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CORDIAL LABELING AND EDGE CORDIAL LABELING FOR SIXTEEN SPROCKET GRAPH

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Abstract: In this paper, a new concept of sixteen sprocket graph is introduced. The Eight sprocket graph is already proven as cordial in graph labeling. Here, we proved that the sixteen sprocket graph is also a cordial. Further, in our study we have investigated some sixteen sprocket graph related families of connected cordial and edge cordial graphs. Also the path union and the cycle of sixteen sprocket graph are cordial.

Keywords: Cordial labeling, Edge cordial, sixteen sprocket graph, path union of graph, cycle of graph.

1.INTRODUCTION

Graph theory is a vital branch of mathematics with a wide range of applications in the real world. Let G=(V,E) be the connected and simple graph, then V(G) and E(G) stand for the vertex set and the edge set of G respectively. The Cardinality of these sets are denoted by |V(G)| and |E(G)| are called the Order and Size of G. One of the most active areas of research in graph theory is graph labeling. If the vertices are assigned by some values or numbers subject to certain rules is known as Graph labeling. In a graph labeling, numerous types of labeling methods can be seen. Cordial labeling is one of method in graph labeling. A binary vertex labeling of a graph G is named as a Cordial labeling if $|\upsilon f(0)-\upsilon f(1)|\leq 1$ and $|\varrho f(0)-\varrho f(1)|\leq 1$. A graph that admits cordial labeling is named as Cordial graph.

The concept of cordial labeling of graph was introduced by Cahit in 1987 and for numbering in graph was defined by S.W.Golomb. It is found from Gallian that many researchers have studied cordialness of several graphs. The eight sprocket graph was introduced by J.C.Kanani and V.J.Kaneria. The same authors have already proved the graceful and cordial labeling for the said graph. The edge cordial and total edge cordial labeling for eight sprocket graph was introduced by A.A.Sathakuthulla, M.G Fajlul Kareem.

Here we introduced a new graph which is called Sixteen sprocket graph. We investigate some sixteen sprocket graph related families of connected cordial and edge cordial graphs. Also thepathunion and the cycle of the sixteen sprocket graph are cordial. In this paper, the notations and definitions are followed from Harary [1].

First of all let us recall some basic definitions, which are useful for the present work.

2. PRELIMINARIES

Definition 2.1.Cordial labeling

A binary vertex labeling of a graph G is called a Cordial labeling of G under f. If $|\upsilon f(0)-\upsilon f(1)|\leq 1$ and $|\varrho f(0)-\varrho f(1)|\leq 1$.A graph G is cordial if it admits cordial labeling.

Where,

$\upsilon f(0)$ = number of vertices of G having label 0 under f.

$\upsilon f(1)$ = number of vertices of G having label 1 under f.

$\varrho f(0)$ = number of edges of G having label 0 under f.

$\varrho f(1)$ = number of edges of G having label 1 under f.

Definition 2.2.Vertex cordial labeling

A function $f:V\rightarrow\{0,1\}$ is called a binary vertex labeling of a graph and f(v) is called label of the vertex of G under f. For e=(u,v), the induced function $f^*:E\rightarrow\{0,1\}$ defined as $f^*(e)=|f(u)-f(v)|$.

Definition 2.3.Edge cordial labeling

A Edge cordial labeling as a binary edge labeling : $E(G)\rightarrow\{0,1\}$, with the induced vertex labeling given by

$$f(v)=\sum_{uv\in E} f(u,v)(mod\ 2)$$

for each υ∈V such that |υf(0)-υf(1)|≤1 and |ϱf(0)-ϱf(1)|≤1 where ϱf(i) and υf(i), (i=0,1,2,3,...) are number of edges and vertices labeled with 0 and 1.

Definition 2.4. Path union and cycle graph

Let G be a graph and G=$G_1,G_2, \dots ,G_n$, n≤2 be n copies of graph G. Let υ∈V(G). Then the graph obtained by joining vertex υ of G^i with the same vertex of G^{i+1} by an edge, for all i=1,2,3,...,n-1 is called a path union of n copies of a graph G. Also if the same vertex υ of G^n join by an edge with υ of G^1 then such graph is known as cycle graph of n copies of G.

These are denoted by P(n.G) and C(n.G). Obviously P(n.k_1)=P_n and C(n.k_n)=C_n.

Definition 2.5.Eight sprocket graph

Eight Sprocket graph [8] is an Union of eight copies of C_{4n}. If $V_{i,j}$(∀i=1,2,3,...,8, ∀j=1,2,3,...,4n) be vertices of i^{th} copy of C_{4n} then we shall combine $V_{1,4n}$ and $V_{2,1}$,$V_{2,4n}$ and $V_{3,1}$, $V_{3,4n}$ and $V_{4,1}$, $V_{4,4n}$ and $V_{5,1}$, $V_{5,4n}$ and $V_{6,1}$, $V_{6,4n}$ and $V_{7,1}$, $V_{7,4n}$ and $V_{8,1}$, $V_{8,4n}$ and $V_{1,1}$ by a single vertex.Wheren∈N-1. So, graph seems like a sprocket shape, and here number of sprockets are eight. Hence named as eight sprocket. It is denoted by $S_{Cn}$of n size. Where |V(S_{Cn})|=16n-8, |E(S_{Cn})|=16n.

3. MAIN SECTION

Definition 3.1.Sixteen Sprocket graph

Sixteen Sprocket graph is an Union of sixteen copies ofC_{8n}. If $V_{i,j}$(∀i=1,2,3,...,16, ∀j=1,2,3,...,8n) be vertices of i^{th}copy of C_{8n} then we shall combine $V_{1,8n}$ and $V_{2,1}$, $V_{2,8n}$ and $V_{3,1}$, $V_{3,8n}$ and$V_{4,1}$,$V_{4,4n}$ and $V_{5,1}$, $V_{5,8n}$ and $V_{6,1}$, $V_{6,8n}$ and$V_{7,1}$, $V_{7,8n}$ and $V_{8,1}$, $V_{8,8n}$ and $V_{9,1}$, $V_{9,8n}$ and $V_{10,1}$, $V_{10,8n}$ and $V_{11,1}$, $V_{11,8n}$ and$V_{12,1}$, $V_{12,8n}$ and $V_{13,1}$, $V_{13,8n}$ and $V_{14,1}$, $V_{14,8n}$ and $V_{15,1}$, $V_{15,8n}$ and $V_{16,1}$, $V_{16,8n}$ and$V_{1,1}$by a single vertex. Where n∈N-1. So, graph seems like a sprocket shape, and here number of sprockets are sixteen. Hence named as sixteen sprocket. It is denoted by S_{Cn} of n size. Where |V(S_{Cn})|=32n-16, |E(S_{Cn})|=32n.

Illustration: Sixteen sprocket graph is shown in figure 1 consisting n=16 sprockets with cordial labeling.

Theorem 3.1. An Sixteen sprocket graph S_{Cn} is a cordial graph, Where n∈N-1.

Proof. Let G=S_{Cn} be any Sixteen sprocket graph of size n, Where n∈N-1.

Each vertices of S_{Cn} like $V_{i,j}$(∀i=1,2,3,4,5,...,16,∀j=1,2,3,4,5,...,8n).

Since the number of vertices in G is p=|V(S_{Cn})|=32n-16 and q=|E(S_{Cn})|=32n.

We define the labeling function f:V(G)→{0,1} as follows

$$f(v_1,j) = \begin{cases} 0, & if\ j = 1,2,5,6, \dots ,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8, \dots ,8n-1,8n. \end{cases}$$

$$f(v_2,j) = \begin{cases} 0, & if\ j = 1,2,5,6, \dots ,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8, \dots ,8n-1,8n. \end{cases}$$

$$f(v_3,j) = \begin{cases} 0, & if\ j = 2,3,6,7, \dots ,8n-2,8n-1 \\ 1, & if\ j = 1,4,5,8, \dots ,8n-3,8n. \end{cases}$$

$$f(v_4,j) = \begin{cases} 0, & if\ j = 3,4,7,8, \dots ,8n-1,8n \\ 1, & if\ j = 1,2,5,6, \dots ,8n-3,8n-2. \end{cases}$$

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$$f(v_{13},j) = \begin{cases} 0, & if\ j = 1,4,5,8, \dots ,8n-3,8n \\ 1, & if\ j = 2,3,6,7, \dots ,8n-2,8n-1. \end{cases}$$

$$f(v_{14},j) = \begin{cases} 0, & if\ j = 1,2,5,6, \dots ,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8, \dots ,8n-1,8n. \end{cases}$$

$$f(v_{15},j) = \begin{cases} 0, & if\ j = 2,3,6,7,\ldots,8n-2,8n-1 \\ 1, & if\ j = 1,4,5,8,\ldots,8n-3,8n. \end{cases}$$

$$f(v_{16},j) = \begin{cases} 0, & if\ j = 3,4,7,8,\ldots,8n-1,8n \\ 1, & if\ j = 1,2,5,6,\ldots,8n-3,8n-2. \end{cases}$$

From the above labeling pattern. Hence, G is a cordial graph.

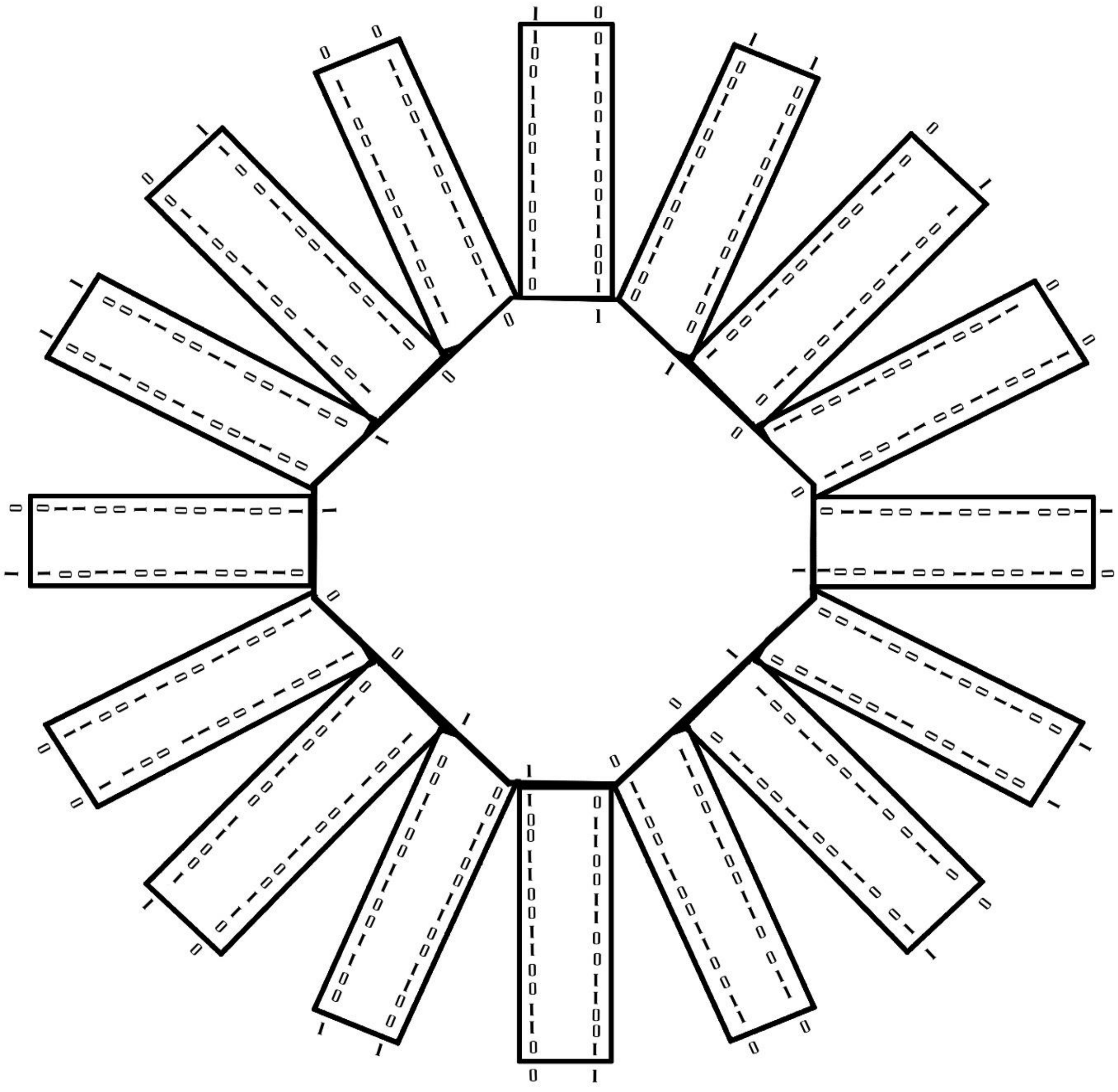

FIGURE 1. Cordial labeling of sixteen sprocket graph with p=498 and q=512.

Theorem 3.2. An Sixteen sprocket graph S_{Cn} is Edge cordial graph, Where n∈N-1.

Proof. Let G=S_{Cn} be any Sixteen sprocket graph of size n, Where n∈N-1.

Each vertices of S_{Cn} like $V_{i,j}$(∀i=1,2,3,4,5,...,16,∀j=1,2,3,4,5,...,8n).

Since the number of vertices in G is p=|V(S_{Cn})|=32n-16 and q=|E(S_{Cn})|=32n.

We define the labeling function f:E(G)→{0,1} as follows

$$f(e_1,j) = \begin{cases} 0, & if\ j = 3,4,7,8,\dots,8n-1,8n \\ 1, & if\ j = 1,2,5,6,\dots,8n-3,8n-2. \end{cases}$$

$$f(e_2,j) = \begin{cases} 0, & if\ j = 1,2,5,6,\dots,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8,\dots,8n-1,8n. \end{cases}$$

$$f(e_3,j) = \begin{cases} 0, & if\ j = 3,4,7,8,\dots,8n-1,8n \\ 1, & if\ j = 1,2,5,6,\dots,8n-3,8n-2. \end{cases}$$

$$f(e_4,j) = \begin{cases} 0, & if\ j = 1,2,5,6,\dots,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8,\dots,8n-1,8n. \end{cases}$$

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$$f(e_{13},j) = \begin{cases} 0, & if\ j = 1,2,5,6,\dots,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8,\dots,8n-1,8n. \end{cases}$$

$$f(e_{14},j) = \begin{cases} 0, & if\ j = 1,2,5,6,\dots,8n-3,8n-2 \\ 1, & if\ j = 3,4,7,8,\dots,8n-1,8n. \end{cases}$$

$$f(e_{15},j) = \begin{cases} 0, & if\ j = 3,4,7,8,\dots,8n-1,8n \\ 1, & if\ j = 1,2,5,6,\dots,8n-3,8n-2. \end{cases}$$

$$f(e_{16},j) = \begin{cases} 0, & if\ j = 3,4,7,8,\dots,8n-1,8n \\ 1, & if\ j = 1,2,5,6,\dots,8n-3,8n-2. \end{cases}$$

From the above labeling pattern. Hence G is an Edge cordial graph.

Illustration: Sixteen sprocket graph is shown in figure 1 consisting n=16 sprockets with edge cordial labeling with p=498 and q=512. Where V_f(0)=V_f(1)=120 and e_f(0)=e_f(1)=124.

Theorem 3.3. Path union of finite copies of the sixteen sprocket graph S_{Cn} is a cordial graph, where n∈N-1.

Proof. Let G=P(r.S_{Cn}) be a path union of r copies for the Sixteen sprocket graph of size n, Where n∈N-1. By theorem 3.1, In graph G,

We see that the vertices p=|V(G)|=r(32n-16) and the edges q=|E(G)|=(r-1)+r32(n).

Let $u_{k,i,j}$ (∀i=1,2,...,16,∀j=1,2,...,8n).

Where the vertices and edges of the k^{th} copy is p=32n-16 and q=32n. Join vertices by an edge to form the path union of r copies of sixteen sprocket graph.

We define the labeling function g:V(G)→{0,1} as follows

$$g(u_{1,i,j}) = \{f(u_{1,i,j})$$

$$g(u_{2,i,j}) = \begin{cases} g(u_{1,i,j})+1, & if\ j = 1,2,5,\dots,8n-3,8n-2 \\ g(u_{1,i,j})-1, & if\ j = 3,4,7,\dots,8n-1,8n \end{cases}$$

$$g(u_{3,i,j}) = \{\ g(u_{2,i,j})$$

$$g(u_{k,i,j}) = \{\ g(u_{k-3,i,j}), if\ k = 4,5,6,\dots,6n+1,6n+2,6n+3$$

From the above labeling pattern. Hence path union of finite copies of sixteen sprocket graph is cordial.

Theorem 3.4. Cycle of r copies of the Sixteen sprocket graph C(r.S_{Cn}) is a cordial graph, Where n∈N-1 and r≡0,3(mod 4).

Proof. Let G=C(r.S_{Cn}) be a cycle of Sixteen sprocket graph of size n, Where n∈N-1.

By theorem 3.1, In graph G, we see that the vertices p=|V(G)|=r(32n-16) and the edges q=|E(G)|=r(32(n)+1).

Let $u_{k,i,j}$(∀i=1,2,...,16,∀j=1,2,...,8n).

Where the vertices and edges of the k^{th} copy is p=32(n)-16 and q=32n. Join vertices by an edge to form the cycle of sixteen sprocket graph.

We define the labeling function g:V(G)→{0,1} as follows

$$g(u_{1,i,j}) = \{ f(u_{,i,j})$$

$$g(u_{2,i,j}) = \begin{cases} g(u_{1,i,j}) + 1, & if\ j = 1,2,5,\ldots,8n-3,8n-2 \\ g(u_{1,i,j}) - 1, & if\ j = 3,4,7,\ldots,8n-1,8n \end{cases}$$

$$g(u_{3,i,j}) = \{ g(u_{2,i,j})$$

$$g(u_{4,i,j}) = \{ g(u_{1,i,j})$$

$$g(u_{k,i,j}) = \{ g(u_{k-3,i,j}), if\ k = 5,6,7,\ldots,8n+1,8n+2,8n+3$$

$$g(u_{k,i,j}) = \{ g(u_{k-4,i,j}), if\ k = 8,12,16,\ldots,8n+4$$

Hence the above labeling pattern give rise a cordial labeling to cycle of r copies for sixteen sprocket graph.

4.CONCLUSION

In this paper, we introduced cordial and edge cordial labeling for sixteensprocket graph and also the path union and cycle of sixteen sprocket graph are cordial. Some theorems are discussed and verify the results through illustrations which provide better understanding to derived results. To investigate similar results for other graph families is an open area of research.

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