

# ON DIRECT SUM OF FIVE INTUITIONISTIC FUZZY GRAPHS

Mr. N. Velmurugan

Assistant Professor, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram-605602, Tamil Nadu, India.

R. Kirubasri

II-M.Sc Mathematics, PG and Research Department of Mathematics, Theivanai Ammal College For Women (Autonomous), Villupuram-605602, Tamil Nadu, India.

**ABSTRACT:** This paper explores that, we illustrate the direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  of five intuitionistic fuzzy graphs (IFGs)  $G_L, G_M, G_N, G_O$  and  $G_P$  is determined. The regular property, connectedness, effectiveness and balanced IFGs on the direct sum of five intuitionistic fuzzy graphs are also examined.

**KEYWORDS AND PHRASES:** Fuzzy graph, direct sum, degree of vertices in an (IFGs). Regular, connected, effective and balanced of an intuitionistic fuzzy graph.

## 1. INTRODUCTION

A graph is a convenient way of representing information involving relationship between objects. The objects are portrayed by vertices and relations by edges. At present a large number and variety of applications have been developed that use graphs for knowledge representation. Generally, an undirected graph is a *symmetric binary relation* on a non-empty vertex set  $V$ . A fuzzy graph (undirected) is also a symmetric binary fuzzy relation on a fuzzy subset.

In 1975, Rosenfeld [1] regarded the fuzzy relation of fuzzy set and developed the theory of fuzzy graphs. Although the first definition of fuzzy graph was specified by Kaufman. R. T. Yeh and S. Y. Banh [6] have also introduced various connectedness concept in fuzzy graphs. L. A. Zadeh [8] in 1965 as a generalisation of classical (crisp) sets.

Intuitionistic fuzzy graph were introduced by Krassimar T. Atanassov [11] and the operations on intuitionistic fuzzy graphs were defined by, R. Parvathi and M. G. Karunambigai [10]. Dr. K. Radha and Mr. S. Arumugam [2] defined the direct sum of two fuzzy graphs. Later on Dr. S. Karthikeyan and Mrs. K. Lakshmi [12] defined the direct sum of two (IFGs). A. Nagoorgani and S. Shajitha Begum [9] gave the various type of degree in IFGs.

In this article, the degree of vertices in the direct sum of Five Intuitionistic Fuzzy Graphs (IFGs) is calculated with an example. The direct sum of five regular, connected, effective and balanced (IFGs) are discussed with an some example and theorem.

## 2. BASIC DEFINITIONS

### 2.1 INTUITIONISTIC FUZZY GRAPH:

An intuitionistic fuzzy graph is the class of  $G=(V,E)$ , where

(i)  $V=\{v_1, v_2, \dots, v_n\}$  such that  $\alpha_1:V \rightarrow [0,1]$  and  $\beta_1:V \rightarrow [0,1]$  denote the degree of membership and non-membership of the element  $v_i \in V$  respectively, and  $0 \leq \alpha_1(v_i) + \beta_1(v_i) \leq 1$ , for every  $v_i \in V$ , ( $i=1,2,3,\dots,n$ ).

(ii)  $E \subset V \times V$  where  $\alpha_2: V \times V \rightarrow [0,1]$  and  $\beta_2: V \times V \rightarrow [0,1]$  are such that,

$$\alpha_2(v_i, v_j) \leq \min \{ \alpha_1(v_i), \alpha_1(v_j) \}, \beta_2(v_i, v_j) \leq \min \{ \beta_1(v_i), \beta_1(v_j) \} \text{ and}$$

$$0 \leq \alpha_2(v_i, v_j) + \beta_2(v_i, v_j) \leq 1.$$

Here the triple  $(e_i, \alpha_{1i}, \beta_{1i})$  indicates the degree of membership and degree on non-membership of the vertex  $v_i$ . The triple  $(e_i, \alpha_{2i}, \beta_{2i})$  denotes the degree of membership and non-membership of the edge  $e_{ij} = (v_i, v_j)$  on  $V$ .

### 2.2 DEGREE OF VERTEX:

Let  $G = (V, E)$  be the intuitionistic fuzzy graph. Then the degree of vertex  $u$  is signified by,

$$d(u) = (d_\alpha(u), d_\beta(u))$$

Where,  $d_\alpha(v) = \sum_{u \neq v} \alpha_2(v, u)$  and

$$d_\beta(v) = \sum_{u \neq v} \beta_2(v, u).$$

### 2.3 REGULAR INTUITIONISTIC FUZZY GRAPH:

Let  $G = (V, E)$  be the intuitionistic fuzzy graph. If all over the vertices have the same degree. Then it is mentioned as regular intuitionistic fuzzy graph.

#### 2.3.1 EXAMPLE:

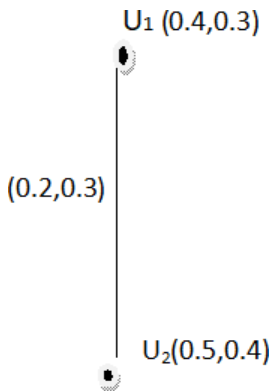


Fig: Regular IFG

### 2.4 CONNECTED INTUITIONISTIC FUZZY GRAPH:

In an intuitionistic fuzzy graph  $G = (V, E)$ . If  $G$  has only one component. Then IFGs is claimed to be connected intuitionistic fuzzy graph. By the another way of explanation, If there is a path between every pair of vertices, then  $G$  is uttered to be a connected intuitionistic fuzzy graph.

#### 2.4.1 EXAMPLE:

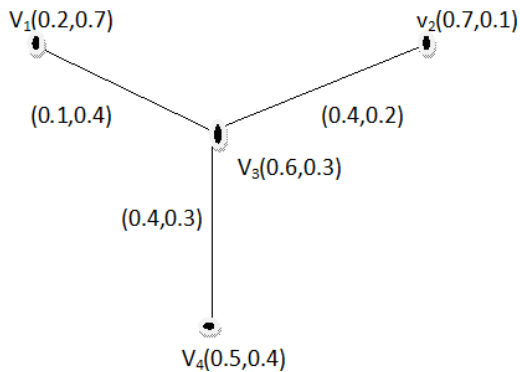


Fig: Connected IFG

### 2.5 EFFECTIVE INTUITIONISTIC FUZZY GRAPH:

In an intuitionistic fuzzy graph  $G = (V, E)$ . The effective intuitionistic fuzzy graph is considered as,

$$\alpha_2(uv) = \alpha_1(u) \wedge \alpha_1(v) \text{ and } \beta_2(uv) = \beta_1(u) \wedge \beta_1(v) \text{ for all } u, v \in E.$$

#### 2.5.1 EXAMPLE:

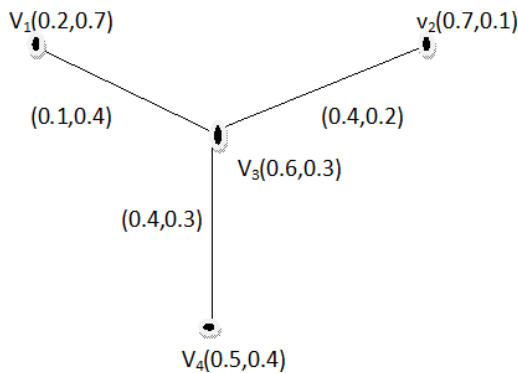


Fig: Effective IFG

## 2.6 DIRECT SUM OF INTUITIONISTIC FUZZY GRAPH:

Let  $G_L = ((v_i, \alpha_{1iL}, \beta_{1iL}), (e_{ij}, \alpha_{1ijL}, \beta_{1ijL}))$  and  $G_M = ((v_i, \alpha_{2iM}, \beta_{2iM}), (e_{ij}, \alpha_{2ijM}, \beta_{2ijM}))$  signifies the two IFGs with the crucial crisp graphs  $G_L^* = (V_1, E_1)$  and  $G_M^* = (V_2, E_2)$  respectively, Let  $v \in V_1 \cup V_2$  and let  $E = \{uv | u, v \in V, uv \in E_1 \text{ or } uv \in E_2 \text{ but not both}\}$ . Define,  $G = G_L \oplus G_M$  by

$$(\alpha_1, \beta_1)_{(u)} = \begin{cases} (\alpha_{1L}, \beta_{1L}) & \text{if } u \in V_1 \\ (\alpha_{1M}, \beta_{1M}) & \text{if } u \in V_2 \\ (\alpha_{1L} \vee \alpha_{1M}, \beta_{1L} \vee \beta_{1M}) & \text{if } u \in V_1 \cap V_2 \end{cases}$$

and

$$(\alpha_2, \beta_2)_{(u) \leq} \begin{cases} (\alpha_{1L}(u) \wedge \alpha_{1L}(v), \beta_{1L}(u) \vee \beta_{1L}(v)) & \text{if } uv \in E_1 \\ (\alpha_{1M}(u) \wedge \alpha_{1M}(v), \beta_{1M}(u) \vee \beta_{1M}(v)) & \text{if } uv \in E_2 \end{cases}$$

Then  $G$  is the direct sum of two IFGs  $G_L$  and  $G_M$ .

## 3. DIRECT SUM OF FIVE INTUITIONISTIC FUZZY GRAPHS:

### 3.1 DEFINITION:

Let  $G_L = ((v_i, \alpha_{1iL}, \beta_{1iL}), (e_{ij}, \alpha_{1ijL}, \beta_{1ijL}))$ ,  $G_M = ((v_i, \alpha_{2iM}, \beta_{2iM}), (e_{ij}, \alpha_{2ijM}, \beta_{2ijM}))$ ,  $G_N = ((v_i, \alpha_{3iN}, \beta_{3iN}), (e_{ij}, \alpha_{3ijN}, \beta_{3ijN}))$ ,  $G_O = ((v_i, \alpha_{4iO}, \beta_{4iO}), (e_{ij}, \alpha_{4ijO}, \beta_{4ijO}))$  and  $G_P = ((v_i, \alpha_{5iP}, \beta_{5iP}), (e_{ij}, \alpha_{5ijP}, \beta_{5ijP}))$  signifies the five intuitionistic fuzzy graphs escorted by the crucial crisp graphs,  $G_L^* = (V_1, E_1)$ ,  $G_M^* = (V_2, E_2)$ ,  $G_N^* = (V_3, E_3)$ ,  $G_O^* = (V_4, E_4)$  and  $G_P^* = (V_5, E_5)$  sequentially. Let  $v \in V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$  and let  $E = \{uv | u, v \in V, uv \in E_1 \text{ or } uv \in E_2 \text{ or } uv \in E_3 \text{ or } uv \in E_4 \text{ or } uv \in E_5\}$

Illustrate,  $G = G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  by

$$(\alpha_1, \beta_1)_{(u)} = \begin{cases} (\alpha_{1L}, \beta_{1L}) & \text{if } u \in V_1 \\ (\alpha_{1M}, \beta_{1M}) & \text{if } u \in V_2 \\ (\alpha_{1N}, \beta_{1N}) & \text{if } u \in V_3 \\ (\alpha_{1O}, \beta_{1O}) & \text{if } u \in V_4 \\ (\alpha_{1P}, \beta_{1P}) & \text{if } u \in V_5 \\ (\alpha_{1L} \vee \alpha_{1M} \vee \alpha_{1N} \vee \alpha_{1O} \vee \alpha_{1P}, \beta_{1L} \wedge \beta_{1M} \wedge \beta_{1N} \wedge \beta_{1O} \wedge \beta_{1P}) & \text{if } u \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \end{cases}$$

and

$$(\alpha_2, \beta_2)_{(uv) \leq} \begin{cases} (\alpha_{1L}(u) \wedge \alpha_{1L}(v), \beta_{1L}(u) \vee \beta_{1L}(v)) & \text{if } uv \in E_1 \\ (\alpha_{1M}(u) \wedge \alpha_{1M}(v), \beta_{1M}(u) \vee \beta_{1M}(v)) & \text{if } uv \in E_2 \\ (\alpha_{1N}(u) \wedge \alpha_{1N}(v), \beta_{1N}(u) \vee \beta_{1N}(v)) & \text{if } uv \in E_3 \\ (\alpha_{1O}(u) \wedge \alpha_{1O}(v), \beta_{1O}(u) \vee \beta_{1O}(v)) & \text{if } uv \in E_4 \\ (\alpha_{1P}(u) \wedge \alpha_{1P}(v), \beta_{1P}(u) \vee \beta_{1P}(v)) & \text{if } uv \in E_5 \end{cases}$$

Therefore, G is called the direct sum of Five Intuitionistic Fuzzy Graphs  $G_L, G_M, G_N, G_O$  and  $G_P$ .

EXAMPLE:

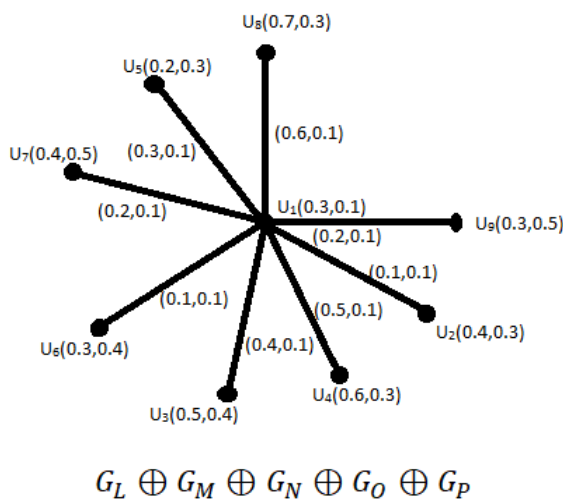
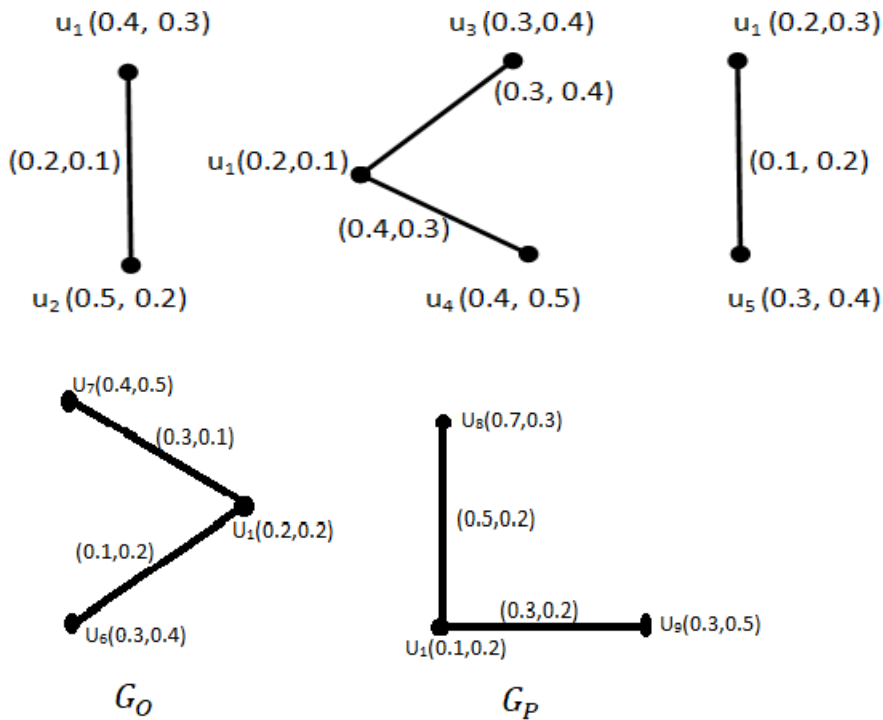


Fig: On Direct Sum of IFG

### 3.1 DEGREE OF VERTICES IN $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$ :

In the present section, we identify the degree of vertices in the direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  of five IFGs  $G_L, G_M, G_N, G_O$  and  $G_P$  in view of the degree of vertices in the IFGs  $G_L, G_M, G_N, G_O$  and  $G_P$ .

#### 3.1.1 THEOREM:

The degree of vertex in  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  which is corresponding to the degree of vertices in  $G_L, G_M, G_N, G_O$  and  $G_P$  is given by,

$$d_{G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P}(u) = \begin{cases} d_{G_L}(u) & \text{if } u \in V_1 \\ d_{G_M}(u) & \text{if } u \in V_2 \\ d_{G_N}(u) & \text{if } u \in V_3 \\ d_{G_O}(u) & \text{if } u \in V_4 \\ d_{G_P}(u) & \text{if } u \in V_5 \\ d_{G_L}(u) + d_{G_M}(u) + d_{G_N}(u) + d_{G_O}(u) + d_{G_P}(u) & \text{if } u \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \text{ and } E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5 \neq \varphi \end{cases}$$

Proof: In  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  for any vertices we poses two cases to evaluate,

Case(i): If either  $u \in V_1$  or  $u \in V_2$  or  $u \in V_3$  or  $u \in V_4$  or  $u \in V_5$  exists. Then the edge prevalence at  $u$  lies in  $E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5$ . If  $u \in V_1$  then ,

$$\begin{aligned} d_{G_L \oplus G_M}(u) &= (d\alpha_L(u), d\beta_L(u)) \\ &= d_{G_L}(u) \end{aligned}$$

Where,  $d\alpha_L(u) = \sum_{u \neq v} \alpha_2(u, v)$  and  $d\beta_L(u) = \sum_{u \neq v} \beta_2(u, v)$

If  $u \in V_2$  then ,

$$\begin{aligned} d_{G_M \oplus G_N}(u) &= (d\alpha_M(u), d\beta_M(u)) \\ &= d_{G_M}(u) \end{aligned}$$

Where,  $d\alpha_M(u) = \sum_{u \neq v} \alpha_2(u, v)$  and  $d\beta_M(u) = \sum_{u \neq v} \beta_2(u, v)$

Similarly, for  $u \in V_3, u \in V_4$  and  $u \in V_5$ .

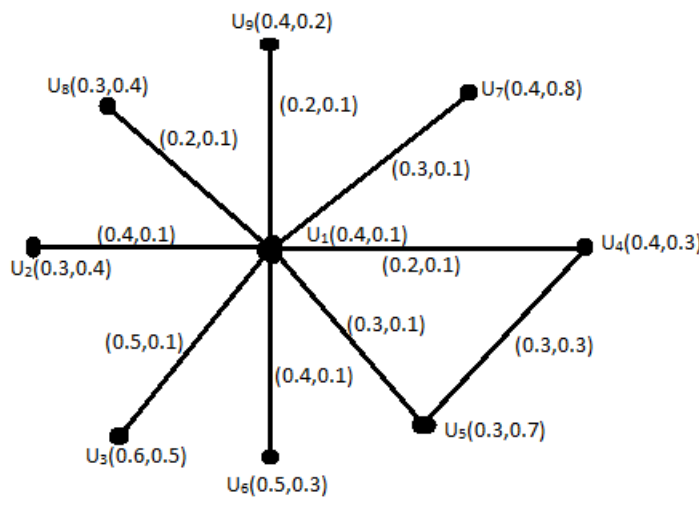
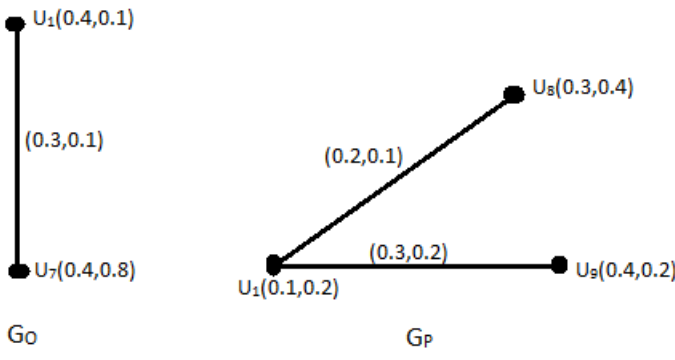
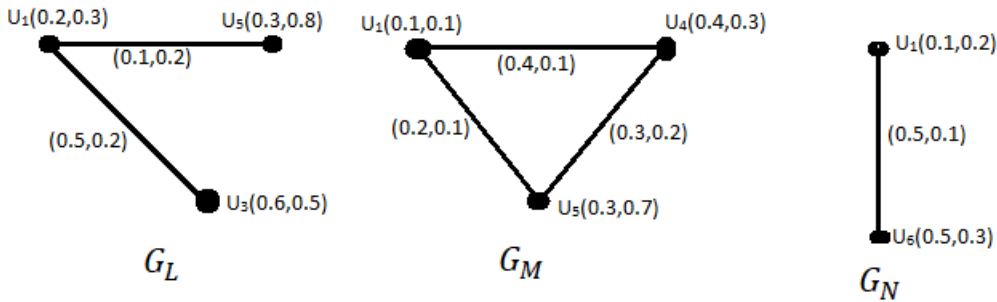
Case(ii): If the edge does not occurrence at  $u$  lies in  $E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5$ . But  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$ . Then any edge occurrence at  $u$  is either in  $E_1$  or in  $E_2$  or in  $E_3$  or in  $E_4$  or in  $E_5$ . Also all these edges are involved in  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  inclined by

$$\begin{aligned} d_{G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P}(u) &= \\ (d\alpha_L(u), d\beta_L(u)) &+ (d\alpha_M(u), d\beta_M(u)) + \\ &(d\alpha_N(u), d\beta_N(u)) + (d\alpha_O(u), d\beta_O(u)) + \\ &(d\alpha_P(u), d\beta_P(u)) \\ &= d_{G_L}(u) + d_{G_M}(u) + d_{G_N}(u) + d_{G_O}(u) + d_{G_P}(u) \end{aligned}$$

Hence the theorem is proved.

### 3.1.2 EXAMPLE:

The subsequent show that the degree of vertices in  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  which the edge set are disjoint.



$G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$

3.2 DIRECT SUM OF FIVE REGULAR IFGs:

If  $G_L = ((v_i, \alpha_{1iL}, \beta_{1iL}), (e_{ij}, \alpha_{1ijL}, \beta_{1ijL}))$ ,  $G_M = ((v_i, \alpha_{2iM}, \beta_{2iM}), (e_{ij}, \alpha_{2ijM}, \beta_{2ijM}))$ ,  $G_N = ((v_i, \alpha_{3iN}, \beta_{3iN}), (e_{ij}, \alpha_{3ijN}, \beta_{3ijN}))$ ,  $G_O = ((v_i, \alpha_{4iO}, \beta_{4iO}), (e_{ij}, \alpha_{4ijO}, \beta_{4ijO}))$  and  $G_P = ((v_i, \alpha_{5iP}, \beta_{5iP}), (e_{ij}, \alpha_{5ijP}, \beta_{5ijP}))$  signifies the five IFGs. Then the direct sum is inessential to the regular intuitionistic fuzzy graphs.

3.2.1 THEOREM: If  $G_L, G_M, G_N, G_O$  and  $G_P$  are regular IFGs along with degree  $K_1, K_2, K_3, K_4, K_5$  sequentially and  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \emptyset$  then  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is regular if and only if  $K_1 = K_2 = K_3 = K_4 = K_5$ .

PROOF: Let  $G_L$  be the  $K_1$  regular IFG with crucial crisp graph  $G^*_L = (V_1, E_1)$ . In the same way, let  $G_M$  be the  $K_2$ ,  $G_N$  be the  $K_3$ ,  $G_O$  be the  $K_4$  and  $G_P$  be the  $K_5$  is the regular IFGs with the crucial crisp graph as  $G^*_M$

$= (V_2, E_2), G_N^* = (V_3, E_3), G_O^* = (V_4, E_4)$  and  $G_P^* = (V_5, E_5)$  sequentially such that,  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \varphi$ . Assume that,  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is regular.

$$d_{G_L} \oplus d_{G_M} \oplus d_{G_N} \oplus d_{G_O} \oplus d_{G_P}(u) = \begin{cases} d_{G_L}(u) & \text{if } u \in V_1 \\ d_{G_M}(u) & \text{if } u \in V_2 \\ d_{G_N}(u) & \text{if } u \in V_3 \\ d_{G_O}(u) & \text{if } u \in V_4 \\ d_{G_P}(u) & \text{if } u \in V_5 \\ d_{G_L}(u) + d_{G_M}(u) + d_{G_N}(u) + d_{G_O}(u) + d_{G_P}(u) & \text{if } u \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \text{ and } E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5 \end{cases}$$

Since,  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \varphi$

$$d_{G_L} \oplus d_{G_M} \oplus d_{G_N} \oplus d_{G_O} \oplus d_{G_P}(u) = \begin{cases} d_{G_L}(u) = K_1 & \text{if } u \in V_1 \\ d_{G_M}(u) = K_2 & \text{if } u \in V_2 \\ d_{G_N}(u) = K_3 & \text{if } u \in V_3 \\ d_{G_O}(u) = K_4 & \text{if } u \in V_4 \\ d_{G_P}(u) = K_5 & \text{if } u \in V_5 \end{cases}$$

Since,  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is regular.

We obtain,  $K_1 = K_2 = K_3 = K_4 = K_5$  conversely assume that,  $G_L, G_M, G_N, G_O$  and  $G_P$  are K-regular fuzzy graphs such that,  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \varphi$ . Then the degree of any vertex in the direct sum is given by,

$$d_{G_L} \oplus d_{G_M} \oplus d_{G_N} \oplus d_{G_O} \oplus d_{G_P}(u) = K \text{ for every } u \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5. \text{ Hence } G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P \text{ is regular IFGs.}$$

### 3.3 DIRECT SUM OF FIVE CONNECTED IFGs:

If  $G_L = ((v_i, \alpha_{1iL}, \beta_{1iL}), (e_{ij}, \alpha_{1ijL}, \beta_{1ijL}))$ ,  $G_M = ((v_i, \alpha_{2iM}, \beta_{2iM}), (e_{ij}, \alpha_{2ijM}, \beta_{2ijM}))$ ,  $G_N = ((v_i, \alpha_{3iN}, \beta_{3iN}), (e_{ij}, \alpha_{3ijN}, \beta_{3ijN}))$ ,  $G_O = ((v_i, \alpha_{4iO}, \beta_{4iO}), (e_{ij}, \alpha_{4ijO}, \beta_{4ijO}))$  and  $G_P = ((v_i, \alpha_{5iP}, \beta_{5iP}), (e_{ij}, \alpha_{5ijP}, \beta_{5ijP}))$  signifies the five disconnected IFGs. And then their the direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is told to be a connected intuitionistic fuzzy graphs.

**3.3.1 THEOREM:** If  $G_L : ((v_i, \alpha_{1iL}, \beta_{1iL}), (e_{ij}, \alpha_{1ijL}, \beta_{1ijL}))$ ,  $G_M : ((v_i, \alpha_{2iM}, \beta_{2iM}), (e_{ij}, \alpha_{2ijM}, \beta_{2ijM}))$ ,  $G_N : ((v_i, \alpha_{3iN}, \beta_{3iN}), (e_{ij}, \alpha_{3ijN}, \beta_{3ijN}))$ ,  $G_O : ((v_i, \alpha_{4iO}, \beta_{4iO}), (e_{ij}, \alpha_{4ijO}, \beta_{4ijO}))$  and  $G_P : ((v_i, \alpha_{5iP}, \beta_{5iP}), (e_{ij}, \alpha_{5ijP}, \beta_{5ijP}))$  represented as five connected IFGs with the crucial crisp graphs  $G_L^* = (V_1, E_1), G_M^* = (V_2, E_2), G_N^* = (V_3, E_3), G_O^* = (V_4, E_4)$  and  $G_P^* = (V_5, E_5)$  sequentially on this wise  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \varphi$  and  $E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5$ . Then their direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is connected intuitionistic fuzzy graphs.

**PROOF:** Since,  $G_L$  is connected IFGs  $((\alpha_{1iL}^\infty(u,v), \beta_{1iL}^\infty(u,v))) > 0, \forall (u,v) \in E_1$ .  $G_M$  is connected IFGs  $((\alpha_{2iM}^\infty(u,v), \beta_{2iM}^\infty(u,v))) > 0, \forall (u,v) \in E_1$ .  $G_N$  is connected IFGs  $((\alpha_{3iN}^\infty(u,v), \beta_{3iN}^\infty(u,v))) > 0, \forall (u,v) \in E_1$ .  $G_O$  is connected IFGs  $((\alpha_{4iO}^\infty(u,v), \beta_{4iO}^\infty(u,v))) > 0, \forall (u,v) \in E_1$ .  $G_P$  is connected IFGs  $((\alpha_{5iP}^\infty(u,v), \beta_{5iP}^\infty(u,v))) > 0, \forall (u,v) \in E_1$ . Also  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \neq \varphi$ . Consequently, there exist atleast one vertex which is in  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$ . Although there is no edge in  $E_1 \cap E_2 \cap E_3 \cap E_4 \cap E_5$ . Hence there exist a path between any two vertices in  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P(v, \alpha, \beta)$  of  $G_L, G_M, G_N, G_O$  and  $G_P$ . This implies that,  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  is connected.

Hence the proof.

### 3.4 DIRECT SUM OF FIVE EFFECTIVE IFGs:



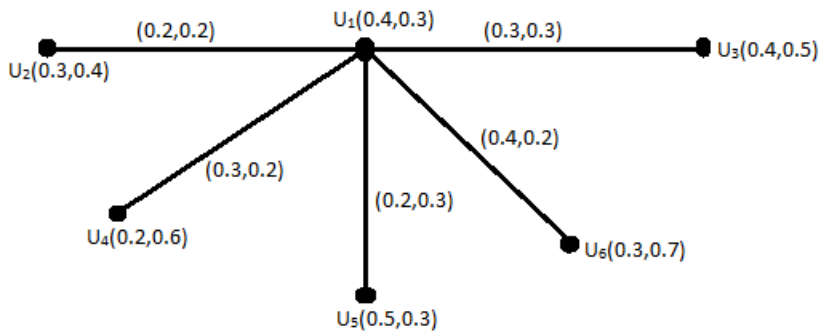
In an intuitionistic fuzzy graph  $G = (V, E)$ . The effective intuitionistic fuzzy graph is considered as,

$$\alpha_2(uv) = \alpha_1(u) \wedge \alpha_1(v) \quad \text{for all } u, v \in E.$$

and

$$\beta_2(uv) = \beta_1(u) \wedge \beta_1(v) \quad \text{for all } u, v \in E.$$

3.4.1 EXAMPLE:



$$G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$$

Fig: Direct Sum of Five Effective IFG

3.4.2 THEOREM: If  $G_L, G_M, G_N, G_O$  and  $G_P$  are five IFGs such that there is no edge of

$G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  carries ends in  $V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$  and every edge  $uv$  of

$G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  including one end  $u \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$  and  $uv \in E_1$  or  $E_2$  or  $E_3$  or  $E_4$  or  $E_5$  such that,

$$\begin{aligned} &\alpha_{1L}(u) \geq \alpha_{1L}(v), \text{ or } \beta_{1L}(u) \geq \beta_{1L}(v), \text{ or } \alpha_{1M}(u) \geq \alpha_{1M}(v), \text{ or} \\ &\beta_{1M}(u) \geq \beta_{1M}(v), \text{ or } \alpha_{1N}(u) \geq \alpha_{1N}(v), \text{ or } \beta_{1N}(u) \geq \beta_{1N}(v), \text{ or} \\ &\alpha_{1O}(u) \geq \alpha_{1O}(v), \text{ or } \beta_{1O}(u) \geq \beta_{1O}(v), \text{ or } \alpha_{1P}(u) \geq \alpha_{1P}(v), \text{ or} \\ &\beta_{1P}(u) \geq \beta_{1P}(v). \end{aligned}$$

PROOF: Let  $u, v$  be an edge of  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  we have consider two cases,

Case(i):  $uv \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$ . Subsequently,  $uv \in V_1$  or  $V_2$  or  $V_3$  or  $V_4$  or  $V_5$ . Finally,  $\alpha_1(u) = \alpha_{1L}(u)$ ,

$\alpha_1(v) = \alpha_{1L}(v)$  and  $\alpha_2(uv) = \alpha_{2L}(uv)$ ,  $\alpha_2(uv) = \alpha_{2L}(uv)$ . Considering

that,  $G_L$  is an effective IFG.  $\alpha_2(uv) = \alpha_{2L}(uv)\beta_2(uv) = \beta_{2L}(uv)$

$$= \alpha_{1L}(u) \wedge \alpha_{1L}(v) = \beta_{2L}(u) \vee \beta_{2L}(v)$$

$$= \alpha_1(u) \wedge \alpha_1(v) = \beta_2(u) \vee \beta_2(v)$$

Case(ii): If  $uv \in V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5 \notin V_1 \cap V_2 \cap V_3 \cap V_4 \cap V_5$  (or vice-versa) indispensable loss of generality, assume that  $v \in V_1$ .

Subsequently,  $\alpha_2(uv) = \alpha_{2L}(uv)$  by the prediction,  $\alpha_{1L}(u) \geq \alpha_{1L}(v)$ , or  $\beta_{1L}(u) \geq \beta_{1L}(v)$ . At the moment,

$$\alpha_1(u) = \alpha_{1L}(u) \vee \alpha_{1L}(v) \beta_1(u) = \beta_{1L}(u) \vee \beta_{1L}(v)$$

$$\geq \alpha_{1L}(u) \geq \beta_{1L}(u)$$

$$\geq \alpha_{1L}(v) \geq \beta_{1L}(v)$$

$$\geq \alpha_1(v) \geq \beta_1(v)$$

So,  $\alpha_1(v) = \alpha_1(u) \wedge \alpha_1(v)$  and  $\beta_1(v) = \beta_1(u) \vee \beta_1(v)$ . Consequently,

$$\alpha_2(uv) = \alpha_{2L}(uv)\beta_2(uv) = \beta_{2L}(uv)$$



$$\begin{aligned}
&= \alpha_{1L}(u) \wedge \alpha_{1L}(v) &= \beta_{2L}(u) \vee \beta_{2L}(v) \\
&= \alpha_{1L}(v) = \beta_{2L}(v) \\
&= \alpha_1(v) &= \beta_2(v) \\
&= \alpha_1(u) \wedge \alpha_1(v) &= \beta_2(u) \vee \beta_2(v)
\end{aligned}$$

Thence,  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  be an effective IFG.

#### CONCLUSION:

In this paper, the direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  of three fuzzy intuitionistic fuzzy graphs  $G_L, G_M, G_N, G_O$  and  $G_P$  is defined. A formula to find the degree of vertices in the direct sum  $G_L \oplus G_M \oplus G_N \oplus G_O \oplus G_P$  of three intuitionistic fuzzy graph  $G_L, G_M, G_N, G_O$  and  $G_P$  and is obtained. Some of the property of the direct sum of regular, connected and effective intuitionistic fuzzy graphs has been illustrated. This operation on intuitionistic fuzzy graph is great tool to consider large fuzzy graph device its properties from those of the small ones. A truly tactical man oeuvre in that direction of the whole is specifically made through this paper.

#### REFERENCES:

1. A.Rosenfeld, fuzzy Graphs, In L.A.Zadeh, K.S.Tanaka, M.Shimura, Fuzzy Sets and their Application to cognitive and Decision processes, Academic Press, New York, (1975), 77-95.
2. K.Radha and S.Arumugam, On Direct Sum of Two Fuzzy Graphs, International Journal of Scientific and Research Publications, 3 (5) (2013).
3. Frank Hararay, Graph Theory, Narosa Addison Wesley, Indian student Edition, (1988).
4. A.Nagorgani and K.Radha, Conjunction of Two Fuzzy Graphs, International Review of Fuzzy Mathematics, 3 (2008), 95-105
5. P. Bhattacharya, Some remarks on fuzzy graphs, Pattern Recognition Letters, vol.6.pp.297-302, 1987.
6. R. T. Yeh and S. Y. Banh, Fuzzy Relations, Fuzzy Graphs an their applications to clustering analysis, In fuzzy sets and their applications to cognitive and decision process, L.A. Zadeh, K.S. Fu, M. Shimura eds: Academic press, New York, 1975.
7. J.N.Mordeson and C.S.Peng, Operations on Fuzzy Graphs, Information Sciences, 79 (1994), 159-170.
8. L.A. Zadeh, Fuzzy sets, Information and control, vol.8,pp.338-353, 1965.
9. A.Nagoorgani, S.Shajitha Begam, Degree, Order and Size in Intuitionistic Fuzzy Graphs, International Journal of Algorithms, Computing and Mathematics, Aug, 2010.
10. M.G.Karunambigai and R.Parvathi, Intuitionistic Fuzzy Graphs, Proceedings of 9th Fuzzy Days International Conference on Computational Intelligence, Advance in soft Computing, Computational Intelligence, Theory and Applications, Springer – Verlag, 20 (2006), 139- 150.
11. K.Atanassov, IFS: Theory and Applications, Physics – Verlag, New York(1999)
12. Dr. S. Karthikeyan, Mrs. K. Lakshmi, On Direct Sum of Intuitionistic Fuzzy Graphs, International Research Journal of Engineering and Technology(IRJET) vol:03, May 2016.