because there is no perfect construction structure for frozen sets. The basic structure were implemented in puncturing and shortening techniques.

For the construction of polar codes with different block sizes, a generalized construction method is proposed. If the length of the polar code is integer powers of 2 then we can use the basic polar code construction method. If not, then use distinct polar code construction method for infinite length of code word. In the generalized construction different block sizes can be developed by combining different size kernels over the same binary alphabet. Construction of polar codes with different block sizes (not only the powers of two) is possible with different kernel sizes at different stages. With this structure a new generalized construction for the polar codes designated as distinct kernel polar codes. Because of distinct kernel structure, the error correcting performance gets increased than the polar codes constructed via puncturing and shortening techniques. Some of the kernels in the distinct kernels may have same structure and size as it results reduction in the encoding complexity to half of the complexity in construction of mother polar codes. Distinct kernel polar codes also have the same complexity as SC decoding of polar codes.

The following sections follows encoding of polar codes I, Distinct kernel polar codes II, Idea of replacing procedure III, Construction of distinct polar codes with kernel substitution IV,

Summary results V, Conclusion VI followed by References VII.

I. Distinct Kernel Polar Codes Encoding procedure:

Distinct kernel polar codes can be distinguished by using a transformation matrix and a frozen set. Let the polar code has the block length N and dimension K is totally determined by using transformation matrix with block length N as $G_N = T_{n1} \otimes \dots \otimes T_{ns}$ and a frozen set F . Depends on the block length, factors of Kroneker product plays vital role. These factors may result in different transformation matrices. Different polarizations may occur because of different transformation matrices cause different frozen sets. The resulting transformation matrix can be used to calculate SNR through density evolution algorithm or Monte-Carlo method. This gives the reliability of the transmitted data. The order of the largest kernel sum can be obtained by adding all the reliabilities of K dimensions. When the order of the kernel is decided with corresponding transformation matrix, then the reliable bits $N-K$ will gives the frozen set F . To simply the notation Code word $X = mG_n$, where X - code word of length- N , m be the message bits, G_n -generator matrix with K information bits are stored according to the frozen set as 1's and the remaining bits as 0's.

Construction example for G_6 :

G_6 matrix can be constructed by using sub-kernels of size T_2 and T_3 . i.e; $G_6 = T_2 \otimes T_3$. Here, the transformation matrices of order 2 and 3 are used. From the knowledge of encoding, code word X of length-6 comprise of, information bits are of K and frozen bits are of $6-K$ bits. With this we know that the distinct code construction can be obtained from the mixing of binary kernels (sub kernels) of sizes 2 and 3. For the construction of distinct kernel polar codes the procedure is same as the construction of polar codes, and it adds different sizes of binary kernels to get large kernel.

Implementation of polar codes with different sizes follows the same procedure as mother code. The component of ministerial term in polar codes can be implemented by using Tanner graph. Different sizes of kernels namely 2 and 3 can be generated using the Tanner graph. The same procedure can be opted for different sizes. Because of Tanner graph, SC decoding is applied to decode distinct polar codes.

For an arbitrary square matrix $n \times n$ ($n \geq 2$) called size- n kernel exists the polarization phenomenon. The generic application for the construction of polar codes with resilient lengths of code called distinct-kernel polar codes. This Distinct Kernel polar codes are employed to construct size-2 and supplemental of $n > 2$ size kernel i.e; the construction method

depends not only on size-2 but also on size-3 as well. In Distinct Kernel polar codes, the code length of polar code will decide the decoding complexity instead of 2^n - bit code length, as a consequence, the complexity involved in SC decoding gets reduced. As go along with size-4 kernels are depends on small size kernels, probably of size-2 and for large size kernels with $n>5$ size can be opted. These structures have the greater decoding complexity while using Successive Cancellation algorithm. For that, it's better to use kernels with smaller sizes ($n<5$) to reduce the decoding complexity. For practical construction only of size-3 and of size-5 kernels are used. Although by using kernels of size-3 and size-5 have the poor polarization phenomenon and hence the distinct polar code construction method using Successive Cancellation decoding is not better. Moreover this complexity is reduced by replacing the large Kernel with two sub-kernels of same size. On excepting the generic construction method for resilient lengths of codes a further techniques are introduced to use the kernels of size- 2^i to upgrade the performance of errors.

The performance of distinct polar code construction gets improved by using 3^n - bit polar codes, as it uses two types of kernels with size-3 on contrasting the sizes of different kernels. Nevertheless, there is no experimental construction procedure and decoding procedure for the kernels of size-3.

For better performance, a narrative and experimental construction for the polar codes are developed for distinct kernel polar code construction with kernel replacing procedure and proportionate decoding algorithm is introduced for generic construction of polar codes. With the use of kernel substitution the decoding complexity is greatly decreased and consequently performance gain could be credible to the Successive Cancellation decoding algorithm on Additive white Gaussian Noise (AWGN) channels.

II. Introduction to Distinct kernel polar codes

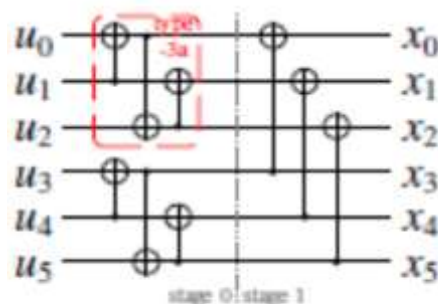
Distinct kernel polar codes are constructed for the flexible length polar codes and these constructions are made from size-2 kernel and type-a kernel. In the demonstration of N-bit generator matrix can be accomplished by the Kronecker product of the distinct kernels.

$G_N = F_{N(i-1)} \otimes F_{N(i-2)} \dots \otimes F_{N(1)} \otimes F_{N(0)}$, where i is the total number of stages and $S_n \geq 2$ and $n \in [0, i-1]$ and the code length N- can be defined as $N = \prod_{j=0}^{i-1} S_j$.

Let the generator matrix G_N with distinct polar code with $N=6$ is shown in figure (a), $S_0=2$ and $S_1=3$. In addition to construction of G_N , Signal flow Graph can also be employed for the construction of generator matrix are shown in figure (b) It has total I number of stages. The bit error rate (BER) of all bits of the generator matrix can be calculated in [1].

$$G_6 = F_2 \otimes F_3 = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

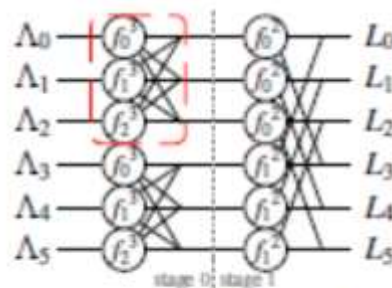
(a) Encoding matrix for $S_0=3$.



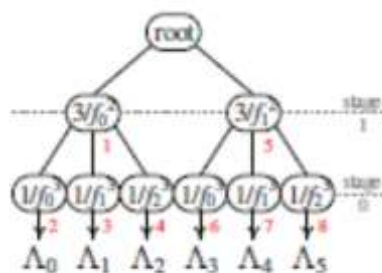
(b) Encoding SFG for $S_0=3$.

To transmit m message bits, it requires K most reliable bits with very low BER is preferred, and a total of N bits are transmitted from the transmitter side. Some of the bits are made permanently to 0 are $N-K$ bits which are known as frozen bits. With the implication of the message and the frozen bits of group are known as message set and frozen set respectively. These are denoted by M and M_c respectively. $R = \frac{K}{N}$ is known as code rate of the polar code.

These encoded data can be decoded by Successive Cancellation decoding algorithm, and by modifying the substitute for the matrix of size- S for large kernel and consequently Signal Flow Graph can be obtained. By replacing the kernel size- S to S nodes will function as S functions are of divergent.



(c) Decoding SFG for $S_0=3$.



(d) Scheduling tree for $S_0=3$.

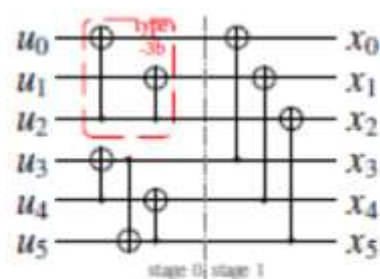
For instant, a 6-bit distinct polar code construction is shown above figure (c). Log likelihood ratio (LLR) for N channel is denoted as $L_n [n \in (0, N-1)]$ are the input to the Signal Flow Graph from the right end. The output of the Signal Flow Graph is at the left side and hard decisions are modeled suitably for $\hat{u}_i = \Theta(\Lambda_i) = \{0 \ \Lambda_i \geq 0 \ 1 \ \Lambda_i < 0 \}$. $\Lambda_i S$ is drawn from $L_i S$ by LLR computations. Min-sum calculation is the general form of decoding function. And the computational complexity of the decoding is calculated from the total number of LLR's used in the decoding are employed to calculate $C_c = N I$.

A scheduling tree of 6-bit, shown above, is used to propose complication in the timing of distinct polar codes. At the decoding, at the distinct stage, on combining the same category of nodes scheduling tree can be acquired. The numbers used and the types of functions are noted at each node in the above figure (d). The indices are arranged in descending order. The previously decoded bits are shown because of the functions, which are depended on the data transmitted. Let, at each node calculations required is done in single step then the timing complexity is given by $C_t = \sum_{n=0}^{i-1} \prod_{j=0}^{i-1} S_j$ that is to decode 6-bit distinct code construction requires 8 stages, and the schedule for decoding of polar codes are shown in figure (d).

III. Idea of replacing procedure

Probability of error for m_{s-1} for type-y kernel is smaller than the size of type-x kernel. Hence, the size of the kernel type-x kernel cannot be chosen for practical applications. Type-b kernel can be chosen to communicate an S-bit with K=1 code word. Finally it is clear that, all the type-x kernels are replaced by type-y kernel to improve error correction performance in distinct polar code construction.

Let distinct polar code can be constructed from $G_6 = F_2 \otimes F_3$ and information set M is $\{2,4,5\}$ and the probability of LBER is enclosed to $p_b \leq \sum_{n \in M} p_e(M_n)$. The block error rate can be improved by replacing type-a kernel with type-b kernel and the information set remains the same and accordingly Signal Flow Graph can be modified as shown in the figure (e).



(e) Encoding SFG after KS for $S_0=3$.

This yields, Probability of error and Block error rate will decrease for u_2 , and u_0 and u_1 are the frozen bits. By changing the BER of these bits, doesn't affect the Block error rate. In contradiction to the remaining bits i.e; u_3, u_4, u_5 are taken for replacing procedure, the probability of error for u_5

may decrease and for u_4 will increase. Finally the total probability doesn't change. In most of the cases, this unreliability exists for large kernels while applied replacing procedure. In the proposed replacing procedure, because of the unreliability, a single type-x kernel have large number of bits cannot be replaced. Kernel replacing procedure can also create a latest environment, less bit error rate than the original information bits. It is clear from the figure (b) that, the information bits are changed to 4 and 5, $A=\{4,5\}$. The same kernel is also replaced by the same kernel with corresponding to $\{u_0, u_1, u_2\}$. The probability error of the bit u_2 may decrease and also it becomes less than the probability error of u_4 after the replacing procedure. In the next scenario the information set $A=\{2,5\}$, the corresponding block error rate get decreased. Later, kernel replacing procedure could be applied to type-a Kernel corresponding to u_3, u_4, u_5 . And applying the same procedure to the u_5 , the probability error for this bit will decrease and accordingly the block error rate will further decrease.

Thus, we can conclude that, after applying replacing procedure, the change in the information set needs to regenerate. The replacing procedure can be applied to distinct kernel polar coding at any stage. If at stage-1, size-3 kernels are used, if the 6-bit generator matrix is defined in the form $G_6 = T_2 \otimes T_3$. Suppose, $Z=2$, and size-3 kernels are interleaved at first stage, then we can verify whether the first s_{i-1} bits are frozen or not. The last Z bits bit error rate can be improved by the corresponding type-akernels are replaced by the type-b kernel. If the same is applied before replacing procedure and there may be extra message bits in the first S_{i-1} bits. This makes the LLR's and the proportionate message bits are having less reliability and as a consequence BLER is very poor.

IV. Construction of distinct polar codes with kernel substitution:

In the generalize construction of distinct polar codes have several stages to construct large Kernel of size-N. In these several stages, some kernels may repeat at different stages. By acquiring all these repetitions and replacing with the same kernel at different stages polar codes are constructed. There may be a confliction that the replacing procedure should starts from initial stage or at the final stage. If the replacing procedure starts from the initial stage, then the new message bits will make kernels at final stages will fail to satisfy the replacing procedure i.e; the first stage itself all the bits may frozen, and hence there may be reduction in the performance gain. If the replacing procedure starts at the final stage, then there is no chance to the initial bits to be frozen. Hence the replacing procedure should be done at the higher stages.

Proposed Algorithm:

1. Depends on Distinct polar codes of 2 in size and type-x kernels, the initial code is generated.
2. At the highest stage, the replacing procedure starts.

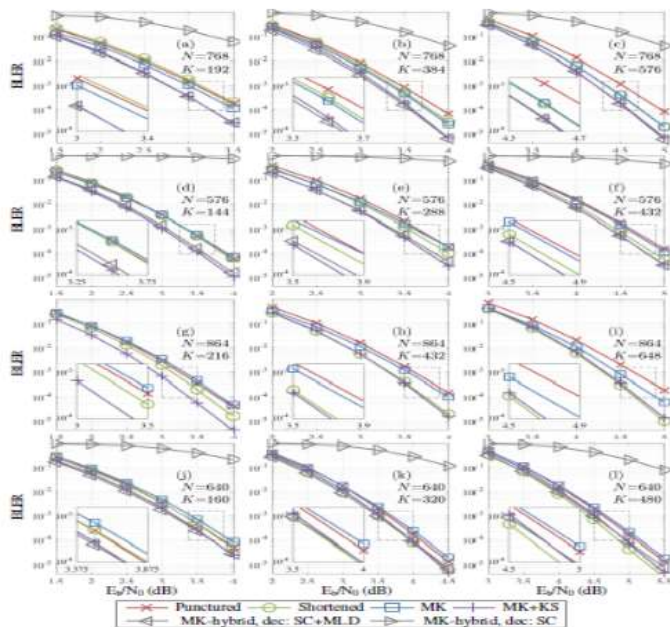
Let the size of Kernel at the highest stage is not equal to 2, then kernel replacing procedure starts at each stage with z number of bits in which first $(S_n - 1) z$ bits are frozen. If first round of replacing procedure finished, then the code set-M is regenerated and the upper bound of Block length error rate is

calculated from the sub kernel matrix. If there is no improvement in block error rate at particular stage, then the replacing procedure stops and proceed for the lower stages. As a result, the generator matrix G_n is revised to new one and the proportional information set M is generated for the target code.

Replacing procedure: The specific type-a kernels on particular stage are substituted with type-b kernel. And the generator matrix is revised correspondingly.

Sub-kernel matrix: in polar code construction, to generate information set M , Monte Carlo simulation algorithm is used for N - bit distinct polar code. From the revised generator matrix K bits are the message bits at the target SNR. Probability of the message bits is calculated by the sum of BER's of message bits, accordingly estimate the BER for the revised generator matrix G_n . this is the procedure to get an idea on kernel replacing procedure to check the procedure improved for decoding performance or not.

V. Summary results:



VI. Conclusion

A method of distinct polar codes with substitution kernel is introduced. A clear-cut construction of polar code is introduced. Simulation results shows that the presented technique is better performed than other construction techniques. As a result construction technique is flexible for different lengths of polar codes with lowest complexity in the decoding.

VII. References

1. E. Arıkan, "Channel polarization: A method for constructing capacityachieving codes for symmetric binaryinputMemoryless channels," IEEE, information theory, vol. 55, pp. 3051–3073, June 2009.
2. H.-P. Lin, S. Lin, and K. Abdel-Ghaffar, "Linear and nonlinear binary kernels of polar codes of small dimensions with maximum exponents," IEEE

- Transactions on Information Theory, vol. 61, pp. 5253–5270, Oct. 2015.
3. R. Wang and R. Liu, "A novel puncturing scheme for polar codes," IEEE Communications Letters, vol. 18, pp. 2081–2084, December 2014.
4. K. Niu et al., "Beyond turbo codes: Rate-compatible punctured polar codes," in Proc. IEEE ICC), 2013, pp. 3423–3427.
5. F. Gabry et al., "Multi-kernel construction of polar codes," in IEEE (ICC). 2017, [7] V. Bioglio and I. Land, "On the marginalization of polarizing kernels,"in IEEE ,ISTC.. IEEE, 2018, pp. 1–5.
6. A. Cavatassi et al., "Asymmetric construction of low-latency and length flexible polar codes," 2019.
7. H.-P. Lin et al., "Linear and nonlinear binary kernels of polar codes of small dimensions with maximum exponents," IEEE Trans. Information Theory.
8. V. Bioglio et al., "Multi-kernel polar codes: Concept and design principles," 2020.
9. N. Presman et al., "Mixed-kernels constructions of polar codes," IEEE communications vol. 34, no. 2, pp. 239–253, February 2016.
10. G. Trofimuk and P. Trifonov, "Reduced complexity window processing of binary polarization kernels," IEEE ,ISIT. IEEE, 2019.
11. V. Guruswami et al., "Arıkan meets shannon: Polar codes with near-optimal convergence to channel capacity," arXiv preprint arXiv:1911.03858, 2019.
12. A. Cavatassi et al., "Fast decoding of multi-kernel polar codes," arXiv preprint arXiv:1902.01922, 2019.
13. G. Coppolino et al., "A multi-kernel multi-code polar decoder architecture," IEEE TCSI, vol. 65, pp. 4413–4422, December 2018.