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Coupled and Common Fixed Points Results in Modified Intuitionistic $\mathcal{M}$-Fuzzy Metric Spaces

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ABSTRACT

Sedghi and Shobe introduced [17] $\mathcal{M}$- fuzzy metric space using idea of D-Metric Space defined by Dhage[8]. Sharma and Diwan [19] presented the new notion of $MI\mathcal{M}-FMS$(modified intuitionistic $\mathcal{M}$- fuzzy metric space) with the help of continuous t-representable. In this paper we prove existence of some coupled and common fixed point theorems in $MI\mathcal{M}-FMS$(modified intuitionistic $\mathcal{M}$- fuzzy metric space) using an implicit function. We also defined E.A property and common E.A property in $MI\mathcal{M}-FMS$ (Modified Intuitionistic $\mathcal{M}$- fuzzy metric space).

Keywords: $\mathcal{M}$- fuzzy metric space, MI $\mathcal{M}$-FMS (Modified Intuitionistic $\mathcal{M}$-fuzzy metric space), implicit relation, t-norm, t- conorm, L^* space, t-representable.

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1. Introduction and preliminaries:

In 1965 Zadeh [24] introduced extremely noteworthy concept of fuzzy sets. Kramosil and Michalek [13] fuzzified metric space and defined FMS (fuzzy metric space). George and Veeramani [10] revised the concept of FMS defined by Kramosil and Michalek [13]. Vasuki [23] proved some common fixed point results for R-weakly commuting maps in fuzzy metric spaces. Dhage [8] extended the metric space as generalized metric space or *D*-metric space. Sedghi and Shobe introduced [17] $\mathcal{M}$- fuzzy metric space using idea of D-Metric Space defined by Dhage[8] and then showed existence of some common fixed point theorems for six mappings.

Atanassov [4] extended the concept of fuzzy sets and established IFS (Intutionistic fuzzy sets). Park [14] introduced the notion of IFMS (intuitionistic fuzzy metric spaces). Alaca et al.[3] established some fixed points results in intuitionistic fuzzy metric spaces, afterward there has been much progress in the field of IFMS by many authors ([11],[15], [18],[20],[21],[22]).

Deschrijver and Kerre [7] defined a new space (L^*, $\leq_{L^*}$) and then showed that the space (L^*, $\leq_{L^*}$) forms a complete lattice. Deschrijver et al. [6] introduced the idea of continuous t-representable on L^*space. Saadati et al. [16] extended the conditions of intuitionistic fuzzy metric space and presented MIFMS (modified intuitionistic fuzzy metric spaces) using continuous t-representable.

Bhaskar and Lakshmikantham established [5] coupled fixed points and mixed monotone property on fuzzy metric space. Aamri and Moutawakil [1] proved common fixed point theorems using E.A property and generalized the concept of non compatible mappings. In 2009, Abbas et al. [2] presented the notion of common property (E.A). Further many authors have done remarkable work on common fixed point theorems.

Recently Grzegrzolka P.[12] defined some asymptotic dimensions of fuzzy metric spaces defined by George and Veeramani[10]. Further they showed that asymptotic dimension is an invariant. Furqan et al.[9]

introduced new notion of fuzzy triple controlled metric space and further generalized some fuzzy metric spaces, such as fuzzy b-metric space, fuzzy rectangular metric space, fuzzy rectangular b-metric space and extended fuzzy b-metric space.

In this paper, we present some coupled and common fixed point results on MI $\mathcal{M}$-FMS (modified intuitionistic $\mathcal{M}$- fuzzy metric space) using (E.A) property and common (E.A) property for coupled maps.

Definition 1.1[8]: "Let X be a nonempty set. A generalized metric (or D-metric) on X is a function: D: $X^3 \to R^+$, that satisfies the following conditions for each x, y, z ∈ X.

(1) D (x, y, z) ≥ 0;

(2) D (x, y, z) = 0 if and only if x = y = z;

(3) D (x, y, z) = D(p{x, y, z}), (symmetry) where p is a permutation function;

(4) D (x, y, z) ≤ D (x, y, a) + D (a, z, z);

The pair (X, D) is called a generalized metric (or D-metric) space".

Some examples of D-metric are:

(a) D (p, q, r) = d(p, q) + d(q, r) + d(r, p)

(b) D(p, q, r) = max {d(p, q), d(q, r), d(r, p)}

Lemma 1.2 [6]: **"**Consider the set L^* and operation $\leq_{L^*}$ defined by:

$$L^* = \{(x_1, x_2): (x_1, x_2) \in [0, 1]^2 \text{ and } x_1 + x_2 \leq 1\}$$

Where, $(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1 \text{ and } x_2 \geq y_2$, for every $(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is a complete lattice".

Definition 1.3 [4]: "An intuitionistic fuzzy set $\mathcal{A}_{\zeta,\eta}$ in a universe U is an object $\mathcal{A}_{\zeta,\eta} = \{\zeta_{\mathcal{A}}(u), \eta_{\mathcal{A}}(u) | u \in U\}$ where , for all $u \in U$, $\zeta_{\mathcal{A}}(u) \in [0,1]$ and $\eta_{\mathcal{A}}(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of U in $\mathcal{A}_{\tau,\eta}$, and furthermore they satisfy $\zeta_{\mathcal{A}}(u) + \eta_{\mathcal{A}}(u) \leq 1$. We denote its units by $0_{L^*} = (0, 1)$ and $1_{L^*} = (1, 0)$".

Definition 1.4 [6]: "A triangular norm (t-norm) on L^* is a mapping $\mathcal{T}$: $(L^*)^2 \to L^*$ satisfying the following conditions:

1. $\mathcal{T}(x, 1_{L^*}) = x, \forall x \in L^*$;
2. $\mathcal{T}(x, y) = \mathcal{T}(y, x), \forall x, y \in (L^*)^2$;
3. $\mathcal{T}(\mathcal{T}(x, y), z) = \mathcal{T}(x, \mathcal{T}(y, z)), \forall x, y, z \in (L^*)^3$;
4. If $x \leq_{L^*} x'$ and $y \leq_{L^*} y'$ then, $\mathcal{T}(x, y) \leq_{L^*} \mathcal{T}(x', y')$, $\forall x, x', y, y' \in (L^*)^4$."

Definition 1.5 [6,7]: "A continuous t-norm $\mathcal{T}$ on L^* is called continuous t- representable if and only if there exist a continuous t-norm ∗ and a continuous t - conorm ◊ on [0, 1] such that, for all $x = (x_1, x_2)$, $y = (y_1, y_2) \in L^*$, $\mathcal{T}(x, y) = (x_1 * y_1, x_2 \lozenge y_2)$".

Definition 1.6 [6,7]: "A negator on L^* is any decreasing mapping N: $L^* \to L^*$ satisfying $N(0_{L^*}) = 1_{L^*}$ and $N(1_{L^*}) = 0_{L^*}$. If N (N (x)) = x, for all $\forall x \in L^*$, then N is called an involutive negator. A negator on [0, 1] is a decreasing mapping N: [0, 1] → [0, 1] satisfying N(0) = 1 and N(1) = 0. N_s denotes the standard negator on [0, 1] and it is defined as, for $\forall x \in [0, 1]$, Ns (ε) = 1 – ε".

Definition 1.7 [19]: "Let M, N are fuzzy sets from $X^3 \times (0, \infty)$ to [0, 1] such that M(x, y, z, t) + N(x, y, z, t) ≤ 1 for all x, y ∈ X and t > 0. The 3-tuple (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) is said to be modified intuitionistic $\mathcal{M}$- fuzzy metric space if X is an arbitrary (non-empty) set, $\mathcal{T}$ is a continuous t- representable and $\mathcal{M}_{M,N}$ is a mapping $X^3 \times (0, \infty) \to L^*$ satisfying the following conditions for every x, y, z ∈ X and t, s > 0:

(a) $\mathcal{M}_{M,N}(x, y, z, t) >_{L^*} 0_{L^*}$;

(b) $\mathcal{M}_{M,N}(x, y, z, t) = 0_{L^*}$ if and only if x = y = z;

(c) $\mathcal{M}_{M,N}(x, y, z, t) = \mathcal{M}_{M,N}(p(x, y, z), t)$, where p is a permutation function;

(d) $\mathcal{M}_{M,N}(x, y, z, t + s) \geq_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, y, a, t), \mathcal{M}_{M,N}(a, a, z, s))$;

(e) $\mathcal{M}_{M,N}(x, y, z, t): (0, \infty) \to L^*$ is continuous;

Here,

$\mathcal{M}_{M,N}(x, y, z, t) = (M(x, y, z, t), N(x, y, z, t))$".

Example1.8[19]:"Suppose (X, d) be a metric space and Let $\mathcal{T}(x,y) = (x'y',\ \min(x''+y'',1))$ *for all* x = $(x',\ x'')$ and y = $(y',\ y'') \in L^*$ and ,

$\mathcal{M}_{M,N}$(x, y, z, t) = (M(x, y, z, t), N(x, y, z, t)) = $\left(\frac{t}{t+D(x,y,z)}, \frac{D(x,y,z)}{t+D(x,y,z)}\right)$

Where D is the D-metric defined on X, then (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) be a modified intuitionistic $\mathcal{M}$- fuzzy metric space."

Lemma 1.9[19]: "In a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$), $\mathcal{M}_{M,N}$(x, y, z, t) is non-decreasing with respect to t, for all x, y, z $\in$ X.

Lemma 1.10[19]: "In a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$),

$\mathcal{M}_{M,N}$(p, q, q, t) = $\mathcal{M}_{M,N}$(p, p, q, t) for all x, y in X and t > 0."

Definition 1.11[19]: "A sequence $\{x_n\}$ is convergent to a point x $\in$ X in the modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) if for each 0 < ε < 1 and t > 0, there exists $n_0 \in$ N such that,

$\mathcal{M}_{M,N}$(x_n, x, x, t) $>_{L^*}$ ($1-\varepsilon$, ε) whenever n ≥ n_0."

Definition 1.12[19]: "A sequence $\{x_n\}$ in a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) is called a Cauchy sequence if for each 0 < ε < 1 and t > 0, there exists $n_0 \in$ N such that,

$\mathcal{M}_{M,N}$ (x_m, y_n, y_n, t) $>_{L^*}$ ($1-\varepsilon$, ε) and for each n, m ≥ n_0."

Definition 1.13[19]: "A modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) is said to be complete if and only if every cauchy sequence in X is convergent."

Lemma 1.14[19]: "Let (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) be a modified intuitionistic $\mathcal{M}$- fuzzy metric space. If

$\mathcal{M}_{M,N}(x_n, x_n, x_{n+1}, t) \geq_L \mathcal{M}_{M,N}(x_0, x_0, x_1, k^n t)$, for some $k > 1$ and n ∈ N. Then $\{x_n\}$ is a Cauchy sequence."

Definition 1.15[19]: " Let (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) be a modified intuitionistic $\mathcal{M}$- fuzzy metric space. $\mathcal{M}$ is said to be continuous on $X^3 \times (0, \infty)$ if,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(x_n, y_n, z_n, t_n) = \mathcal{M}_{M,N}$ (x, y, z, t), whenever $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$, $\lim_{n\to\infty} z_n = z$ and $\lim_{n\to\infty} \mathcal{M}_{M,N}(x, y, z, t_n) = \mathcal{M}_{M,N}$ (x, y, z, t)."

Definition 1.16[19]: "Let A and S be mappings from a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is, Ax = Sx implies that ASx = SAx."

Definition 1.17[19]: "Let A and S be mappings from a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) into itself. Then the mappings are said to be compatible if,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(ASx_n, ASx_n, SAx_n, t_n) = 1_{L^*}$, for every t > 0, whenever x_n is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = x ∈ X."

Lemma 1.18[19]: "If self-mappings A and S of a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) are compatible, then they are weak compatible, but the converse is not true."

Definition 1.19: Let A and B be two self-mappings of a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$). We say that A and B satisfy the property (E. A.), if there exists a sequence $\{x_n\}$ such that,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(Ax_n, x, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(Bx_n, x, x, t) = 1_{L^*}$, for some x $\in$ X and t > 0.

Definition 1.20: Two pairs (A, S) and (B, T) of two self-mappings of a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$) share the common property (EA), if there exist two sequences $\{x_n\}$and $\{y_n\}$ such that,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(Ax_n, x, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(Sx_n, x, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(By_n, x, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(Ty_n, x, x, t) = 1_{L^*}$, for some x $\in$ X and t > 0.

2. Main Results:

Definition 2.1:

Let A: X × X → X and B: X → X be two mappings of a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$). A and B are said to be satisfy the property (E. A.), if there exists two sequences $\{x_n\}$ and $\{y_n\}$ such that,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(A(x_n, y_n), x, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(Bx_n, x, x, t) = 1_{L^*}$,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(A(y_n, x_n), y, y, t)$

$$= \lim_{n\to\infty} \mathcal{M}_{M,N}(By_n, y, y, t) = 1_{L^*}$$

for some x, y ∈ X and t > 0.

Definition 2.2:

Let A: X × X → X , B: X × X → X , S: X → X and T: X → X be mappings of a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$). The pairs (A, S) and (B, T) are said to be share the common property (E. A.), then there exist sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$ and $\{v_n\}$ such that,

$\lim_{n\to\infty} \mathcal{M}_{M,N}(A(x_n, y_n), x, x, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(Sx_n, x, x, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(B(u_n, v_n), x, x, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(Tu_n, x, x, t) = 1_{L^*}$

and

$\lim_{n\to\infty} \mathcal{M}_{M,N}(A(y_n, x_n), y, y, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(Sy_n, y, y, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(B(v_n, u_n), y, y, t)$

$= \lim_{n\to\infty} \mathcal{M}_{M,N}(Tv_n, y, y, t) = 1_{L^*}$

for some x, y ∈ X and t > 0.

Theorem 2.3:

Let A, B, F, G, S and T be self-mappings on a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$), which satisfying following conditions:

I) Let $\alpha: L^* \to L^*$, be such that $\alpha(a) \geq a$, for every a ∈ L^*.

$\mathcal{M}_{M,N}(A(x,y), B(u,v), B(u,v), t)$

$\geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FTx, GSu, GSu, kt), \mathcal{M}_{M,N}(A(x,y), FTx, FTx, kt),$

$\mathcal{M}_{M,N}(B(u,v), GSu, GSu, kt), \mathcal{M}_{M,N}(A(x,y), GSu, GSu, kt))$

For every x, y ∈ X and k > 0.

II) A(X×X) ⊂ GS(X) and B(X×X) ⊂ FT(X)

III) One of the pair (A, FT) or (B, GS) satisfies (E.A) property.

If one of the A(X×X) , GS(X) , B(X×X) and FT(X) is a complete subspace of X then the pair (A, FT) and (B, GS) have coupled coincident point. Further, if the pairs (A, FT) and (B, GS) are weakly compatible, then A, B, FT and GS have unique common fixed point in X.

Proof: Let the pair (B, GA) satisfies (E.A) property, then there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that,

$\mathcal{M}_{M,N}(B(x_n, y_n), x, x, t) = \mathcal{M}_{M,N}(GSx_n, x, x, t) = 1_{L^*}$ and $\mathcal{M}_{M,N}(B(y_n, x_n), y, y, t) = \mathcal{M}_{M,N}(GSy_n, y, y, t) = 1_{L^*}$, for some x, y ∈ X. -----(1)

From condition (II) there exist two sequences $\{u_n\}$ and $\{v_n\}$in X, such that,

$B(x_n, y_n) = FTu_n$ and $B(y_n, x_n) = FTv_n$ ----- (2)

Taking n→∞ and using equation (1)

$$x = \lim_{n\to\infty} FTu_n \quad \text{and} \quad y = \lim_{n\to\infty} FTv_n$$ ----- (3)

From condition (I)

$\mathcal{M}_{M,N}(A(u_n, v_n), B(x_n, y_n), B(x_n, y_n), t)$

$\geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FT u_n, GS x_n, GS x_n, kt), \mathcal{M}_{M,N}(A(u_n, v_n), FTu_n, FTu_n, kt),$

$\mathcal{M}_{M,N}(B(x_n, y_n), GS x_n, GS x_n, kt), \mathcal{M}_{M,N}(A(u_n, v_n), GS x_n, GS x_n, kt)$).

Taking n→∞

$\mathcal{M}_{M,N}(A(u_n, v_n), x, x, t)$

$\geq_{L^*} \alpha(\mathcal{M}_{M,N}(A(u_n, v_n), x, x, kt) >_{L^*} \mathcal{M}_{M,N}(A(u_n, v_n), x, x, kt)$

Hence $A(u_n, v_n) = x$, similarly $A(v_n, u_n) = y$.

Let FT(X) is a complete subspace of X, there exist y, z ∈ X, such that,

$\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} FTu_n = \lim_{n\to\infty} B(u_n, v_n) = \lim_{n\to\infty} GSx_n$= x = FTy

$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} FTv_n = \lim_{n\to\infty} B(v_n, u_n) = \lim_{n\to\infty} GSy_n$= x = FTz. -----(5)

From condition I)

$\mathcal{M}_{M,N}(A(y,z), B(x_n, y_n), B(x_n, y_n), t))$

$\geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FTy, GSx_n, GSx_n, kt), \mathcal{M}_{M,N}(A(y,z), FTy, FTy, kt),$

$\mathcal{M}_{M,N}(B(x_n, y_n), GS x_n, GS x_n, kt), \mathcal{M}_{M,N}(A(y,z), GS x_n, GS x_n, kt)$).

Taking n→∞ and equation (5)

A(y, z) = FTy , and A(z, y) = FTz -----(6)

Hence the pair (A, FT) has unique coupled coincident point in X.

Since (A, FT) are weak compatible then,

FT(A(y, z)) = A(FTy, FTz) and FT(A(z, y)) = A(FTz, FTy) ----(7)

As A(X × X) ⊆ GS(X) there exists x_1, x_2 ∈ X, such that

$A(y, z) = GSx_1$ and $A(z, y) = GSx_2$ ----(8)

By inequality (I) and using equation (6) and (8)

$\mathcal{M}_{M,N}(A(y,z), B(x_1, x_2), B(x_1, x_2), t)$

$\geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FT y, GS x_1, GS x_1, kt),$

$\mathcal{M}_{M,N}(A(y,z), FTy, FTy, kt),$

$\mathcal{M}_{M,N}(B(x_1, x_2), GSx_1, GSx_1, kt),$

$\mathcal{M}_{M,N}(A(y,z), GSx_1, GSx_1, kt))$

Hence,

$GSx_1 = B(x_1, x_2)$, and $GSx_2 = A(x_1, x_2)$ ----(9)

Hence the pair (A, FT) has coupled coincident point in X.
Hence let,

$A(y,z) = FTy = GSx_1 = B(x_1, x_2) = y_1$ and

$A(z,y) = FTz = GSx_2 = B(x_1, x_2) = y_2$ ----(10)

From equation (7) we have

$FTy_1 = A(y_1, y_2)$ and $FTy_2 = A(y_2, y_1)$ ----(11)

Since (B, GS) are weak compatible then,

$GS(B(x_1, x_2)) = B(GSx_1, GSx_2)$, $GS(B(x_2, x_1)) = B(GSx_2, GSx_1)$.

Therefore,

$GS\, y_1 = B(y_1, y_2)$ and $GS\, y_2 = B(y_2, y_1)$ ----(12)

Hence from inequality (I)

$$\mathcal{M}_{M,N}(A(y_1, y_2), B(x_1, x_2), B(x_1, x_2), t) \geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FT\, y_1, GS\, x_1, GS\, x_1, kt), \mathcal{M}_{M,N}(A(y_1, y_2), FTy_1, FTy_1, kt), \mathcal{M}_{M,N}(B(x_1, x_2), GSx_1, GSx_1, kt), \mathcal{M}_{M,N}(A(y_1, y_2), GS\, x_1, GS\, x_1, kt))$$

From equation (11) $A(y_1, y_2) = y_1$ and $A(y_2, y_1) = y_2$

Hence, $A(y_1, y_2) = y_1 = GSy_1$ and $A(y_2, y_1) = y_2 = GSy_2$ ----(13)

Using inequality (I) and using equations (11) and (12)

$$\mathcal{M}_{M,N}(A(y,z), B(y_1, y_2), B(y_1, y_2), t) \geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FT\, y, GS\, y_1, GS\, y_1, kt), \mathcal{M}_{M,N}(A(y,z), FTy, FTy, kt), \mathcal{M}_{M,N}(B(y_1, y_2), GSy_1, GSy_1, kt), \mathcal{M}_{M,N}(A(y,z), GSy_1, GSy_1, kt))$$

Hence, $B(y_1, y_2) = y_1 = GSy_1$ and $B(y_2, y_1) = y_2 = GSy_2$.

Hence (A, FT) and (B, GS) have common coupled fixed point. -----(14)

$$\mathcal{M}_{M,N}(y_1, y_2, y_2, t) = \mathcal{M}_{M,N}(A(y_1, y_2), B(y_2, y_1), B(y_2, y_1), t)$$

$$\geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(FTy_1, GS\, y_2, GS\, y_2, kt), \mathcal{M}_{M,N}(A(y_1, y_2), FTy_1, FTy_1, kt), \mathcal{M}_{M,N}(B(y_2, y_1), GSy_2, GSy_2, kt), \mathcal{M}_{M,N}(A(y_1, y_2), GSy_2, GSy_2, kt))$$

$$>_{L^*} \mathcal{M}_{M,N}(y_1, y_2, y_2, k\, t)$$

Therefore, $y_1 = y_2$.

Hence A, B, FT, GS have common fixed point in X.
Similarly, the uniqueness of the fixed point can be proved.

Theorem 2.4:

Let A, B, F, and G be self-mappings on a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$), which satisfying following conditions:

I) Let $\alpha: L^* \to L^*$, be such that $\alpha(a) \geq a$, for every $a \in L^*$.

$$\mathcal{M}_{M,N}(A(x,y), B(u,v), B(u,v), t) \geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(Fx, Gu, Gu, kt), \mathcal{M}_{M,N}(A(x,y), Fx, Fx, kt), \mathcal{M}_{M,N}(B(u,v), Gu, Gu, kt), \mathcal{M}_{M,N}(A(x,y), Gu, Gu, kt))$$

II) $A(X\times X) \subset GS(X)$ and $B(X\times X) \subset FT(X)$

III) The pair (A, F) and (B, G) satisfies (E.A) property.

If one of the $A(X\times X)$, G(X) , $B(X\times X)$ and F(X) is a complete subspace of X then the pair (A, F) and (B, G) have coupled coincident point. Further, if the pair (A, FT) and (B, GS) are weakly compatible, then A, B, F and G have unique common fixed point in X.

Proof: Taking Sx = Tx = Ix , where I is the identity mapping in the theorem 2.3 , the above theorem can be proved.

Theorem 2.5:

Let $A: X\times X \to X$, $B: X\times X \to X$, $F: X \to X$, $G: X \to X$ mappings on a modified intuitionistic $\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}_{M,N}$, $\mathcal{T}$), which satisfying following conditions:

I) Let, $\alpha: L^* \to L^*$, be such that $\alpha(a) \geq a$, for every $a \in L^*$.

$$\mathcal{M}_{M,N}(A(x,y), B(u,v), B(u,v), t) \geq_{L^*} \alpha(\min(\mathcal{M}_{M,N}(Fx, Gu, Gu, kt), \mathcal{M}_{M,N}(A(x,y), Fx, Fx, kt), \mathcal{M}_{M,N}(B(u,v), Gu, Gu, kt), \mathcal{M}_{M,N}(A(x,y), Gu, Gu, kt))$$

II) F(X) and G(X) are closed subsets of X.

III) The pair (A, F) and (B, G) share common (E. A) property,

Then (A, F) and (B, G) have coupled coincident point. Further, if (A, F) and (B, G) are weakly compatible then A, B, F and G have unique common fixed point in *X*.

Proof: Since the pair (A, F) and (B, G) share common (E. A) property hence there exist sequences $\{x_n\}$, $\{y_n\}$, $\{u_n\}$, $\{v_n\}$such that, for some $x_1, x_2 \in X$,

$$\mathcal{M}_{M,N}(A(x_n, y_n), x_1, x_1, t) = \mathcal{M}_{M,N}(Gx_n, x_1, x_1, t) = \mathcal{M}_{M,N}(B(u_n, v_n), x_1, x_1, t) = \mathcal{M}_{M,N}(Fu_n, x_1, x_1, t) = 1_{L^*}$$

----(1)

And

$$\mathcal{M}_{M,N}(A(y_n, x_n), x_2, x_2, t) = \mathcal{M}_{M,N}(Gy_n, x_2, x_2, t) = \mathcal{M}_{M,N}(B(v_n, u_n), x_2, x_2, t) = \mathcal{M}_{M,N}(Fv_n, x_2, x_2, t) = 1_{L^*}$$

----(2)

Since G(X) is a closed subspace of *X*, then there exist $y_1, y_2 \in X$, such that

$Gy_1 = x_1$ and $Gy_2 = x_2$.

----(3)

From the condition(I),

$\mathcal{M}_{M,N}$ $(A(u_n, v_n), B(y_1, y_2), B(y_1, y_2), t)$

$\geq_{L^*}$ $\alpha($ $\min($ $\mathcal{M}_{M,N}$ (Fu_n, Gy_1, Gy_1, kt), $\mathcal{M}_{M,N}$ $(A(u_n, v_n), Fu_n, Fu_n, kt)$,

$\mathcal{M}_{M,N}$ $(B(y_1, y_2), Gy_1, Gy_1, kt)$, $\mathcal{M}_{M,N}$ $(A(u_n, v_n), Gy_1, Gy_1, kt)$)

Taking n→∞ and from equations (1), (2) and (3),

$B(y_1, y_2) = Gy_1 = x_1$ and $B(y_2, y_1) = Gy_2 = x_2$

----(4)

Hence (B, G) has coupled fixed point.

Since F(X) is a closed subspace of X, there exists $z_1, z_1 \in X$ such that,

$Fz_1 = x_1$ and $Fz_2 = x_2$

----(5)

From condition (I)

$\mathcal{M}_{M,N}$ $(A(z_1, z_2), B(x_n, y_n), B(x_n, y_n), t)$

$\geq_{L^*}$ $\alpha($ $\min($ $\mathcal{M}_{M,N}$ (Fz_1, Gx_n, Gx_n, kt), $\mathcal{M}_{M,N}$ $(A(z_1, z_2), Fz_1, Fz_1, kt)$,

$\mathcal{M}_{M,N}$ $(B(x_n, y_n), Gx_n, Gx_n, kt)$, $\mathcal{M}_{M,N}$ $(A(z_1, z_2), Gx_n, Gx_n, kt)$)

Using equation (1) and (5)

$A(z_1, z_2) = Fz_1 = x_1$ and $A(z_2, z_1) = Fz_2 = x_2$

----(6)

Hence (A, F) has coupled coincident point.

Let (B, G) is weakly compatible such that,

$G(B(y_1, y_2)) = B(Gy_1, Gy_2)$ and $G(B(y_2, y_1)) = B(Gy_2, Gy_1)$

From condition (I) and equations (1 to 4),

$\mathcal{M}_{M,N}$ $(A(z_1, z_2), B(x_1, x_2), B(x_1, x_2), t)$

$\geq_{L^*}$ $\alpha($ $\min($ $\mathcal{M}_{M,N}$ (Fz_1, Gx_1, Gx_1, kt), $\mathcal{M}_{M,N}$ $(A(z_1, z_2), Fz_1, Fz_1, kt)$,

$\mathcal{M}_{M,N}$ $(B(x_1, x_2), Gx_1, Gx_1, kt)$, $\mathcal{M}_{M,N}$ $(A(z_1, z_2), Gx_1, Gx_1, kt)$)

Hence, $B(x_1, x_2) = Gx_1 = x_1$ and $B(x_2, x_1) = Gx_2 = x_2$

----(7)

Since (A, F) is weakly compatible such that,

$F(A(z_1, z_2)) = A(Fz_1, Fz_2)$ and $F(A(z_2, z_1)) = A(Fz_2, Fz_1)$

----(8)

Using condition (I)

$\mathcal{M}_{M,N}$ $(A(x_1, x_2), B(y_1, y_2), B(y_1, y_2), t)$

$\geq_{L^*}$ $\alpha($ $\min($ $\mathcal{M}_{M,N}$ (Fx_1, Gy_1, Gy_1, kt), $\mathcal{M}_{M,N}$ $(A(x_1, x_2), Fx_1, Fx_1, kt)$,

$\mathcal{M}_{M,N}$ $(B(y_1, y_2), Gy_1, Gy_1, kt)$, $\mathcal{M}_{M,N}$ $(A(x_1, x_2), Gy_1, Gy_1, kt)$)

Hence, $A(x_1, x_2) = Fx_1 = x_1$ and $A(x_2, x_1) = Fx_2 = x_2$

----(9)

Therefore (A, F) and (B, G) have common coupled fixed point.

Now we show that $x_1 = x_2$,

$\mathcal{M}_{M,N}(x_1, x_2, x_2, t)$

$= \mathcal{M}_{M,N}$ $(A(x_1, x_2), B(x_2, y_2), B(y_1, y_2), t)$

$\geq_{L^*}$ $\alpha($ $\min($ $\mathcal{M}_{M,N}$ (Fx_1, Gx_2, Gx_2, kt), $\mathcal{M}_{M,N}$ $(A(x_1, x_2), Fx_1, Fx_1, kt)$,

$\mathcal{M}_{M,N}$ $(B(x_2, x_1), Gx_2, Gx_2, kt)$, $\mathcal{M}_{M,N}$ $(A(x_1, x_2), Gx_2, Gx_2, kt)$)

Hence, $A(x_1, x_2) = Fx_1 = x_1$ and $A(x_2, x_1) = Fx_2 = x_2$

Hence, $\mathcal{M}_{M,N}(x_1, x_2, x_2, t) \geq_{L^*} \alpha\,(\mathcal{M}_{M,N}(x_1, x_2, x_2, kt)\ ) >_{L^*} \mathcal{M}_{M,N}(x_1, x_2, x_2, kt)$

Hence, $x_1 = x_2$.

Therefore A, B, F and T have common fixed point in X.

Similarily the uniqueness of the fixed point can be proved.

3. Conclusion:

In this paper we proved some coupled and common fixed point theorems in $MI\mathcal{M}-FMS$ (modified intuitionistic $\mathcal{M}$- fuzzy metric space) using an implicit function. We also defined the properties (E. A) and common (E. A) for the mappings defined in the modified intuitionistic $\mathcal{M}$- fuzzy metric space.

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