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An Application of Pythagorean Fuzzy Set-In Machine Sequencing Problem

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Abstract

All decision-making processes involves uncertainty in some aspects. There are many theories to tackle this uncertainty in real life models. Fuzzy set, Intuitionistic Fuzzy set, Neutrosophic set, Pythagorean Fuzzy set, etc. are some of them. This paper uses a Pythagorean Fuzzy set approach in solving a machine sequencing problem. This approach is illustrated using numerical example.

Keywords: Fuzzy set, Intuitionistic Fuzzy set, Neutrosophic set, Pythagorean Fuzzy set (PFS), Pythagorean Fuzzy number (PFN).

1. Introduction

Decision making is one of the foremost human activities in our real life. We are often faced with situations where we have to make a decision among the available criteria and alternatives. Real life situations are not certain and usually accompanied with impreciseness and vagueness. Fuzzy theory introduced by Zadeh [26] is one of the most important tools to tackle the uncertainty prevailing in our daily life. Fuzzy set contains a membership function μ used to assign the degree of membership of an element from the universe of discourse to the unit interval [0,1]. Atanassov [1] thought that Fuzzy set was incomplete as it contains only the membership function. He introduced the concept of Intuitionistic Fuzzy set in 1986 which contains both membership and non-membership function. In 1995 Smarandache [17] introduced the concept of Neutrosophic set which contains the truth membership, falsity membership and intermediate membership of an element. In 2013. Yager RR [23] introduced the Pythagorean Fuzzy set which is now applied to solve many Science and Engineering problems. Yager [24], Zhang and Xu [25] applied the Pythagorean membership grades in MCDM techniques. Peng and Yang [15] applied PFS in MADM models. Weietal [19], WeiandLi [20] discussed the application of Pythagorean Interval-valued Fuzzy maclaurin symmetric mean operators in MADM problems. Garg applied PFN in decision making models [4,6] using the concept of Linear programming [5]. Li and Lu [12] discussed the similarity and distance measures of PFS and their applications. Kumar et.al [11] applied PFS Transportation models.

A job sequencing problem arise where we have situations in which a number of tasks to be performed in a minimum duration. Sequencing problems are one of the most important applications of Operations research. The main objective of sequencing problem is to determine the optimal sequence of jobs on the machines such that the total elapsed time is minimum, Johnson's [10] work was one of the foremost and renowned in the field of production scheduling. In his algorithm he minimizes the total elapsed time of all the jobs and the total idle time of the machines.

Smith and Dudek [17] provided a general algorithm for the solution of N jobs and M-machines sequencing problem. In recent years' Fuzzy sets developed by Zadeh [26] has been applied extensively in real life problems involving uncertainty. A large number of applications are found in the literature of Fuzzy sequencing. Some novel approaches are included in [2,7,8,9,11,14,18,19].

The rest of the paper is organized as follows: section 2 comprises the basic definitions and preliminaries. Section 3 presents the Mathematical formulation of Sequencing problem in Fuzzy environment. Section 4 compares the Pythagorean Fuzzy Sequencing problem with Fuzzy and intuitionistic Fuzzy Sequencing problem for two and three machines.

2 Basic Definitions

2.1 Fuzzy Set

Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A Fuzzy set A with the membership function $\mu_A(x)$ is defined as A={(x, $\mu_A(x)$; x ϵ A} and $\mu_A(x)$ ϵ [0.1].

2.2 Intuitionistic Fuzzy Set

Let X be a given set. An Intuitionistic Fuzzy Set B in X is given by, B = {(x, $\mu_B(x)$,$\vartheta_B(x)$) | x ϵ X} where μ_B, ϑ_B : X→[0,1], where $\mu_B(x)$ is the degree of membership of the element x in B and $\vartheta_B(x)$is the degree of non-membership of x in B and0≤ $\mu_B(x)$+ $\vartheta_B(x) \leq 1$.

For each x ϵ X, $\pi_B(x) = 1 - \mu_B(x) - \vartheta_B(x)$ is the degree of hesitation.

2.3 Pythagorean Fuzzy set

Let X be a universal set. Then, a Pythagorean Fuzzy set $\widetilde{A}^{p_y}$, which is a set of ordered pairs over X, is defined as:

$$\widetilde{A}^{p_y} = \{(x, \mu_{\widetilde{A}^{p_y}}(x), \vartheta_{\widetilde{A}^{p_y}}(x) : x \epsilon X\}$$

where $\mu_{\widetilde{A}^{p_y}} : X \rightarrow [0, 1]$ and $\vartheta_{\widetilde{A}^{p_y}} : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership, respectively, of the element x ∈ X to A, which is a subset of X, and for every x ∈ X: $0 \leq (\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2 \leq 1$.

Let $(\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2 \leq 1$, then there is a degree of indeterminacy of x ∈ X to A given by $\pi_{\widetilde{A}^{p_y}}(x) = \sqrt{1 - [(\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2]}$ and $\pi_{\widetilde{A}^{p_y}}(x) \in [0,1]$

Then it follows that $(\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2 + (\pi_{\widetilde{A}^{p_y}}(x))^2 = 1$.

Otherwise $(\pi_{\widetilde{A}^{p_y}}(x)) = 0$ whenever $(\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2 = 1$

2.4 Pythagorean Fuzzy Number [PFN]

A pair denoted by $\widetilde{A}^{p_y} = \{(\mu_{\widetilde{A}^{p_y}}(x), \vartheta_{\widetilde{A}^{p_y}}(x)\}$ is called Pythagorean Fuzzy number.

The degree of indeterminacy is given by

$$\pi_{\widetilde{A}^{p_y}}(x) = \sqrt{1 - [(\mu_{\widetilde{A}^{p_y}}(x))^2 + (\vartheta_{\widetilde{A}^{p_y}}(x))^2]}$$

2.5 Arithmetic Operations of PFN:

Let $\widetilde{A}^{p_y} = (\mu, \vartheta\}$, $\widetilde{A_1}^{p_y} = (\mu_1, \vartheta_1\}$ and $\widetilde{A_2}^{p_y} = (\mu_2, \vartheta_2\}$ be three PFNs, then we have

1) $\widetilde{A_1}^{p_y} \cap \widetilde{A_2}^{p_y} = (\min(\mu_1, \mu_2), \max(\vartheta_1, \vartheta_2))$
2) $\widetilde{A_1}^{p_y} \cup \widetilde{A_2}^{p_y} = (\max(\mu_1, \mu_2), \min(\vartheta_1, \vartheta_2))$
3) $\widetilde{A_1}^{p_y} \oplus \widetilde{A_2}^{p_y} = (\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \vartheta_1\vartheta_2)$
4) $\widetilde{A_1}^{p_y} \otimes \widetilde{A_2}^{p_y} = (\mu_1\mu_2, \sqrt{\vartheta_1^2 + \vartheta_2^2 - \vartheta_1^2\vartheta_2^2})$

2.6 Ranking Technique

Let $\widetilde{A}^{p_y} = \{(\mu_{\widetilde{A}^{p_y}}(x), \vartheta_{\widetilde{A}^{p_y}}(x)\}$ be a PFN. The ranking of $\widetilde{A}^{p_y}$ on the set of PFN is defined by:

$$R(\widetilde{A}^{p_y}) = \left(\frac{(\mu_{\widetilde{A}^{p_y}})^2 + (\vartheta_{\widetilde{A}^{p_y}})^2}{2}\right)$$

3. Mathematical Formulation of Fuzzy Sequencing Problem

3.1 Algorithm for solving Fuzzy sequencing problem

3.1.1 Processing 'n' jobs through TWO machines

The simplest possible Fuzzy sequencing decision problem is that of 'n' jobs two machine Fuzzy sequencing problem which determine the sequence in which 'n' jobs should be processed through two machines so as to minimize the total elapsed time 'T'. This type of problem can be completely described as:

i) Only two machines Ã and B̃ are involved
ii) Each job is processed in the order AB, and
iii) The expected Fuzzy processing time Ãi and B̃i where i = 1,2,3…n expressed as Fuzzy numbers.

The procedure for the solution of the above problem is described as follows:

Step 1: Using ranking function the Fuzzy sequencing problem is defuzzified into crisp sequencing problem

Step 2: The optimal sequence for the crisp sequencing problem is determined using Johnson's algorithm

Step 3: After finding the optimal sequence by Johnson's algorithm, the total elapsed (Fuzzy) time and also the Fuzzy idle times on machines Ã and B̃ are determined as follows

Step 4: Total elapsed Fuzzy time = the Fuzzy time between starting the first job in the optimal sequence on machine Ã and completing the last job in the optimal sequence on machine B̃.

Step 5: Fuzzy idle time on machine Ã = (Fuzzy time when the last job in the optimal sequences is completed on Machine B̃) – (Fuzzy time when the last job in the optimal sequence completed on machine Ã)

Step 6: Fuzzy idle time on machine B = (Fuzzy time when the first job in the optimal sequence completed on machine Ã) + $\sum_{k=2}^{m}$ [(Fuzzy time when k th job starts on machine B̃) – (Fuzzy time k – 1)st job finished on machine B̃)]

3.1.2 Processing 'n' jobs through THREE machines

This type of problem can be completely described as:

i) only two machines Ã, B̃ and C̃ are involved
ii) each job is processed in the order ÃB̃C̃
iii) no passing of jobs is permitted, and the expected Fuzzy processing time Ãi , B̃i, C̃i where i = 1,2,3…n expressed as Fuzzy numbers.

The procedure for the solution of the above problem is described as follows:

Step 1: using ranking function the Fuzzy sequencing problem is defuzzified into crisp sequencing problem

Step 2: the optimal sequence for the crisp sequencing problem of Processing 'n' jobs through three machines is determined using Johnson's algorithm.

The resulting optimal sequence will also be optimal for the original problem of 3 machines and n jobs. The total elapsed Fuzzy time and the Fuzzy idle times on machines Ã, B̃ and C̃ are determined as follows:

Step 3: Total elapsed Fuzzy time = the Fuzzy time between starting the first job in the optimal sequence on machine Ã and completing the last job in the optimal sequence on machine C̃.

Step 4: Fuzzy idle time on machine Ã = (Fuzzy time when the job J1 is completed on Machine C̃) – (Fuzzy time when the last job is completed on machine Ã)

Step5: Fuzzy idle time on machine B = (Fuzzy time when the first job J̃3 is completed on machine Ã) + $\sum_{k=2}^{m}$ [(Fuzzy time when k th job on machine B̃) – (Fuzzy time out for (k – 1)st job on machine B̃)] + [Fuzzy time last job is completed on C̃ – Fuzzy time last job is completed on B̃]

Step 6: Fuzzy idle time on machine C = (Fuzzy time when the first job J̃3 is completed on machine B̃) +

$$\sum_{k=2}^{m} [(\text{Fuzzy time when } k\text{ th job on machine } \tilde{C}) - (\text{Fuzzy time out for } (k-1)\text{st job on machine } \tilde{C})]$$

4. Numerical Illustration

4.1. Numerical Example-1

A two-machine sequencing problem is considered in the Fuzzy environment and Pythagorean Fuzzy environment. The processing times are taken as trapezoidal Fuzzy numbers and Pythagorean Fuzzy numbers in the respective environment.

A book binder has one printing press, one binding machine and manuscripts of a number of different books. The time required to perform the printing and binding operation for each book are shown below. It is required to determine the order in which books should be processed, in order to minimize the total time (in hrs.) required to turn out all the books.

Processing time as TrFN

MACHINES	BOOK 1	BOOK 2	BOOK 3	BOOK 4	BOOK 5
M1	(0.08, 0.10, 0.12, 0.16)	(0.02, 0.02, 0.02, 0.02)	(0.16, 0.18, 0.20, 0.24)	(0.04, 0.06, 0.08, 0.12)	(0.18, 0.20, 0.22, 0.26)
M2	(0.02, 0.04, 0.06, 0.10)	(0.10, 0.12, 0.16, 0.20)	(0.12, 0.14, 0.16, 0.20)	(0.14, 0.16, 0.18, 0.22)	(0.06, 0.08, 0.10, 0.14)

Using the defuzzification measure R = $\frac{(2a+2b+d-c)}{4}$ the given sequencing problem is converted to crisp problem as follows:

MACHINES	BOOK 1	BOOK 2	BOOK 3	BOOK 4	BOOK 5
MACHINE 1	0.1	0.02	0.18	0.06	0.2
MACHINE 2	0.04	0.12	0.14	0.16	0.08

The optimal sequence of the books is given by:

B2	B4	B3	B5	B1

Calculation of Total elapsed time:

	MACHINE 1		MACHINE 2	
BOOKS	TIME-IN	TIME-OUT	TIME-IN	TIME-OUT
B2	0	0.02	0.02	0.14
B4	0.02	0.08	0.14	0.30
B3	0.08	0.26	0.30	0.44
B5	0.26	0.46	0.46	0.54
B1	0.46	0.56	0.56	0.60

Total elapsed time = 0.60 hrs
Idle time for machine A = 0.04 hrs
Idle time for machine B = 0.02+0.02+0.02
= 0.06 hrs
Idle time for the system = 0.04+0.06
= 0.08 hrs

Processing time as PFN

MACHINES	BOOK 1	BOOK 2	BOOK 3	BOOK 4	BOOK 5
MACHINE 1	(0.08, 0.16)	(0.02, 0.02)	(0.16, 0.24)	(0.04, 0.12)	(0.18, 0.26)
MACHINE 2	(0.02, 0.10)	(0.10, 0.20)	(0.12, 0.20)	(0.14, 0.22)	(0.06, 0.14)

Using the defuzzification measure R= $\frac{\left[\left(\mu_{\tilde{A}^{p_y}}\right)^2+\left(\vartheta_{\tilde{A}^{p_y}}\right)^2\right]}{2}$, the given sequencing problem is converted to crisp problem as follows:

BOOKS	BOOK 1	BOOK 2	BOOK 3	BOOK 4	BOOK 5
PRINTING TIME	0.016	0.0004	0.0416	0.008	0.05
BINDING TIME	0.0052	0.025	0.0272	0.034	0.0116

The optimal sequence of the books is given by:

B2	B4	B3	B5	B1

Calculation of Total elapsed time:

BOOKS	MACHINE 1		MACHINE 2	
	TIME-IN	TIME-OUT	TIME-IN	TIME-OUT
B2	0	0.0004	0.0004	0.0254
B4	0.0004	0.0084	0.0254	0.0594
B3	0.0084	0.05	0.0594	0.0866
B5	0.05	0.10	0.10	0.1116
B1	0.10	0.116	0.116	0.1212

Total elapsed time = 0.1212 hrs
Idle time for machine A = 0.0052 hrs
Idle time for machine B = 0.0004+0.0134+0.0044
= 0.0182 hrs
Idle time for the system = 0.0052+0.0182
= 0.0234 hrs

From the result we infer that the total elapsed time and the idle time of the Machines are lesser in Pythagorean Fuzzy case than the Fuzzy case even though the optimal sequence of Machines remains same in both the cases.

4.2 Numerical Example-2

A three-machine sequencing problem is considered in the Intuitionistic Fuzzy environment and Pythagorean Fuzzy environment. The processing times are taken as Triangular Intuitionistic Fuzzy number and Pythagorean Fuzzy number in the respective environment.

Processing time as TIFN

MACHINES	JOB 1	JOB 2
MACHINE A	(0.045,0.05,0.055) (0.04,0.05,0.06)	(0.065,0.07,0.075) (0.06,0.07,0.08)
MACHINE B	(0.015,0.02,0.025) (0.01,0.02,0.03)	(0.01, 0.01, 0.01) (0.01, 0.01, 0.01)
MACHINE C	(0.025,0.03,0.035) (0.02,0.03,0.04)	(0.06, 0.07, 0.08) (0.05, 0.07, 0.09)

JOB 3	JOB 4	JOB 5
(0.055,0.06,0.065) (0.05,0.06,0.07)	(0.085,0.09,0.095) (0.08,0.09,0.10)	(0.045,0.05,0.055) (0.04,0.05,0.06)
(0.035,0.04,0.045) (0.03,0.04,0.05)	(0.045,0.05,0.055) (0.04,0.05,0.06)	(0.025,0.03,0.035) (0.02,0.03,0.04)
(0.045,0.05,0.055) (0.04,0.05,0.06)	(0.055,0.06,0.065) (0.05,0.06,0.07)	(0.065,0.07,0.075) (0.06,0.07,0.08)

Using the defuzzification measure R = $\frac{a_1+4a_2+a_3+a_1^*+a_3^*}{8}$, the given sequencing problem is converted to crisp problem as follows:

MACHINES	JOB 1	JOB 2	JOB 3	JOB 4	JOB 5
MACHINE A	0.05	0.07	0.06	0.09	0.05
MACHINE B	0.02	0.01	0.04	0.05	0.03
MACHINE C	0.03	0.07	0.05	0.06	0.07

Now let, Machine X = Machine A + Machine B
Machine Y = Machine B + Machine C

MACHINES	JOB 1	JOB 2	JOB 3	JOB 4	JOB 5
MACHINE X	0.07	0.08	0.10	0.135	0.08
MACHINE Y	0.05	0.08	0.09	0.11	0.10

The optimal sequence of the jobs is given by:

J2	J5	J4	J3	J1

Calculation of Total elapsed time:

NO: OF JOBS	MACHINE 1		MACHINE 2		MACHINE 3	
	TIME -IN	TIME -OUT	TIME -IN	TIME -OUT	TIME -IN	TIME -OUT
J2	0	0.07	0.07	0.08	0.08	0.15
J5	0.07	0.12	0.12	0.15	0.15	0.22
J4	0.12	0.21	0.21	0.26	0.26	0.32
J3	0.21	0.27	0.27	0.31	0.32	0.37
J1	0.27	0.32	0.32	0.34	0.37	0.40

Total elapsed time = 0.40 hrs
Idle time for machine A = 0.08 hrs
Idle time for machine B = (0.07+0.04+0.06+0.01+0.01+0.06)
= 0.25 hrs
Idle time for machine C = 0.08+0.04
= 0.12 hrs
Idle time for the system = 0.08+0.25+0.12
= 0.45 hrs

Processing time as PFN

MACHINES	JOB 1	JOB 2	JOB 3	JOB 4	JOB 5
MACHINE A	(0.04, 0.06)	(0.06, 0.08)	(0.05, 0.07)	(0.08, 0.10)	(0.04, 0.06)
MACHINE B	(0.01, 0.03)	(0.01, 0.01)	(0.03, 0.05)	(0.04, 0.06)	(0.02, 0.04)
MACHINE C	(0.02, 0.04)	(0.05, 0.09)	(0.04, 0.06)	(0.05, 0.07)	(0.06, 0.08)

Using defuzzification measure R= $\frac{\left[(\mu_{\tilde{A}p_y})^2+(\vartheta_{\tilde{A}p_y})^2\right]}{2}$, the given sequencing problem is converted to crisp problem as follows:

MACHINES	JOB 1	JOB 2	JOB 3	JOB 4	JOB 5
MACHINE A	0.0026	0.005	0.0037	0.0082	0.0026
MACHINE B	0.0005	0.0001	0.0017	0.0026	0.001
MACHINE C	0.001	0.0053	0.0026	0.0037	0.005

Now let,
Machine G = Machine A + Machine B
Machine H = Machine B + Machine C

MACHINES	JOB 1	JOB 2	JOB 3	JOB 4	JOB 5
MACHINE G	0.0031	0.0051	0.0054	0.0108	0.0036
MACHINE H	0.0015	0.0054	0.0043	0.0063	0.006

The optimal sequence of the jobs is given by:

J5	J2	J4	J3	J1

Calculation of Total elapsed time:

NO: OF JOBS	MACHINE A		MACHINE B		MACHINE C	
	TIME -IN	TIME - OUT	TIME -IN	TIME -OUT	TIME -IN	TIME -OUT
J5	0	0.0026	0.0026	0.0126	0.0126	0.0176
J2	0.0026	0.0076	0.0126	0.0127	0.0176	0.0229
J4	0.0076	0.0158	0.0158	0.0184	0.0229	0.0266
J3	0.0158	0.0195	0.0195	0.0212	0.0266	0.0292
J1	0.0195	0.0221	0.0221	0.0226	0.0292	0.0302

Total elapsed time = 0.0302 hrs
Idle time for machine A = 0.0081 hrs
Idle time for machine B =(0.0026+0.0031+0.0009+0.0076)
= 0.0153 hrs
Idle time for machine C = 0.0126 hrs
Idle time for the system = (0.0081+0.0153+0.0126)
= 0.036 hrs

From the results, we note that the optimal sequence is different in both the cases. The total elapsed time and the idle time of the Machines are much lesser in the Pythagorean Fuzzy case.

5. Conclusion

In this paper, a machine sequencing problem in Pythagorean Fuzzy environment is considered. The processing time are considered as PFN and the optimal sequence is obtained using Johnson's algorithm. Numerical examples comparing the Pythagorean Fuzzy case with the Fuzzy and Intuitionistic Fuzzy case shows that the advantage of applying PFN in machine sequencing model. Further aspects lie on applying PFS in other Optimization and multistage decision-making problems in Science and Engineering.

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