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On Fundamental Algebraic Attributes of $\omega - Q -$Fuzzy Subring, Normal Subring and Ideal

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Abstract

In this paper, the introduced the new notion of on Fundamental Algebraic attributes of $\omega - Q - FSR$ and $\omega - Q - FI$ are defined and discussed. The Homomorphism of $\omega - Q - FSR$, $\omega - Q - FNSR$ and $\omega - Q - FI$ and their inverse images has been obtained. Some related results have been discussed in their paper.

Keywords:

Fuzzy set (FS), Fuzzy subrings (FSR), Fuzzy normal subring (FNSR), $\omega - Q -$Fuzzy subrings $(\omega - Q - FSR)$, $\omega - Q -$fuzzy normal subring $(\omega - Q - FNSR)$, Fuzzy ideal(FI) , $\omega - Q -$fuzzy ideal $(\omega - Q - FI)$.

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I. Introduction

The pioneering work of L A Zadeh on fuzzy subsets of a set in[17]. A. B Chakranarty et al. invented the theory of fuzzy homomorphism and algebraic structures in 1993[1]. The concept of Prime fuzzy ideals in ring was established by T.K. Mukhrjee et al. in 1989[8]. V.N. Dixit et al. introduced the concept of Fuzzy rings in 1992[3]. In 1996, explored the notion of K-fuzzy ideals in semi rings by B. K Chang et al.[2]. A. Prasanna et.al.[9], introduced the concept on Fundamental Algebraic attributes of $\chi -$Fuzzy Subring, Normal Subring and Ideal. R. Kumar, described the new notion of fuzzy algebra in 1993[4]. In 1982, Wang-Jin Liu[16] ,the concept of fuzzy invariant subgroups and fuzzy ideals. A. Rosenfeld[11], explored the new notation of fuzzy groups in 1971. P.K. Sharma[13], explored the α - Anti Fuzzy Subgroups in 2012. In addition , more recent development, α- Fuzzy subgroups in 2013[11] . D.S. Malik et al. derived from the extension of fuzzy subrings and fuzzy ideals in 1992[6]. In 1992, R. Kumar[5], introduced the new notion of Certain fuzzy ideals of rings. A. Prasanna et.al[10], introduced the new concept on Elemental Algebraic characteristic of $\omega -$Fuzzy Subring, Normal Subring and Ideal in 2020. V. Veeramani et al. derived from the Some Properties of Intuitionistic Fuzzy Normal Subrings in 2010[15]. T.K. Mukhrjee et al. propsed by the concept of on fuzzy ideals of a ring in 1987[7]. A. Solairaju and R. Nagarajan[14] , described the notation of a structure and Construction of Q-fuzzy Groups in 2009.

The research article is arranged as follows, section II contains the elementary basic concept of definitions related to the results which are thoroughly crucial to this research. In section III, we introduce fundamental algebraic attributes of $\omega - Q -$fuzzy subring$(\omega - Q - FSR)$ and ideal$(\omega - Q - FI)$ and section IV, describe the algebraic structures on homomorphism of $\omega - Q -$fuzzy subrings$(\omega - Q - FSR)$, normal subrings$(\omega - Q - FSNR)$ and ideals $(\omega - Q - FI)$.

II. Preliminaries

Definition: 2.1 [7]

Let R be a ring. A function $A: R \to [0,1]$ is said to be a FSR of R if

$(i) A(x-y) \geq min\{\mu_A(x), \mu_A(y)\}$

$(ii) A(xy) \geq min\{\mu_A(x), \mu_A(y)\}, \forall\, x, y \in R.$

Definition: 2.2 [13]

A FSR A of a ring R is said to be a $FNSR$ of R if $A(xy) = A(yx), \forall\, x, y \in R.$

Definition: 2.3 [14]

Let R be a ring. A function $A: R \to [0,1]$ is said to be a

(a) Fuzzy Left Ideal of R if
$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq A(y), \forall\, x, y \in R$

(b) Fuzzy Right Ideal of R if
$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq A(x), \forall\, x, y \in R$

(c) Fuzzy Ideal of R if

$(i) A(x-y) \geq min\{A(x), A(y)\}$
$(ii) A(xy) \geq max\{A(x), A(y)\}, \forall\, x, y \in R$

Theorem: 2.4 [6]

If A be a FSR of the ring R then

$(i)\ A(0) \geq A(x)$

$(ii) A(-x) = A(x), \forall\, x \in R$

(iii) If R is ring with unity 1, then $A(1) \geq A(x), \forall\, x \in R$.

Definition: 2.5 [1]

Let X and Y be two non-empty sets and $f: X \to Y$b e a mapping. Let A and B be FS of X and Y respectively. Then the image of A under the map f is denoted by $f(A)$ and is defined as $f(A)(y) = \begin{cases} Sup\{A(x): x \in f^{-1}(y)\} \\ 0: otherwise \end{cases}, \forall\, y \in Y$

Also the pre-image of B under f is denoted by $f^{-1}(B)$ and defined

$$f^{-1}(B)(x) = Bf(x), \forall\, x \in X.$$

Definition: 2.6 [1]

The mapping $f: R_1 \to R_2$ from the ring R_1 into a ring R_2 is called a ring homomorphism if

$$(i) f(x+y) = f(x) + f(y)$$

$(ii) f(xy) = f(x)f(y), \forall\, x, y \in R_1$.

Definition:2.7[14]

Let Q and G a set and a group respectively. A mapping $\mu: G \times Q \to [0,1]$ is named $Q - FS$ in G. For any $Q - FS$ μ in G and $t \in [0,1]$ we define the set $U(\mu; t) = \{x \in G \;/\; \mu(x,q) \geq t, q \in Q\}$ which is named an upper cut of "μ" and may be use to the characterization of μ.

III. On Fundamental Algebraic attributes of $\omega - Q -$Fuzzy Subring($\omega - Q - FSR$) and $\omega - Q -$Ideal($\omega - Q - FI$)

Definition: 3.1

Let Ặ be a fuzzy set (FS) of a ring τ, $and\ Q -$ fuzzy subset (FSb) of a set τ. Let $\omega \in [0,1]$. Then the $Q -$fuzzy set $Ặ^{\omega}$ of τ is called the $\omega - Q -$fuzzy subset($\omega - Q -$FS) of τ with respect to FS Ặ and is defined by

$$Ặ^{\omega}(\theta, q) = \{Ặ(\theta, q) \wedge \omega\}, \qquad \forall \omega \in [0,1] and\ q \in Q.$$

Definition: 3.2

Let Ặ be a FS of a ring τ , $Q -$ fuzzy subset (FSb) of a set τ and $\in [0,1]$. Then Ặ is called $\omega - Q -$Fuzzy Subring ($\omega -$FSR) of τ if $Ặ^{\omega}$ is FSR of τ i.e. if the following conditions hold

$$(i) Ặ^{\omega}((\theta - \varphi), q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}$$

$$(ii) Ặ^{\omega}(\theta\varphi, q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}, \forall\, \theta, \varphi \in \tau\ and\ q \in Q.$$

In other words, Ặ is $\omega - Q -$FSR of τ if $Ặ^{\omega}$ is FSR of τ.

Definition: 3.3

Let Ặ be a FS of a ring τ and $Q -$ fuzzy subset (FSb) of a set τ . Let $\omega \in [0,1]$. Then Ặ is called $\omega - Q -$Fuzzy Left Ideal of τ ($\omega -$FLI) if

$(i)\ Ặ^{\omega}((\theta - \varphi), q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}$

$(ii)\ Ặ^{\omega}(\theta\varphi, q) \geq Ặ^{\omega}(\varphi, q), \forall\, \theta, \varphi \in \tau\ and\ q \in Q$.

Definition: 3.4

Let Ặ be a FS of a ring $\tau\ and\ Q -$ fuzzy subset (FSb) of a set τ. Let $\omega \in [0,1]$. Then Ặ is called $\omega - Q -$Fuzzy Right Ideal of τ ($\omega -$FRI) if

$(i)\ Ặ^{\omega}((\theta - \varphi), q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}$

$(ii)\ Ặ^{\omega}(\theta\varphi, q) \geq Ặ^{\omega}(\theta, q), \forall\, \theta, \varphi \in \tau\ and\ q \in Q$.

Definition: 3.5

Let Ặ be a FS of a ring τ, $and\ Q -$ fuzzy subset (FSb) of a set τ. Let $\omega \in [0,1]$. Then Ặ is called $\omega - Q -$Fuzzy Ideal of τ ($\omega -$FI) if

$(i)\ Ặ^{\omega}((\theta - \varphi), q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}$

$(ii)\ Ặ^{\omega}(\theta\varphi, q) \geq \{Ặ^{\omega}(\theta, q) \wedge Ặ^{\omega}(\varphi, q)\}, \forall\, \theta, \varphi \in \tau\ and\ q \in Q$.

Proposition: 3.6

Let $Ặ^{\omega}$ and $Ƀ^{\omega}$ be two $\omega - Q -$FS of a ring τ. Then $(Ặ \cap Ƀ)^{\omega} = Ặ^{\omega} \cap Ƀ^{\omega}$.

Proof:

Let, $\theta \in \tau\ and\ q \in Q$ be any element, then

$$(Ặ \cap Ƀ)^{\omega}(\theta, q) = \{(Ặ \cap Ƀ) \wedge \omega\}$$

$$= (\{Ặ(\theta, q) \wedge Ƀ(\varphi, q)\} \wedge \omega)$$

$$= (\{Ặ(\theta, q) \wedge \omega\} \wedge \{Ƀ(\varphi, q) \wedge \omega\})$$

$$= \{Ặ^{\omega}(\theta, q) \wedge Ƀ^{\omega}(\varphi, q)\}$$

$$= (Ặ^{\omega} \cap Ƀ^{\omega})(\theta, q)$$

Hence $(Ặ \cap Ƀ)^{\omega}(\theta, q) = (Ặ^{\omega} \cap Ƀ^{\omega})(\theta, q), \forall\, \theta \in \tau\ and\ q \in Q$.

Proposition: 3.7

Let $g: \alpha \to \beta$ be a mapping. Let Ặ and Ƀ are two FS of α and β respectively, $and\ Q -$ fuzzy subset (FSb) of a set τ , then

$(i)\ g^{-1}(Ƀ^{\omega}) = (g^{-1}(Ƀ))^{\omega}$

$(ii)\ g(Ặ^{\omega}) = (g(Ặ))^{\omega}, \forall\, \omega \in [0,1]\ and\ q \in Q$.

Proof:

Let $g: \alpha \to \beta$ be a mapping.

Let Ặ and Ƀ are two FS of α and β respectively, $and\ Q -$ fuzzy subset (FSb) of a set τ.

$$(i) g^{-1}(Ƀ^{\omega})(\varphi, q) = Ƀ^{\omega}(g(\varphi, q))$$

$$= \{Ƀ(g(\varphi, q)) \wedge \omega\}$$

$$= \{g^{-1}(Ƀ)(\varphi, q) \wedge \omega\}$$

$$= (g^{-1}(Ƀ))^{\omega}\ (\varphi, q)$$

$$\Rightarrow g^{-1}(Ƀ^{\omega})(\varphi, q) = (g^{-1}(Ƀ))^{\omega}$$

$$(ii)\ g(Ặ^{\omega})(\theta, q) = Sup\{Ặ^{\omega}(\varphi, q): g(\varphi, q) = (\theta, q)\}$$

$$= sup(\{Ặ(\varphi, q) \wedge \omega\}: g(\varphi, q) = (\theta, q))$$

$$= \left(Sup\{Ặ(\varphi,q): g(\varphi,q) = (\theta,q)\}\wedge\omega\right)$$
$$= \{g(Ặ)(\theta,q)\wedge\omega\}$$
$$= \left(g(Ặ)\right)^{\omega}(\theta,q)$$
$$\Rightarrow g(Ặ^{\omega}) = \left(g(Ặ)\right)^{\omega}. \forall\, \omega \in [0,1]\ and\ q \in Q.$$

Theorem: 3.8

Let Ặ is FSR of a ring τ, $and\ \ Q-$ fuzzy subset (FSb) of a set τ , then Ặ is also $\omega - Q$ −FSR of τ.

Proof:

Let $\theta,\varphi \in \tau$ be any element of the ring τ, $and\ \ Q-$ fuzzy subset (FSb) of a set τ.

Now,

$$(i)\ Ặ^{\omega}\left((\theta-\varphi),q\right) = \{Ặ^{\omega}((\theta-\varphi),q)\wedge\omega\}$$
$$\geq \left(\{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}\wedge\omega\right)$$
$$= \left(\{Ặ^{\omega}(\theta,q)\wedge\omega\}\wedge\{Ặ^{\omega}(\varphi,q)\wedge\omega\}\right)$$
$$= \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}$$
$$\RightarrowẶ^{\omega}\left((\theta-\varphi),q\right) \geq \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}.$$
$$(ii)\ Ặ^{\omega}(\theta\varphi,q) = \{Ặ(\theta\varphi,q)\wedge\omega\}$$
$$\geq \left(\{Ặ(\theta,q)\wedgeẶ(\varphi,q)\}\wedge\omega\right)$$
$$= (\{Ặ(\theta,q)\wedge\omega\}\wedge\{Ặ(\varphi,q)\wedge\omega\})$$
$$= \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}$$
$$\RightarrowẶ^{\omega}(\theta\varphi,q) \geq \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}, \forall\, \omega \in [0,1]\ and\ q \in Q.$$

Therefore, Ặ is $\omega - Q$ −FSR of τ.

Proposition: 3.9

The Converse of above theorem (3.8) need not be a true.

Example: 3.9.1

Let us consider the ring $(Z_5, +_5, \times_5)$, where $Z_5 = \{0,1,2,3,4,5\}$.

Define the Q −fuzzy set Ặ of Z_5 by

$$Ặ(\theta,q) = \begin{cases} 0.7;\ \ if\ x = 0 \\ 0.5;\ if\ x = 1,3 \\ 0.2;\ if\ x = 2,4 \end{cases}$$

It is easy to verify that Ặ is not $\omega - Q$ −FSR of Z_5.

However, if we take $\omega = 0.1$, then $Ặ^{\omega}(\theta,q) = 0.1, \forall\, \theta \in Z_5$.

Now, it can be easily proved that $Ặ^{\omega}$ is FSR of Z_5 and hence Ặ is $\omega - Q$ −FSR of Z_5.

Lemma: 3.10

Let Ặ be a FS of the ring τ. $and\ \ Q-$ fuzzy subset (FSb) of a set τ . Let $\omega \leq L$, where $L = \{Ặ(\theta,q): \forall\, \theta \in \tau\ and\ q \in Q\}$. Then Ặ is $\omega - Q$ −FSR of τ.

Proof:

Let Ặ be a FS of the ring τ , $Q-$ fuzzy subset (FSb) of a set τ and $\omega \leq L$

Since $\omega \leq L \Rightarrow L \geq \omega$

Implies that $\{Ặ(\theta,q): \forall\, \theta \in \tau\ and\ q \in Q\} \geq \omega$

$$\RightarrowẶ(\theta,q) \geq \omega, \forall\, \theta \in \tau\ and\ q \in Q.$$

$\therefore Ặ^{\omega}((\theta-\varphi),q) \geq \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}$ and $Ặ^{\omega}(\theta\varphi,q) \geq \{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}$,

Hence Ặ is $\omega - Q$ −FSR of τ.

Theorem: 3.11

Let the intersection of two $\omega - Q$ −FSR's of a ring τ $and\ \ Q-$ fuzzy subset (FSb) of a set τ is also ω −FSR of τ.

Proof:

Let $\theta,\varphi \in \tau$ be any element of the ring τ and $q \in Q$.

Then,

$$(i)\left(Ặ\cap Ƀ\right)^{\omega}\left((\theta-\varphi),q\right) = \{\left(Ặ\cap Ƀ\right)\left((\theta-\varphi),q\right)\wedge\omega\}$$
$$= \left(\{Ặ((\theta-\varphi),q)\wedge Ƀ((\theta-\varphi),q)\}\wedge\omega\right)$$
$$= \left(\{Ặ((\theta-\varphi),q)\wedge\omega\}\wedge\{Ƀ((\theta-\varphi),q)\wedge\omega\}\right)$$
$$= \{Ặ^{\omega}((\theta-\varphi),q)\wedge Ƀ^{\omega}((\theta-\varphi),q)\}$$
$$\geq \left(\{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}\wedge\{Ƀ^{\omega}(\theta,q)\wedge Ƀ^{\omega}(\varphi,q)\}\right)$$
$$= \left(\{Ặ^{\omega}(\theta,q)\wedge Ƀ^{\omega}(\theta,q)\}\wedge\{Ặ^{\omega}(\varphi,q)\wedge Ƀ^{\omega}(\varphi,q)\}\right)$$
$$= \{\left(Ặ^{\omega}\cap Ƀ^{\omega}\right)(\theta,q)\wedge\left(Ặ^{\omega}\cap Ƀ^{\omega}\right)(\varphi,q)\}$$
$$= \{\left(Ặ\cap Ƀ\right)^{\omega}(\theta,q)\wedge\left(Ặ\cap Ƀ\right)^{\omega}(\varphi,q)\}$$
$$\Rightarrow\left(Ặ\cap Ƀ\right)^{\omega}\left((\theta-\varphi),q\right) \geq \{\left(Ặ\cap Ƀ\right)^{\omega}(\theta,q)\wedge\left(Ặ\cap Ƀ\right)^{\omega}(\varphi,q)\}.$$
$$(ii)\left(Ặ\cap Ƀ\right)^{\omega}(\theta\varphi,q) = \{(Ặ\cap Ƀ)(\theta\varphi,q)\wedge\omega\}$$
$$= \left(\{Ặ(\theta\varphi,q)\wedge Ƀ(\theta\varphi,q)\}\wedge\omega\right)$$
$$= \left(\{Ặ(\theta\varphi,q)\wedge\omega\}\wedge\{Ƀ(\theta\varphi,q)\wedge\omega\}\right)$$
$$= \{Ặ^{\omega}(\theta\varphi,q)\wedge Ƀ^{\omega}(\theta\varphi,q)\}$$
$$\geq \left(\{Ặ^{\omega}(\theta,q)\wedgeẶ^{\omega}(\varphi,q)\}\wedge\{Ƀ^{\omega}(\theta,q)\wedge Ƀ^{\omega}(\varphi,q)\}\right)$$
$$= \left(\{Ặ^{\omega}(\theta,q)\wedge Ƀ^{\omega}(\theta,q)\}\wedge\{Ặ^{\omega}(\varphi,q)\wedge Ƀ^{\omega}(\varphi,q)\}\right)$$
$$= \{(Ặ^{\omega}\cap Ƀ^{\omega})(\theta,q)\wedge\left(Ặ^{\omega}\cap Ƀ^{\omega}\right)(\varphi,q)\}$$
$$= \{\left(Ặ\cap Ƀ\right)^{\omega}(\theta,q)\cap\left(Ặ\cap Ƀ\right)^{\omega}(\varphi,q)\}$$
$$\Rightarrow\left(Ặ\cap Ƀ\right)^{\omega}(\theta\varphi,q) \geq \{\left(Ặ\cap Ƀ\right)^{\omega}(\theta,q)\cap\left(Ặ\cap Ƀ\right)^{\omega}(\varphi,q)\}, \forall\, \theta,\varphi \in \tau\ and\ q \in Q.$$

Therefore $Ặ\cap Ƀ$ is $\omega - Q$ −FSR of τ.

Theorem: 3.12

Let Ặ be FNSR of a ring τ $and\ \ Q-$ fuzzy subset (FSb) of a set τ . Then Ặ is also $\omega - Q$ −FNSR of τ.

Proof:

Let $\theta,\varphi \in \tau$ be any element of the ring τ $and\ \ Q-$ fuzzy subset (FSb) of a set τ.

Then $Ặ^{\omega}(\theta\varphi,q) = \{Ặ(\theta\varphi,q)\wedge\omega\}$

$$= \{Ặ(\varphi\theta,q)\wedge\omega\}$$

$= Ặ^{\omega}(\varphi\theta,q), \forall\, \theta,\varphi \in \tau\ and\ q \in Q.$

Therefore Ặ is $\omega - Q$ −FNSR of τ.

Lemma: 3.13

Let Ặ is FLI of a ring τ, $and\ \ Q-$ fuzzy subset (FSb) of a set τ , then Ặ is also $\omega-Q$ −FLI of τ.

Proof:

In this theorem(3.12), we need only to prove that

$Ặ^{\omega}(\theta\varphi,q)\geq Ặ^{\omega}(\varphi,q),\ \forall\,\theta,\varphi\in\tau\ and\ q\in Q.$

$$Ặ^{\omega}(\theta\varphi,q)=\{Ặ(\theta\varphi,q),\omega\}$$
$$\geq\{Ặ(\varphi,q)\wedge\omega\}$$
$$=Ặ^{\omega}(\varphi,q)$$

Implies that $Ặ^{\omega}(\theta\varphi,q)\geq Ặ^{\omega}(\varphi,q),\forall\,\theta,\varphi\in\tau\ and\ q\in Q.$

∴ Ặ is $\omega-Q$ −FLI of τ.

Lemma: 3.14

Let Ặ is FRI of a ring τ, $and\ \ Q-$ fuzzy subset (FSb) of a set τ then Ặ is also $\omega-Q$ −FRI of τ.

Proof:

In this theorem(3.14), we need only to prove that

$Ặ^{\omega}(\theta\varphi,q)\geq Ặ^{\omega}(\theta,q),\ \forall\,\theta,\varphi\in\tau\ and\ q\in Q.$

$$Ặ^{\omega}(\theta\varphi,q)=\{Ặ(\theta\varphi,q),\omega\}$$
$$\geq\{Ặ(\theta,q)\wedge\omega\}$$
$$=Ặ^{\omega}(\theta,q)$$

Implies that $Ặ^{\omega}(\theta\varphi,q)\geq Ặ^{\omega}(\theta,q),\forall\,\theta,\varphi\in\tau\ and\ q\in Q.$

∴ Ặ is $\omega-\ Q$ −FRI of τ.

Theorem: 3.15

Let Ặ is FI of a ring τ, $and\ \ Q-$ fuzzy subset (FSb) of a set τ then Ặ is also $\omega-Q$ −FI of τ.

Proof:

Follows from Lemma (3.13) and Lemma (3.14)

IV. Algebraic Structures on Homomorphism of ω −Fuzzy Subrings, Normal Subrings and Ideals

Theorem: 4.1

Let $g:\tau_1\rightarrow\tau_2$ be a ring homomorphism from the ring τ_1 into a ring τ_2, $and\ \ Q-$ fuzzy subset (FSb) of a set τ .Let $\mathcal{B}$ be $\omega-Q$ −FSR of τ_2. Then $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FSR of τ_1.

Proof:

Let $\mathcal{B}$ $\omega-Q$ −FSR of τ_2.

Let $\theta_1,\theta_2\in\tau_1$ be any element $and\ \ Q-$ fuzzy subset (FSb) of a set τ.

Then

(i) $g^{-1}(\mathcal{B}^{\omega})\big((\theta_1-\theta_2),q\big)=\mathcal{B}^{\omega}\big(g((\theta_1-\theta_2),q)\big)$

$$=\mathcal{B}^{\omega}\big(g(\theta_1,q)-g(\theta_2,q)\big)$$
$$\geq\{\mathcal{B}^{\omega}(g(\theta_1,q))\wedge\mathcal{B}^{\omega}(g(\theta_2,q))\}$$
$$=\{g^{-1}(\mathcal{B}^{\omega})(\theta_1,q)\wedge g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)\}$$
$$\Rightarrow g^{-1}(\mathcal{B}^{\omega})((\theta_1-\theta_2),q)\geq\{g^{-1}(\mathcal{B}^{\omega})(\theta_1,q)\wedge g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)\}.$$

(ii) $g^{-1}(\theta_1\theta_2,q)=\mathcal{B}^{\omega}\big(g(\theta_1\theta_2,q)\big)$

$$=\mathcal{B}^{\omega}\big(g(\theta_1,q)-g(\theta_2,q)\big)$$
$$\geq\{\mathcal{B}^{\omega}\big(g(\theta_1,q)\big)\wedge\mathcal{B}^{\omega}\big(g(\theta_2,q)\big)\}$$
$$=\{g^{-1}(\mathcal{B}^{\omega})(\theta_1,q)\wedge g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)\}$$
$$\Rightarrow g^{-1}(\theta_1\theta_2,q)\geq\{g^{-1}(\mathcal{B}^{\omega})(\theta_1,q)\wedge g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)\},\ \forall\,\theta,\varphi\in\tau\ and\ q\in Q.$$

Therefore $g^{-1}(\mathcal{B}^{\omega})=(g^{-1}(\mathcal{B}))^{\omega}$ is FSR of τ_1 and hence $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FSR of τ_1.

Lemma: 4.2

A function $g:\tau_1\rightarrow\tau_2$ be a ring homomorphism from the ring τ_1 into a ring τ_2, $and\ \ Q-$ fuzzy subset (FSb) of a set τ . Let $\mathcal{B}$ be $\omega-Q$ −FNSR of τ_2. Then $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FNSR of τ_1.

Proof:

Let $\mathcal{B}$ be $\omega-Q$ −FNSR of τ_2 , $\forall\,\theta_1,\theta_2\in\tau_1\ and\ q\in Q.$

$$g^{-1}(\mathcal{B}^{\omega})(\theta_1\theta_2,q)=\mathcal{B}^{\omega}\big(g(\theta_1\theta_2,q)\big)$$
$$=\mathcal{B}^{\omega}\big(g(\theta_1,q)g(\theta_2,q)\big)$$
$$=\mathcal{B}^{\omega}\big(g(\theta_2,q)g(\theta_1,q)\big)$$
$$=\mathcal{B}^{\omega}(g(\theta_1\theta_2,q))$$
$$=g^{-1}(\mathcal{B}^{\omega}(\theta_1\theta_2,q))$$

$\Rightarrow g^{-1}(\mathcal{B}^{\omega})=(g^{-1}(\mathcal{B}))^{\omega}$ is FNSR of τ_1 and hence $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FNSR of τ_1.

Theorem: 4.3

Let $g:\tau_1\rightarrow\tau_2$ be a ring homomorphism from the ring τ_1 into a ring τ_2, $and\ \ Q-$ fuzzy subset (FSb) of a set τ. Let $\mathcal{B}$ be $\omega-Q$ −FLI of τ_2. Then $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FLI of τ_1.

Proof:

Let $\mathcal{B}$ be $\omega-Q$ −FLI of τ_2, $\forall\,\theta_1,\theta_2\in\tau_1\ and\ q\in Q.$

Then, in view of the theorem(4.1),

We have only prove that

$$g^{-1}(\mathcal{B}^{\omega})(\theta_1\theta_2,q)\geq g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)$$

Now,

$$g^{-1}(\mathcal{B}^{\omega})(\theta_1\theta_2,q)=\mathcal{B}^{\omega}(g(\theta_1\theta_2,q))$$
$$=\mathcal{B}^{\omega}\big(g(\theta_1,q)g(\theta_2,q)\big)$$
$$\geq\mathcal{B}^{\omega}(g(\theta_2,q))$$
$$=g^{-1}(\mathcal{B}^{\omega})(\theta_2,q)$$

Thus implies that $g^{-1}(\mathcal{B}^{\omega})(\theta_1\theta_2,q)\geq g^{-1}(\mathcal{B}^{\omega})(\theta_2,q),\forall\ \theta_1,\theta_2\in\tau_1\ and\ q\in Q.$

Thus implies also $g^{-1}(\mathcal{B}^{\omega})=(g^{-1}(\mathcal{B}))^{\omega}$ is FLI of τ_1 and hence $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FLI of τ_1.

Theorem: 4.4

Let $g:\tau_1\rightarrow\tau_2$ be a ring homomorphism from the ring τ_1 into a ring τ_2, $and\ \ Q-$ fuzzy subset (FSb) of a set τ. Let $\mathcal{B}$ be $\omega-Q$ −FRI of τ_2. Then $g^{-1}(\mathcal{B})$ is $\omega-Q$ −FRI of τ_1.

Proof:

It can be easily to prove that, can be obtained similar to theorem(4.3)

Theorem: 4.5

Let $g:\tau_1 \rightarrow \tau_2$ be a ring homomorphism from the ring τ_1 into a ring $\tau_2, and\ \ Q-$ fuzzy subset (FSb) of a set τ. Let $\mathcal{B}$ be $\omega - Q$ −FI of τ_2. Then $g^{-1}(\mathcal{B})$ is $\omega - Q$ −FI of τ_1.

Proof:

It can be easily to follows from the above theorem (4.3) and Theorem (4.4)

Theorem: 4.6

Let $g:\tau_1 \rightarrow \tau_2$ be Surjective ring homomorphism and Ặ be $\omega - Q$ −FSR of $\tau_1, and\ \ Q-$ fuzzy subset (FSb) of a set τ. Then g(Ặ) is $\omega - Q$ −FSR of τ_2.

Proof:

Let Ặ be $\omega - Q - Q$ −FSR of τ_1.

Let $\varphi_1, \varphi_2 \in \tau_2$ be any element, $and\ \ Q-$ fuzzy subset (FSb) of a set τ. Then there exist some $\theta_1, \theta_2 \in \tau_1\ and\ q \in Q$ such that $g(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$. (Since that θ_1, θ_2 need not be unique)

$$(i) g(\text{Ặ}^{\omega})\big((\varphi_1 - \varphi_2), q\big) = \big(g(\text{Ặ})\big)\big((\varphi_1 - \varphi_2), q\big)$$

$$= \big\{g(\text{Ặ})(g(\theta_1, q) - g(\theta_2, q)) \wedge \omega\big\}$$

$$= \big\{g(\text{Ặ})(g((\theta_1 - \theta_2), q)) \wedge \omega\big\}$$

$$\geq \big\{\text{Ặ}\big((\theta_1 - \theta_{2),q}\big) \wedge \omega\big\}$$

$$= \text{Ặ}^{\omega}\big((\theta_1 - \theta_2), q\big)$$

$$\geq \big\{\text{Ặ}^{\omega}(\theta_1, q) \wedge \text{Ặ}^{\omega}(\theta_2, q)\big\},$$

For all $\theta_1, \theta_2 \in \tau_1\ and\ q \in Q$ such that $g(\theta_1, q) = (\varphi_1, q)$ and $g(\theta_2, q) = (\varphi_2, q)$

$$= \Big\{\big\{\text{Ặ}^{\omega}(\theta_1, q): g(\theta_1, q) = (\varphi_1, q)\big\} \wedge \big\{\text{Ặ}^{\omega}(\theta_2, q): g(\theta_2, q) = (\varphi_2, q)\big\}\Big\}$$

$$= \big\{g(\text{Ặ}^{\omega})(\varphi_1, q) \wedge g(\text{Ặ}^{\omega})(\varphi_2, q)\big\}$$

Thus implies that $g(\text{Ặ}^{\omega})((\varphi_1 - \varphi_2), q) \geq \big\{g(\text{Ặ}^{\omega})(\varphi_1, q) \wedge g(\text{Ặ}^{\omega})(\varphi_2, q)\big\}$.

$$(ii)\ g(\text{Ặ}^{\omega})(\varphi_1\varphi_2, q) = \big(g(\text{Ặ})\big)^{\omega}(\varphi_1\varphi_2, q)$$

$$= \big\{g(\text{Ặ})\big(g(\theta_1, q) g(\theta_2, q)\big) \wedge \omega\big\}$$

$$= \big\{g(\text{Ặ})(g(\theta_1\theta_2, q)) \wedge \omega\big\}$$

$$\geq \big\{\text{Ặ}(\theta_1\theta_2, q) \wedge \omega\big\}$$

$$= \text{Ặ}^{\omega}(\theta_1\theta_2, q)$$

$$\geq \big\{\text{Ặ}^{\omega}(\theta_1, q) \wedge \text{Ặ}^{\omega}(\theta_2, q)\big\}$$

For all $\theta_1, \theta_2 \in \tau_1\ and\ q \in Q$ such that $g(\theta_1, q) = (\varphi_1, q)$ and $g(\theta_2, q) = (\varphi_2, q)$

$$= \Big\{\big\{\text{Ặ}^{\omega}(\theta_1, q): g(\theta_1, q) = (\varphi_1, q)\big\} \wedge \big\{\text{Ặ}^{\omega}(\theta_2, q): g(\theta_2, q) = (\varphi_2, q)\big\}\Big\}$$

$$= \big\{g(\text{Ặ}^{\omega})(\varphi_1, q) \wedge g(\text{Ặ}^{\omega})(\varphi_2, q)\big\}$$

Thus implies that $g(\text{Ặ}^{\omega})(\varphi_1\varphi_2, q) \geq \big\{g(\text{Ặ}^{\omega})(\varphi_1, q) \wedge g(\text{Ặ}^{\omega})(\varphi_2, q)\big\}$.

Thus $g(\text{Ặ}^{\omega}) = \big(g(\text{Ặ})\big)^{\omega}$ is FSR of τ_2 and hence g(Ặ) is $\omega - Q$ −FSR of τ_2.

Theorem: 4.7

Let $g:\tau_1 \rightarrow \tau_2$ be Surjective ring homomorphism and Ặ be ω −FNSR of $\tau_1, and\ \ Q-$ fuzzy subset (FSb) of a set τ . Then g(Ặ) is $\omega - Q$ −FNSR of τ_2.

Proof:

Let Ặ be $\omega - Q$ −FNSR of τ_1.

Let $\varphi_1, \varphi_2 \in \tau_2$ be any element, $and\ \ Q-$ fuzzy subset (FSb) of a set τ. Then there exist some $\theta_1, \theta_2 \in \tau_1\ and\ q \in Q$ such that $g(\theta_1) = \varphi_1$ and $g(\theta_2) = \varphi_2$. (Since that θ_1, θ_2 need not be unique)

In this view of theorem(4.6), we need only to prove that $(g(\text{Ặ}))^{\omega}(\varphi_1\varphi_2, q) = g(\text{Ặ}^{\omega})(\varphi_2\varphi_1, q)$

$$(g(\text{Ặ}))^{\omega}(\varphi_1\varphi_2, q) = g(\text{Ặ}^{\omega})(g(\theta_1, q) g(\theta_2, q))$$

$$= g(\text{Ặ}^{\omega})(g(\theta_1\theta_2, q))$$

$$= \big\{\text{Ặ}^{\omega}(\theta_1\theta_2, q): g(\theta_1\theta_2, q) = (\varphi_1\varphi_2, q)\big\}$$

$$= \big\{\text{Ặ}^{\omega}(\theta_2\theta_1, q): g(\theta_1\theta_2, q) = (\varphi_1\varphi_2, q)\big\}$$

$$= g(\text{Ặ}^{\omega})(g(\theta_2, q) g(\theta_1, q))$$

$$= \big(g(\text{Ặ})\big)^{\omega}(\varphi_2\varphi_1, q)$$

Thus implies that $\big(g(\text{Ặ})\big)^{\omega}$ is FNSR of τ_2 and hence g(Ặ) is $\omega - Q$ −FNSR of τ_2.

Theorem: 4.8

Let $g:\tau_1 \rightarrow \tau_2$ be bijective ring homomorphism and Ặ be $\omega - Q$ −FLI of $\tau_1, and\ \ Q-$ fuzzy subset (FSb) of a set τ. Then g(Ặ) is $\omega - Q$ −FLI of τ_2.

Proof:

Let Ặ be $\omega - Q$ −FLI of τ_1.

Let $\varphi_1, \varphi_2 \in \tau_2$ be any element, $and\ \ Q-$ fuzzy subset (FSb) of a set τ. Then there exist some $\theta_1, \theta_2 \in \tau_1$ such that $g(\theta_1, q) = (\varphi_1, q)$ and $g(\theta_2, q) = (\varphi_2, q)$.

In this view of theorem(4.6), we need only to prove that $(g(\text{Ặ}))^{\omega}(\varphi_1\varphi_2, q) \geq (g(\text{Ặ}))^{\omega}(\varphi_2, q)$

$$(g(\text{Ặ}))^{\omega}(\varphi_1\varphi_2, q) = \big\{g(\text{Ặ})\big(g(\theta_1, q) g(\theta_2, q)\big) \wedge \omega\big\}$$

$$= \big\{g(\text{Ặ})(g(\theta_1\theta_2, q)) \wedge \omega\big\}$$

$$= \big\{\text{Ặ}(\theta_1\theta_2, q) \wedge \omega\big\}$$

$$= \text{Ặ}^{\omega}(\theta_1\theta_2, q)$$

$$\geq \text{Ặ}^{\omega}(\theta_2, q)$$

$$= \big\{\text{Ặ}^{\omega}(\theta_2, q) \wedge \omega\big\}$$

$$= \big\{g(\text{Ặ})(g(\theta_2, q)) \wedge \omega\big\}$$

$$= \big\{g(\text{Ặ})(\varphi_2, q) \wedge \omega\big\}$$

$$= (g(\text{Ặ}))^{\omega}(\varphi_2, q)$$

Thus implies that $(g(\text{Ặ}))^{\omega}(\varphi_1\varphi_2, q) \geq (g(\text{Ặ}))^{\omega}(\varphi_2, q)$

Hence $(g(\text{Ặ}))^{\omega}$ is FLI of τ_2 and hence g(Ặ) is $\omega - Q$ −FLI of τ_2.

Theorem: 4.9

Let $g: \tau_1 \rightarrow \tau_2$ be bijective ring homomorphism and Ặ be $\omega - Q$ −FRI of $\tau_1, and\ Q -$ fuzzy subset (FSb) of a set τ. Then g(Ặ) is $\omega - Q$ −FRI of τ_2.

Proof:

In this view of prof it can be obtained similar to theorem(4.8)

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Conflict of interest

All authors declare no conflict of interest in this paper.

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