ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

International Journal of Mechanical Engineering

SOME SPECIAL GRACEFUL LABELING RESULTS OF HEXAGONAL PYRAMIDAL GRACEFUL GRAPHS

N. Velmurugan

Assistant Professor, PG and Research Department of Mathematics, Theivanai Ammal Collage for Women (Autonomous), Villupuram -605202, Tamil Nadu India.

S. Umamaheshwari

II-M.Sc Mathematics, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram -605202, Tamil Nadu India.

ABSTRACT:

In this paper, talks about the hexagonal pyramidal Graceful Graphs and the Numbers of the form $\frac{n(n+1)(4n-1)}{6}$ for all n≥1 are called hexagonal pyramidal numbers. Let G be a graph with p vertices and q edges. Let ϕ : V(G) →{0, 1, 2… M_k} where M_k is the k th hexagonal pyramidal number be an injective function. Define the function ϕ* :E(G) →{1,7,22,50,….,M_k } such that ϕ* (uv) = | ϕ (u)- ϕ (v)| for all edges uv∊E(G). If ϕ * (E (G)) is a sequence of distinct consecutive hexagonal pyramidal numbers {$M_1, M_2 \ldots\ldots M_k$}, then the function ϕ is said to be hexagonal pyramidal graceful labeling and the graph which admits such a labeling is called a hexagonal pyramidal graceful graph. In this paper, hexagonal graceful labeling of some graph is studied.

Keywords**:** Hexagonal pyramidal graceful number, hexagonal pyramidal graceful labeling, hexagonal pyramidal graceful graphs.

INTRODUCTION:

Graphs considered in this paper are finite, undirected and (simple) without loops or multiple edges. Let G = (V, E) be a graph with p vertices and q edges. Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. Graph labeling was first introduced in 1960's.

A graph labeling is an assignment of integers to the vertices (edges / both) subject to certain conditions. If the domain of the mapping is the set of vertices (edges / both) then the labeling is called the vertex (edge / total) labeling. For number theoretic terminology, we refer to [1] and [2].

Terms not defined here are used in the sense of Parthasarathy [3] and Bondy and B. R. Murthy [4]. function (labeling) f a β-valuation of a graph G with q edges if f is an injection from the vertices of G to the set $\{0,1,2, \ldots , q\}$ such that each edge xy in G is assigned the label $|f(x) - f(y)|$, the resulting edge labels are distinct consecutive numbers and Golomb [6] called it as graceful labeling. Acharya [7] constructed certain infinite families of graceful graphs.

Labeled graphs are becoming an increasing useful family of mathematical models for a broad range of application like designing X-Ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory etc. For more information related to graph labeling and its applications, see [8- 49].

There are several types of graph labeling and a detailed survey is found in [50]. The following definitions are necessary for present study.

Definition 1:

Let G be a (p, q) graph. A 1 to 1 function ϕ :V(G) →{0,1,2,…,q}is called a graceful labeling of G if the induced edge labeling f ': E(G) →{1,2,…,q} defined by f '(e) = |f (u)-f (v)| for each e = uv of G is also one to one. The graph G graceful labeling is called graceful graph.

Definition 2:

Bistar is the graph obtaining by joining the apex vertices of two copies of star $K_{1,n}$.

Definition 3:

Let $v_1, v_2, \ldots\ldots v_n$ be the n vertices of a path P_n. From each vertex v_i, i = 1,2,…,n there are m_i, , i = 1,2,…,n pendent vertices sayv_{i1}, v_{i2}, v_{imi},. The result graph is a caterpillar and is denoted by B ($m_1, m_2, \ldots . m_n$).

Definition 4:

A coconut tree CT(n, m) is the graph obtained from the path p_m by appending n new pendant edges at an end vertex of p_m.

Definition 5:

A path p_n is obtained by joining u_i to the consecutive vertices u_{i+1} for $1 \le i \le n-1$.

Definition 6:

Let G be a graph with p vertices and q edges. Let ϕ : V(G) →{0, 1, 2… M_k} where Mk is the k th hexagonal pyramidal number be an injective function. Define the function ϕ* : E(G) →{1,7,22,50,.., M_K} such that ϕ * (uv) = |ϕ(u) - ϕ (v) | for all edges uv ∈ E(G). If ϕ* (E (G)) is a sequence of distinct consecutive pentagonal pyramidal numbers {$M_1, M_2, \ldots . M_K$}, then the function ϕ is said to be hexagonal pyramidal graceful labeling and the graph which admits such a labeling is called a hexagonal pyramidal graceful graph.

Definition 7:

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set and the edge set of G are denoted by V(G) and E(G) respectively. The number of elements of V(G) = p is called the order of G and the number of elements of E(G) = q is called the size of G. A graph of order p and size q is called a (p,q) - graph. If e = uv is an edges of G, we say that u and v are adjacent and that u and v are incident with e.

Definition 8:

The degree of a vertex v in a graph G is defined to be the number of edges incident on v and is denoted by deg(v). A graph is called r-regular if deg(v) = r for each v∈ V(G). The minimum of {deg v : v∈ V(G) } is denoted by δ and maximum of {deg v : v ∈ V(G)} is denoted by △. A vertex of degree 0 is called an isolated vertex, a vertex of degree is called a pendant vertex or an end vertex.

Theorem 1:

Let G be a path with m vertices. Then G is Hexagonal pyramidal graceful for all m ≥ 3.

Proof:

Let G be a path with m vertices.

Let V (G) = {v_i: 1≤ i ≤ m} be the vertex set of G and

E(G) = {$v_i v_{i+1}$:1 ≤ i ≤ m-1} be the edge set of G.

Hence G has m vertices and m-1 edges.

Let k = m-1.

Define a function ϕ: V(G) →{0,1,2,…,M_k} as follows

ϕ (v_1) = 0

ϕ (v_2) = M_K

ϕ (v_I) = ϕ (v_{i-1}) - $M_k-(i-2)$ if i is odd and 3 ≤ i ≤ m.

ϕ (v_i) = ϕ (v_{i-1}) + $M_k-(i-2)$ if i is even and 3 ≤ i ≤ m.

Let ϕ * be the induced edge labeling of f.

Then ϕ ($v_1 v_2$)=M_k

ϕ *($v_i v_{i+1}$) = $M_k-(i-1)$; 2 ≤ i ≤ m-1.

The induced edge lables $M_1, M_2, \ldots, M_k$ are distinct and consecutive hexagonal pyramidal numbers .

Hence the graph G is a hexagonal pyramidal graceful.

Example : Hexagonal pyramidal graceful labeling of H_6 is given,

Theorem: 2

Coconut tree CT(n,m) is hexagonal pyramidal graceful for all $n \geq 1$, $m \geq 2$.

Proof:

Let G be the graph CT(n,m).

Let V(G) = {v, $v_i u_j$: $1 \leq i \leq n$, $1 \leq j \leq m$-1} and

E(G) = {$vv_i vu_1$,$u_j u_{j+1}$: $1 \leq i \leq n$, $1 \leq j \leq m$-1}.

G has $n + m$ vertices and $n + m - 1$ edges.

Let k= $n + m - 1$.

Let ϕ : V(G)→{0,1,2,…,M_k} be defined as follows

$\phi(v) = 0$

$\phi(v_i) = M_{k-i+1}$; $1 \leq i \leq n$

$\phi(u_1) = M_{k-n}$

$\phi(u_j) = \phi(u_{j-1}) + M_{k-n-(j-1)}$ if j is odd and $2 \leq j \leq m$-1

$\phi(u_j) = \phi(u_{j-1}) - M_{k-n-(j-1)}$ if j is even and $2 \leq j \leq m$-1

Let ϕ * be the induced edge labeling of ϕ.

Then $\phi^*(vv_i) = M_{k-i+1}$; $1 \leq i \leq n$.

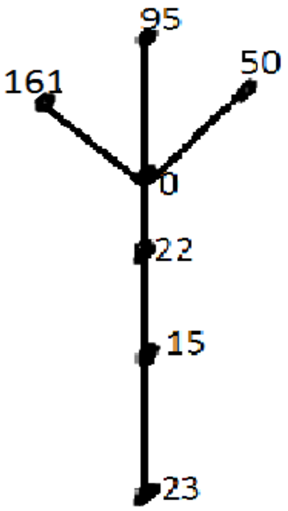
$\phi^*(vu_1) = M_{k-n.}$

$\phi^*(u_j u_{j+1}) = M_{k-n-j}$; $1 \leq j \leq m$-2.

The induced edge labels $M_1, M_2, \dots M_k$ are distinct and consecutive hexagonal pyramidal numbers.

Hence Coconut tree is hexagonal pyramidal graceful.

Example: Hexagonal Pyramidal graceful labeling of CT(3,4) is given in fig



THEOREM :3

The bistar B($n_{,1}$ n_2) where $n_{1\geq1}$ and $n_{2\geq1}$ is hexagonal pyramidal graceful.

Proof:

Let P_2be a path on two vertices and let v_1 and $v_2$be the vertices of P_2

From v_1 there are n_1 pendent vertices say $v_{11,}v_{12,}$ .... v_{1n} and from v_2, there are $n_2$pendent vertices say v_{21} v_{22} v_{2n2} .

The resulting graph is a bistar B($n_1 n_2$) .

Let G = (V,E) be the bistar B($n_{1,}$ n_2).

Let V(G) = {v_i : i=1,2} ∪ {v_{1j}: 1≤ j ≤ n_1} ∪ {v_{2j} : 1≤ j≤ n_2} and

E(G) = {$v_1 v_2$} ∪ {$v_1 v_{1j}$: 1≤ j ≤ n_1} ∪{$v_2 v_{2j}$: 1≤ j≤ n_2}.

Then G has $n_1 + n_2 + 2$ vertices and $n_1 + n_2 + 1$ edges.

Let $n_1 + n_2 + 1$ = k(say)

Now label the vertices v_1, v_2 of P_2 as 0 and 1.

Then label the n, vertices adjacent to $v_1$other than $v_2$as M_k, M_{k-1}, M_{k-2},…, M_{k-n_1}+1

Next label the n_2 vertices adjacent to v_2 other than vv_1 as $M_{k-n_1} + 1$,…, $M_{k-n_1} - n_{2+1} + 1$

We shall prove that G admits hexagonal pyramidal graceful labeling.

From the definition, it is clear that max ϕ(v)϶{0,1,2,…, M_k} for all v ∈V(G)

Also from the definition, all the vertices of G have different labeling.

Hence ϕ is one to one.

It remains to show that the edges values are of the form {M_1,M_2, ... , M_k}.

The induced edges function ϕ*:E(G) →{1,7,22,…, M_k} is defined as follows

ϕ*($v_i v_{ij}$) = $M_{k-(j-1)}$ if i = 1 and 1 ≤ j≤ n_1

ϕ *($v_i v_{ij}$) = $M_{k-(n_{1+j-1})}$ if i = 2 and 1≤ j ≤ n_2.

And ϕ *($v_1 v_2$) = M_1.

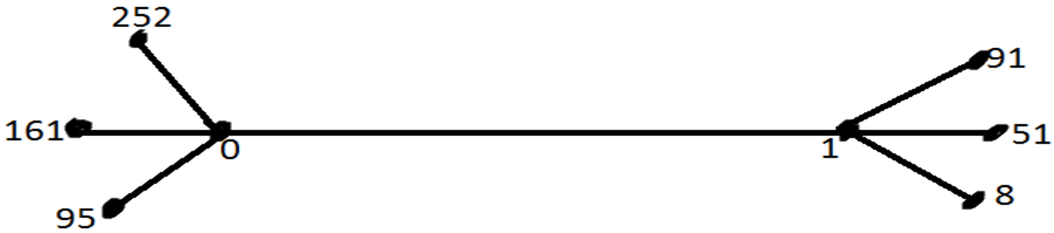
Clearly ϕ * is one to one and ϕ*(E(G)) = {M_1,$M_{2,....}$$M_k$}.

Therefore G is admits hexagonal pyramidal graceful labeling.

Hence the graph B(n_1, n_2) is hexagonal pyramidal graceful.

Example :

The hexagonal pyramidal graceful labeling of B(3,3) is given in Fig.



Theorem : 4

The caterpillar B (n_1,0 ,n_2) is hexagonal pyramidal graceful for all $n_1, n_2 \geq 1$

Proof: *Let v_1 ,v_2, v_3 be the three vertices of P_3.*

From v there are n_1 pendent vertices say u_1 ,u_2,…u_n1 and from v_3, there are n_2 pendent vertices say $W_1, W_{2,....}W_{n2}$.

The resulting graph is denoted as B (n_1,0 ,n_2).

Let it be G = (V,E).

Then G has $n_1 + n_2 + 3$ vertices and $n_1 + n_2 + 2$ edges.

Let k= $n_1 + n_2 + 2$

Define ϕ: V(G) →{0,1,2,…,Mk} as follows.

ϕ (v_1) = M_k

ϕ (v_2) = 0

ϕ (v_3) = $M_{k-}n_{1-1}$

ϕ(u_i) = $M_{k-}M_{k-i}$where 1 ≤ i ≤ n_1 ,

ϕ(w_j) = $M_{k-n_{1-1}} + M_j$, where 1 ≤ j ≤ n_2.

We shall prove that G admits hexagonal pyramidal graceful labeling.

From the definition, it is clear that max ϕ (v) is M_k for all vϵV(G) and ϕ (v)ϵ{0,1,2,…, M_k}.

Also from the definition, all the vertices of G have different labeling.

Hence ϕ is one to one.

It remains to show that the edge values are of the form {$M_1, M_{2,....}M_k$}.

The induced edge function ϕ *: E (G) →{1,6,…, M_k}. } is defined as follows

ϕ *($v_1 v_2$) = . M_k

ϕ *($v_2 v_3$) = $M_{k-n_1} - 1$

ϕ * ($v_1 u_i$) = $M_{k-i}$where 1 ≤ i ≤ n_1 .

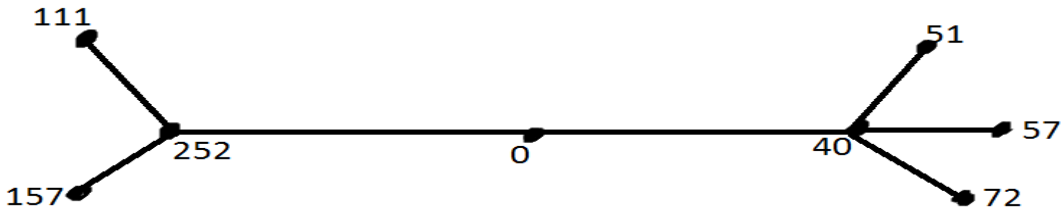
ϕ*($v_3 w_j$) = M_j , where 1 ≤ j ≤n_2 .

Clearly ϕ * is one to one and ϕ*(E(G)) = {$M_1, M_{2,....}M_k$}.

Therefore G admits hexagonal pyramidal graceful labeling.

Hence the graph B (n_1,0 ,n_2) is hexagonal pyramidal graceful for all $n_1, n_2 \geq 1$

Example : The hexagonal pyramidal graceful labeling of B(2,0,3) is given in Fig



CONCLUSIONS:

In this paper, we have briefly discussed about the concept of hexagonal pyramidal graceful labeling graphs and graceful labeling of some graphs. This work contributes several new results to the theory of graph labeling. The hexagonal pyramidal graceful can be verified for many other graphs. In future it is easy to introduce Nanogonal pyramidal graphs.

REFERENCES:

[1] M. Apostal, Introduction to Analytic Number Theory, Narosa Publishing House, Second Edition, 1991.

[2] I. Niven and Herbert S. Zuckerman, An Introduction to the Theory of Numbers, Wiley Eastern Limited, Third Edition, 1991

[3] K. R. Parthasarathy, Basic Graph Theory, Tata Mcgraw Hill Publishing Company Limited, 1994

[4] J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, Macmillan Press, London, 1976

[5] A. Rosa, On Certain Valuations of the Vertices of a Graph, Theory of Graphs (Proc. Internat. Symposium, Rome, 1966), Gordon and Breach, N. Y and Dunad Paris (1967), 349-355

[6] S. W. Golomb, How to Number a Graph, Graph Theory and Computing, R. C. Read, Academic Press, New York (1972), 23-37

[7] B. D. Acharya, Construction of Certain Infinite Families of sGraceful Graphs from a given graceful graph. Def. Sci. J. 32(3), (1982) 231-236

[8] J. C. Berbond, Graceful Graphs, Radio Antennae and French Wind Mills, Graph Theory and Combinatories, Pitman, London, (1979), 13-17

[9] K. Murugan, Square graceful labeling of Some Graphs. International Journal of Innovative Research in Science, Engineering and Technology, 4(2) (2015) 511-520

[10] K. Murugan and A. Subramanian. Skolem Difference Mean Graphs. Mapana J Sci 11(4) (2012) 109-120