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SOME NEW LABELING ON CYCLE (C_n) WITH ZIGZAG CHORDS

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Abstract

The main aim of this paper is to discuss or identify the labeling on the graph which is related to cycle graph (C_n), where cycle with *n* length and the chord is formed by the two non-adjacent vertices connected by an edge

In this article, we prove the graph (G) cycle with zigzag chords is vertex even mean graph, vertex odd mean graph, square sum graph, square difference graph with appropriate illustrations. Also, we prove that the graph, cycle with zigzag chords admits cube sum labeling, cube difference labeling, strongly multiplicative labeling and strongly *labeling with related examples.

Keywords: Graph labeling, cycle with zigzag chords, cube sum, cube difference, vertex even mean, vertex odd mean, square sum, square difference, strongly multiplicative, strongly* graph

INTRODUCTION

Labelling is an effective area of research in graph theory and also serve as useful models for a broad range of applications in coding theory, communication network, channel assignment problems and soon on. If the vertices and edges or both assigned by the integers with subject to certain constrains is mean to be graph labeling. A dynamic survey of graph labeling can be seen in Electronic Journal of Combinatorics by J.A.Gallian[1]. This paper deals with graph which is finite undirected graph. Rosa[2] is defined β-valuations later, by Golomb[3] it was renamed as graceful. S.Somasundram and R.Ponarj[4] introduced mean labeling. N.Revathi[5] introduced vertex even mean and vertex odd mean labeling some results can be seen in [20][21][23][25][26][27]. V. Ajitha et.al[6] introduced square sum labeling and some results can be seen in[22]. J.Shiama[7] introduce square difference labeling some results can be seen in[8][19]. Shiama[9] introduce cube difference labeling. K.Srinivasan et .al[10] introduced the cube sum labeling. Beineke and Hegde[11] started strongly multiplicative labeling some results can be seen in[24]. Adiga and somashekara[12] introduce the strongly* graph and some graphs are proved in[13][14]. Mathew Varkey T.K et.al[15] introduce absolute difference of cube sum and square sum (ADCSS) labeling and many families admits can be seen in[16]. Elumalai and Anand Ephremnath[17] proved cycle with zigzag chord are graceful and in[18] they proved cycle with chord Hamiltonian path is harmonious and elegant.

So far there are vast amount of literature is available on different of graphs labeling and more research papers have been published in past five decades.

In this paper, we prove that every cycle$C_n (n \geq 8)$ with zigzag chord admits some new labelings were vertex even mean and vertex odd mean, square sum and square difference labeling, Cube sum and cube difference labeling, strongly multiplicative and strongly* graph and also admits ADCSS-labeling.

Definition 1.1: [17]

Cycle with zigzag chords is consider as a graph G, it acquired from cycle $C_n (n \geq 8)$: $u_0, u_1, u_2, \ldots, u_{n-1} u_0$ be the vertices and by joining the vertex of cycle $u_1 u_{n-1}, u_{n-1} u_3, \ldots, u_\alpha u_\beta$ arrives zigzag chords.

If (i) $\alpha = \frac{n-2}{2}$ and $\beta = \frac{n+2}{2}$ if $n \equiv 0 \ (mod\ 4)$, (ii) $\alpha = \frac{n-3}{2}$ and $\beta = \frac{n+3}{2}$ if $n \equiv 1 \ (mod\ 4)$, (iii) $\alpha = \frac{n+4}{2}$ and $\beta = \frac{n}{2}$ if $n \equiv 2 \ (mod\ 4)$, (iv) $\alpha = \frac{n+5}{2}$ and $\beta = \frac{n-1}{2}$ if $n \equiv 3 \ (mod\ 4)$.

G has M number of edges, $\frac{3n-2}{2}$ if n is even $\frac{3n-3}{2}$ if n is odd.

The generalized graph G, cycle with zigzag chords is given below shown in Fig.1(a) and (b)

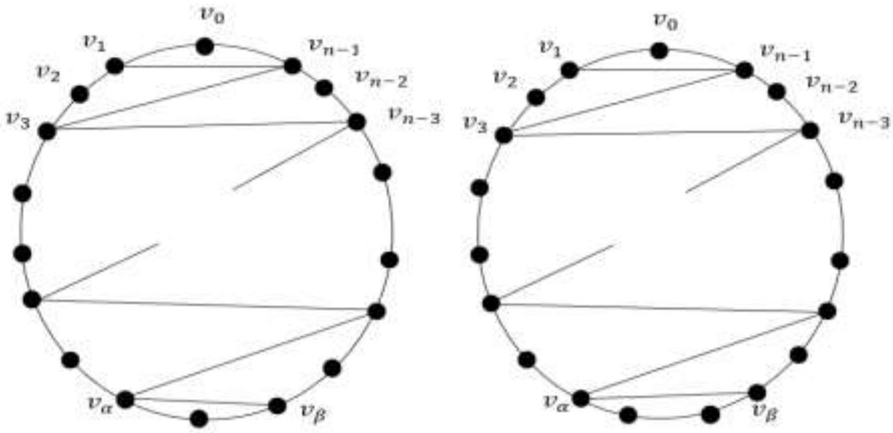

Fig.1 (a) if n is even Fig.1 (b) if n is odd

Definition 1.2: [5]
A graph is known to be vertex even mean graph, if there exist injection $f: V(G) \to \{2,4,\ldots 2q\}$ such that the induced mapping $f^*: E(G) \to I\{Set\ of\ positive\ intergers\}$ is given by $f^*(xy) = \frac{f(x)+f(y)}{2}$ are distinct.

Definition 1.3: [5]
A graph is known to be vertex odd mean graph, if there exist injection $f: V(G) \to \{1,3,\ldots 2q-1\}$ such that the induced mapping $f^*: E(G) \to I\{Set\ of\ positive\ intergers\}$ is given by $f^*(xy) = \frac{f(x)+f(y)}{2}$ are distinct.

Definition 1.4: [6]
A graph is known to be square sum graph, if there exist bijective $f: V(G) \to \{0, 1, 2 \ldots, p-1\}$ such that the induced mapping $f^*: E(G) \to I\{Set\ of\ positive\ intergers\}$ is given by $f^*(xy) = [f(x)]^2 + [f(y)]^2$ are injective and distinct.

Definition 1.5: [8]
A graph is known to be square difference graph, if there exist bijective $f: V(G) \to \{0, 1, 2 \ldots, p-1\}$ such that the induced mapping $f^*: E(G) \to Z$ is given by $f^*(xy) = |[f(x)]^2 - [f(y)]^2|$ and distinct.

Definition 1.6:[10]
A graph is known to be cube sum graph, if there exist bijective $f: V(G) \to \{0, 1, 2 \ldots, p-1\}$ such that the induced mapping $f^*: E(G) \to Z$ is given by $f^*(xy) = [f(x)]^3 + [f(y)]^3$ are distinct.

Definition 1.7: [9]
A graph is known to be cube difference graph, if there exist bijective $f: V(G) \to \{0, 1, 2 \ldots, p-1\}$ such that the induced mapping $f^*: E(G) \to Z$ is given by $f^*(xy) = |[f(x)]^3 - [f(y)]^3|$ are distinct.

Definition 1.8: [15]
A graph is known to be Absolute difference of cubic and square sum (ADCSS) graph, if there exist bijective $f: V(G) \to \{1, 2 \ldots, p\}$ such that the induced mapping $f^*: E(G) \to 2Z$ is given by $f^*(xy) = |[f(x)]^3 + [f(y)]^3 - ([f(x)]^2 + [f(y)]^2)|$ are injective and distinct.

Definition 1.9: [9]
A graph is known to be strongly multiplicative graph, if there exist bijective $f: V(G) \to \{1, 2, 3, \ldots, p\}$ such that the induced mapping $f^*: E(G) \to N$ is given by $f^*(xy) = f(x)f(y)$ are distinct.

Definition 1.10:[12]
A graph is known to be strongly*graph, if there exist bijective $f: V(G) \to \{1, 2, 3, \ldots, p\}$ such that the induced mapping $f^*: E(G) \to N$ is given by $f^*(xy) = f(x) + f(y) + f(x)f(y)$ are distinct.

MAIN RESULTS

Theorem 1: *For $n \geq 8$, all cycle C_n with zigzag chords is vertex even mean graph*

Proof: Let us imagine the graph, C_n cycle has the vertex $V = \{v_k; 1 \leq k \leq n\}$. In cycle C_n the set of vertices$\{v_2v_3, v_3v_6, v_6v_7, v_7v_{10}, \ldots, v_\gamma v_\delta\}$ for (i) $\gamma = n-2\ and\ \delta = n-1\ if\ n \equiv 0(mod4)$, (ii) $\gamma = n-3\ and\ \delta = n-2\ if\ n \equiv 1(mod4)$, (iii) $\gamma = n-3\ and\ \delta = n\ if\ n \equiv 3(mod4)$, (iv) $\gamma = n-4\ and\ \delta = n-1\ if\ n \equiv 4(mod4)$ forms the zigzag chords.

The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \to \{2,4,6, \ldots, 2n\}$ is stated as below,

$f(v_k) = 2k; 1 \leq k \leq n$

We can notice that all vertices are labeled with separate values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_6$

$E_1 = \{g(v_1v_2)\}$, $E_2 = \left\{g\left(v_{2k}v_{2(k+1)}\right), 1 \leq k \leq \left\lfloor\frac{n-2}{2}\right\rfloor\right\}$

$E_3 = \left\{g(v_{2k-1}v_{2k+1}), 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor\right\}$, $E_4 = \left\{g(v_{4k-2}v_{4k-1}), 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor\right\}$

$E_5 = \left\{g(v_{4k-1}v_{4k+2}), 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor\right\}$, $E_6 = \{g(v_{n-1}v_n)\}$

Labeling the induced function of edges $g: E(G) \rightarrow I\ (set\ of\ positive\ integers)$

$g(v_1v_2) = 3$

$g(v_{2k}v_{2(k+1)}) = 4k + 2;\ 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor$

$g(v_{2k-1}v_{2k+1}) = 4k; 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor$

$g(v_{4k-2}v_{4k-1}) = 8k - 3;\ 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor$

$g(v_{4k-1}v_{4k+2}) = 8k + 1; 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor$

$g(v_{n-1}v_n) = 2n - 1$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be vertex even mean graph shown in Fig.2

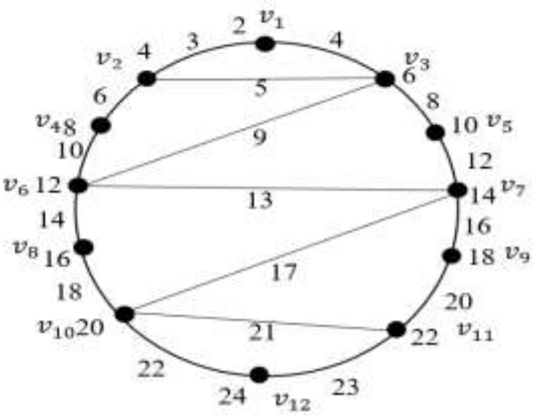

Fig.2 C_{12} with zigzag chords is vertex even mean graph

Theorem 2: *For $n \ge 8$, all cycle C_n with zigzag chords is vertex odd mean graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 1 \le k \le n\}$. The Zigzag chords in the cycle formed by the vertices $\{v_2v_3, v_3v_6, v_6v_7, v_7v_{10}, \dots, v_\gamma v_\delta\}$, (values of γ and δ are given stated in theorem.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \rightarrow \{1,3,5,\dots,2n-1\}$ is stated as below,

$f(v_k) = 2k - 1; 1 \le k \le n - 1$

We can notice that all vertices are labeled with separate values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \dots \cup E_6$

$E_1 = \{g(v_1v_2)\}$,

$E_2 = \left\{g(v_{2k}v_{2(k+1)}), 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor\right\}$

$E_3 = \left\{g(v_{2k-1}v_{2k+1}), 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor\right\}$

$E_4 = \left\{g(v_{4k-2}v_{4k-1}), 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor\right\}$

$E_5 = \left\{g(v_{4k-1}v_{4k+2}), 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor\right\}$

$E_6 = \{g(v_{n-1}v_n)\}$

Labeling the induced function of edges $g: E(G) \rightarrow I\ (set\ of\ positive\ integers)$

$g(v_1v_2) = 2$

$g(v_{2k}v_{2(k+1)}) = 4k + 1;\ 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor$

$g(v_{2k-1}v_{2k+1}) = 4k - 1; 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor$

$g(v_{4k-2}v_{4k-1}) = 4(2k - 1);\ 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor$

$g(v_{4k-1}v_{4k+2}) = 8k; 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor$

$g(v_{n-1}v_n) = 2(n - 1)$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be vertex odd mean graph shown in Fig.2

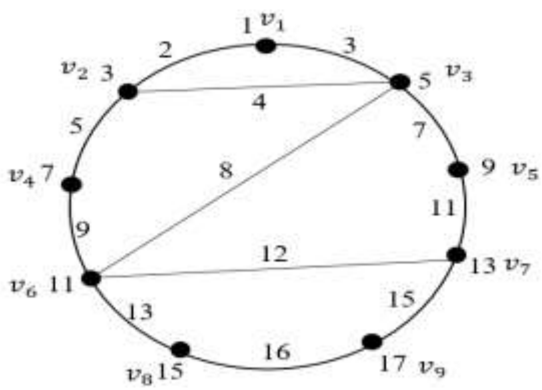

Fig.3 C_9 with zigzag chords vertex odd mean graph

Theorem 3: *For $n \ge 8$, all cycle C_n with zigzag chords is strongly multiplicative graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \le k \le n - 1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1v_{n-1}, v_{n-1}v_3, v_3v_{n-3}, \dots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \rightarrow \{1,2,3,\dots,n\}$ is stated as below,

$f(v_0) = 1$

$f(v_k) = 2k; 1 \le k \le \left\lfloor\frac{n}{2}\right\rfloor$

$f(v_{n-k}) = 2k + 1; 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \dots \cup E_7$

$E_1 = g(v_0 v_1)$ $\quad E_2 = g(v_0 v_{n-1})$

$E_3 = \left\{g(v_k v_{k+1}); 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor\right\}$

$E_4 = \{g(v_{n-k} v_{n-k-1}); 1 \le k \le \lfloor\frac{n-3}{2}\rfloor\}$,

$E_5 = \left\{g(v_{2k-1} v_{n-2k+1}); 1 \le k \le \left\lfloor\frac{n}{2}\right\rfloor\right\}$

$E_6 = \{g(v_{2k+1} v_{n-2k+1}); 1 \le k \le \lfloor\frac{n-2}{4}\rfloor\}$,

$$E_7 = \begin{Bmatrix} g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right), if\ n \equiv 0 (mod\ 2) \\ g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right), if\ n \equiv 1 (mod\ 2) \end{Bmatrix}$$

Labeling the induced function of edges $g: E(G) \to N$

$g(v_0 v_1) = 2,$

$g(v_0 v_{n-1}) = 3$

$g(v_k v_{k+1}) = 4k(k+1); 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor$

$g(v_{n-k} v_{n-k-1}) = 4k(k+2) + 3; 1 \le k \le \left\lfloor\frac{n-3}{2}\right\rfloor$

$g(v_{2k-1} v_{n-2k+1}) = 4k(4k-3) + 2; 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor$

$g(v_{2k+1} v_{n-2k+1}) = 4k(k+1) - 2; 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor$

$g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right) = n(n-1), if\ n \equiv 0 (mod\ 2)$

$g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right) = n(n-1), if\ n \equiv 1 (mod\ 2)$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be strongly multiplicative graph shown in Fig.4

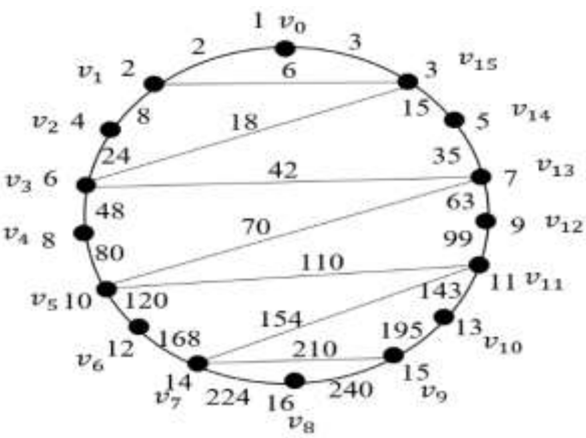

Fig.4 C_{16} with zigzag chords is a strongly multiplicative graph

Theorem 4: *For* $n \ge 8$, *all cycle* C_n *with zigzag chords is square sum graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \le k \le n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1 v_{n-1}, v_{n-1} v_3, v_3 v_{n-3}, \dots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \to \{0,1,2,3, \dots, n-1\}$ is stated as below

$f(v_0) = 0$

$f(v_k) = 2k - 1; 1 \le k \le \left\lfloor\frac{n}{2}\right\rfloor$

$f(v_{n-k}) = 2k; 1 \le k \le \left\lfloor\frac{n-1}{2}\right\rfloor$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \dots \cup E_7$

$E_1 = \{g(v_0 v_1)\}$

$E_2 = \{g(v_0 v_{n-1})\}$

$E_3 = \left\{g(v_k v_{k+1}); 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor\right\}$

$E_4 = \left\{g(v_{n-k} v_{n-k-1}); 1 \le k \le \left\lfloor\frac{n-3}{2}\right\rfloor\right\}$

$E_5 = \left\{g(v_{2k-1} v_{n-2k+1}); 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor\right\}$

$E_6 = \left\{g(v_{2k+1} v_{n-2k+1}); 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor\right\}$

$$E_7 = \begin{Bmatrix} g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right), if\ n \equiv 0 (mod\ 2) \\ g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right), if\ n \equiv 1 (mod\ 2) \end{Bmatrix}$$

Labeling the induced function of edges $g: E(G) \to I$

$g(v_0 v_1) = 1$

$g(v_0 v_{n-1}) = 4$

$g(v_k v_{k+1}) = 2(4k^2 + 1); 1 \le k \le \left\lfloor\frac{n-2}{2}\right\rfloor$

$g(v_{n-k} v_{n-k-1}) = 4k(2k+1) + 4; 1 \le k \le \left\lfloor\frac{n-3}{2}\right\rfloor$

$g(v_{2k-1} v_{n-2k+1}) = 8k(4k-5) + 13; 1 \le k \le \left\lfloor\frac{n}{4}\right\rfloor$

$g(v_{2k+1} v_{n-2k+1}) = 8k(4k-1) + 5; 1 \le k \le \left\lfloor\frac{n-2}{4}\right\rfloor$

$g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right) = 2n(n-3) + 5, if\ n \equiv 0 (mod\ 2)$

$g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right) = 2n(n-3) + 5, if\ n \equiv 1 (mod\ 2)$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be square sum graph shown in Fig.5

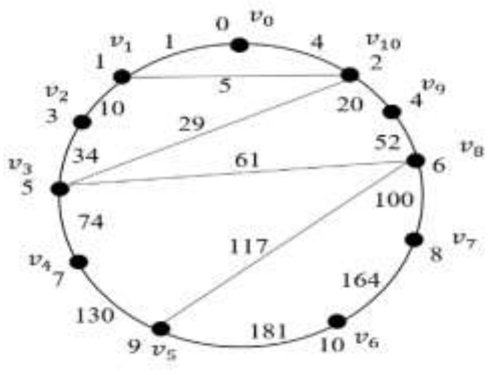

Fig.5 C_{11} with zigzag chords is a square sum graph

Theorem 5: *For $n \geq 8$, all cycle C_n with zigzag chords is square difference graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \leq k \leq n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1v_{n-1}, v_{n-1}v_3, v_3v_{n-3}, \ldots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \to \{0,1,2,3, \ldots., n-1\}$ is stated as below

$f(v_k) = k; 0 \leq k \leq n-1$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_4$

$E_1 = g(v_0v_{n-1})$

$E_2 = \{g(v_kv_{k+1});\ 0 \leq k \leq n-2\}$

$E_3 = \left\{g(v_{2k-1}v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor\right\}$

$E_4 = \left\{g(v_{2k+1}v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor\right\}$

Labeling the induced function of edges $g: E(G) \to Z$

$g(v_0v_{n-1}) = n(n-2)+1$

$g(v_kv_{k+1}) = 2k+1;\ 0 \leq k \leq n-2$

$g(v_{2k-1}v_{n-2k+1}) = 8k(k-1) + n(n-4k+2) + 2; 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor$

$g(v_{2k+1}v_{n-2k+1}) = n^2 - 4k(n+2) + 2n; 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be square Difference graph shown in Fig.6

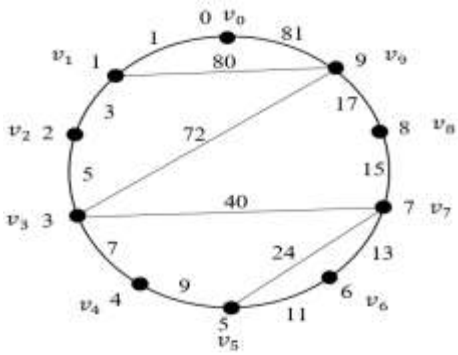

Fig.6 C_{10} with zigzag chords is a square difference graph

Theorem 7: *For $n \geq 8$, all cycle C_n with zigzag chords is cube sum graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \leq k \leq n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1v_{n-1}, v_{n-1}v_3, v_3v_{n-3}, \ldots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \to \{0,1,2,3, \ldots., n-1\}$ is stated as below

$f(v_k) = k; 0 \leq k \leq n-1$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_4$

$E_1 = \{g(v_0v_{n-1})\}$

$E_2 = \{g(v_kv_{k+1});\ 0 \leq k \leq n-2\}$

$E_3 = \left\{g(v_{2k-1}v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor\right\}$

$E_4 = \left\{g(v_{2k+1}v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor\right\}$

Labeling the induced function of edges $g: E(G) \to Z$

$g(v_0v_{n-1}) = (n-1)^3$

$g(v_kv_{k+1}) = 2k^3 + 3k(k+1) + 1;\ 0 \leq k \leq n-2$

$g(v_{2k-1}v_{n-2k+1}) = (2k-1)^3 + (n-2k+1)^3; 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor$

$g(v_{2k+1}v_{n-2k+1}) = (2k+1)^3 + (n-2k+1)^3; 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be cube sum graph shown in Fig.8

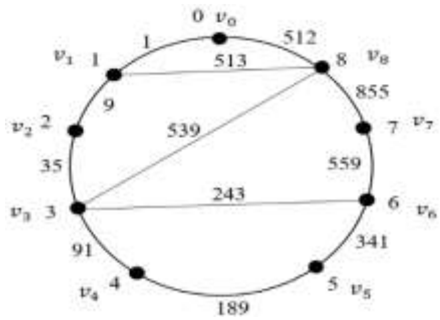

Fig.8 C_9 with zigzag chords is cube sum graph

Theorem 8: *For $n \geq 8$, all cycle C_n with zigzag chords is cube difference graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \leq k \leq n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1v_{n-1}, v_{n-1}v_3, v_3v_{n-3}, \ldots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as *f* and the edges are labeling as *g* respectively.

Labeling a vertex $f: V(G) \to \{0,1,2,3, \ldots., n-1\}$ is stated as below

$f(v_k) = k; 0 \leq k \leq n-1$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_4$

$E_1 = \{g(v_0 v_{n-1})\}$

$E_2 = \{g(v_k v_{k+1});\ 0 \leq k \leq n-2\}$

$E_3 = \left\{g(v_{2k-1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor\right\}$

$E_4 = \left\{g(v_{2k+1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor\right\}$

Labeling the induced function of edges $g: E(G) \rightarrow Z$

$g(v_0 v_{n-1}) = (n-1)^3$

$g(v_k v_{k+1}) = 3k(k+1) + 1;\ 0 \leq k \leq n-2$

$$g(v_{2k-1} v_{n-2k+1}) = (n-4k+2)[n(n-2k+1) + 4k(k-1) + 1]; 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$g(v_{2k+1} v_{n-2k+1}) = (n-4k)[2k(2k-n) + n(n+3) + 3]; 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor$$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be cube Difference graph shown in Fig.9

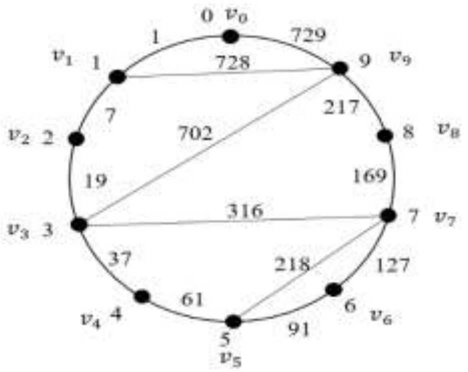

Fig.9 C_{10} with zigzag chords is a cube difference graph

Theorem 9: *For* $n \geq 8$, *all cycle* C_n *with zigzag chords is absolute difference of cube sum and square difference graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \leq k \leq n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1 v_{n-1}, v_{n-1} v_3, v_3 v_{n-3}, \ldots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as f and the edges are labeling as g respectively.

Labeling a vertex $f: V(G) \rightarrow \{1,2,3, \ldots., n\}$ is stated as below

$f(v_{k-1}) = k; 1 \leq k \leq n$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_4$

$E_1 = \{g(v_0 v_{n-1})\}$

$E_2 = \{g(v_{k-1} v_k);\ 1 \leq k \leq n\}$

$E_3 = \left\{g(v_{2k-1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor\right\}$

$E_4 = \left\{g(v_{2k+1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor\right\}$

Labeling the induced function of edges $g: E(G) \rightarrow$ $Multiples\ of\ 2\mathrm{Z}$

$g(v_0 v_{n-1}) = n^2(n-1)$

$g(v_{k-1} v_k) = k(2k^2 + k + 1);\ 1 \leq k \leq n$

$$g(v_{2k-1} v_{n-2k+2}) = 4k^2(2k-1) + (n-2k+2)^2(n-2k + 1); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor$$

$$g(v_{2k+1} v_{n-2k+2}) = (2k+2)^2(2k+1) + (n-2k+2)^2(n-2k+1); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor$$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be absolute Difference of css-graph shown in Fig.10

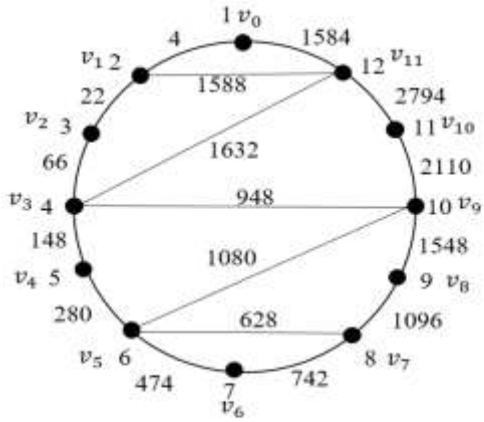

Fig.10 C_{12} with zigzag chords is Absolute difference of CSS-graph

Theorem 10: *For* $n \geq 8$, *all cycle* C_n *with zigzag chords is Strongly * graph*

Proof: Let us consider the cycle C_n with vertex $V = \{v_k; 0 \leq k \leq n-1\}$. The Zigzag chords in the cycle formed by the vertices $\{v_1 v_{n-1}, v_{n-1} v_3, v_3 v_{n-3}, \ldots, v_\alpha v_\beta\}$, α and β stated in Def.1. The vertices are labeling as f and the edges are labeling as g respectively.

Labeling a vertex $f: V(G) \rightarrow \{1,2,3, \ldots., n\}$ is stated as below

$f(v_0) = 1$

$f(v_k) = 2k; 1 \leq k \leq \left\lfloor \frac{n}{2} \right\rfloor$

$f(v_{n-k}) = 2k+1; 1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$

All vertices are independently labeled with individual values and they distinct.

The edge set E(G) is given by $E(G) = E_1 \cup E_2 \cup \cdots \cup E_7$

$E_1 = \{g(v_0 v_1)\}$

$E_2 = \{g(v_0 v_{n-1})\}$

$E_3 = \left\{g(v_k v_{k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{2} \right\rfloor\right\}$

$E_4 = \left\{g(v_{n-k} v_{n-k-1}); 1 \leq k \leq \left\lfloor \frac{n-3}{2} \right\rfloor\right\}$

$E_5 = \left\{g(v_{2k-1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor\right\}$

$E_6 = \left\{g(v_{2k+1} v_{n-2k+1}); 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor\right\}$

$$E_7 = \left\{\begin{array}{l} g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right), if\ n \equiv 0 (mod\ 2) \\ g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right), if\ n \equiv 1 (mod\ 2) \end{array}\right\}$$

Labeling the induced function of edges $g: E(G) \to N$

$g(v_0 v_1) = 5$

$g(v_0 v_{n-1}) = 7$

$g(v_k v_{k+1}) = 4k(k+2) + 2; 1 \leq k \leq \left\lfloor \frac{n-2}{2} \right\rfloor$

$g(v_{n-k} v_{n-k-1}) = 4k(k+3) + 7; 1 \leq k \leq \left\lfloor \frac{n-3}{2} \right\rfloor$

$g(v_{2k-1} v_{n-2k+1}) = 2k(2k+3) + 1; 1 \leq k \leq \left\lfloor \frac{n}{4} \right\rfloor$

$g(v_{2k+1} v_{n-2k+1}) = 4k(4k+3) - 1; 1 \leq k \leq \left\lfloor \frac{n-2}{4} \right\rfloor$

$g\left(v_{\frac{n}{2}} v_{\frac{n}{2}+1}\right) = n(n+1) - 1, if\ n \equiv 0 (mod\ 2)$

$g\left(v_{\frac{n-1}{2}} v_{\frac{n+1}{2}}\right) = n(n+1) - 1, if\ n \equiv 1 (mod\ 2)$

From the labeling of vertex and edges it is noticed that the values are distinct for the graph(G) and it is recognized to be strongly *graph shown in Fig.11

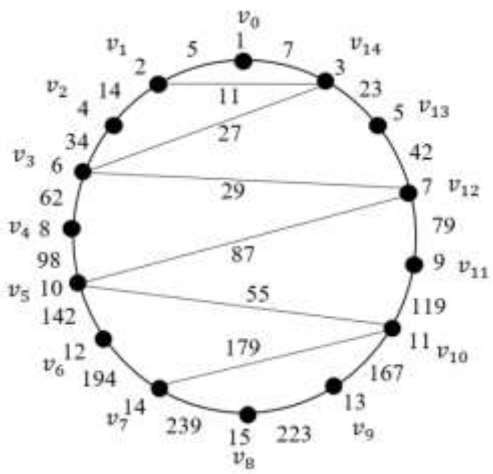

Fig.11 C_{15} with zigzag chords is Strongly * graph

Conclusion:

In this paper we prove, cycle with zigzag chords is vertex even mean graph, vertex odd mean graph, square sum graph, square difference graph with illustrations. Also, we prove that the graph, cycle with zigzag chords admits cube sum labeling, cube difference labeling, strongly multiplicative labeling, absolute difference of cube sum and square sum and strongly*labeling.

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