

# VERTEX EVEN MEAN LABELING OF SOME GRAPHS WITH PENDANT EDGES

**P. Kavitha**

Assistant Professor, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram-605 602, Tamilnadu, India.

**B. Manimegalai**

M.Sc Mathematics, PG and Research Department of Mathematics, Theivanai Ammal College for Women (Autonomous), Villupuram-605 602, Tamil Nadu, India.

## ABSTRACT

In this paper, we investigate vertex even mean labelling with pendant edges. We prove any cycle and complete graph is a vertex even mean labelling. Similar way of plotting and labelling methodology is called a vertex even mean labelling. Here  $Q_m + K_n$  and  $K_3 + C_n$  is a graph we used for plotting in vertex even mean labelling.

**Keywords:** Mean labeling, Vertex even mean labeling, Join of Graphs.

## INTRODUCTION

All graphs in this paper are finite, simple and undirected.  $V(G)$  represented vertex set and  $E(G)$  represent edge set. A vertex labelling is a function of  $V$  to a set of labels. A Graph with such a vertex labelling function is defined as vertex labelled graph. The concept of mean labelling was explicated and introduced in the work by Somasundaram and Ponraj [3]. Even mean labelling of some graphwork was presented by the author Revathi N in her work[5]. With the above reference and analysis we tried and investigated the possibilities of even mean labelling with  $Q_m + K_n$  and  $K_3 + C_n$  with pendant edges.

## PRELIMINARIES

### DEFINITION 2.1 MEAN LABELING OF GRAPH

A Graph  $G$  With  $(p, q)$  is a mean graph if there is injective function  $f$  from the vertices of  $G$  to  $\{0, 1, 2, \dots, q\}$  such that when each edge  $uv$  is labeled with  $(f(u)+f(v))/2$  if  $f(u) + f(v)$  is even and  $(f(u)+f(v)+1)/2$  if  $f(u) + f(v)$  is odd then the ensuring edges are different.

### DEFINITION 2.2 VERTEX EVEN MEAN LABELING

A Graph  $G$  with  $q$  edges to be an vertex even mean graph if there is an injective function  $f$  from the vertices of  $G$  to  $\{2, 4, 6, \dots, 2q\}$  such that the edge labels are given by  $f(u) + f(v) / 2$  are distinct. such a function is called a vertex mean labeling.

### DEFINITION 2.3 JOIN OF GRAPHS

The join of graphs  $k_3$  and  $C_n$ ,  $K_3 + C_n$  is obtained by joining a vertex of  $k_3$  with every vertex of  $C_n$  with an edge.

### DEFINITION 2.4 PENDANT EDGE

An edge of a graph is said to be a pendant edge if and only if one of its vertices is a pendant edge.

## THEOREM 3.1:

### MAIN RESULT

The graph obtained by adding 4 pendant edges to each vertex of  $k_n$  in the graph  $Q_m + k_n$  admits vertex even mean labeling.

### PROOF:

The order and size of the graph  $G$  obtained by adding 4 pendant edges to each vertex of  $k_n$  in the graph respectively.

Let  $v_1$  and  $v_2$  be the vertices of  $Q_m$ ,  $u_j$  ( $1 \leq j \leq n$ ) be the vertices of  $k_n$ . Obviously it  $u_{j+t}$  ( $1 \leq t \leq 4n$ ) will be the pendant vertices corresponding to  $u_j$ .

Define a vertex labeling function:

$f: (Q_m + k_n) \rightarrow \{2, 4, 6, \dots, 2q\}$  by as follows

$$f(v_1) = 2f(v_2) = 4f(v_3) = 46$$

$$f(v_n) = 6j + 10, j=7,8,9\dots$$

$$10j - 4t, t = 1, 1 \leq j \leq 4n$$

$$10j - 2t + 2, t = 2, 1 \leq j \leq 4n$$

$$f(u_{jt}) =$$

$$\{$$

$$10j - t + 3, t = 3, 1 \leq j \leq 4n$$

$$10j - t + 6, t = 4, 1 \leq j \leq 4n$$

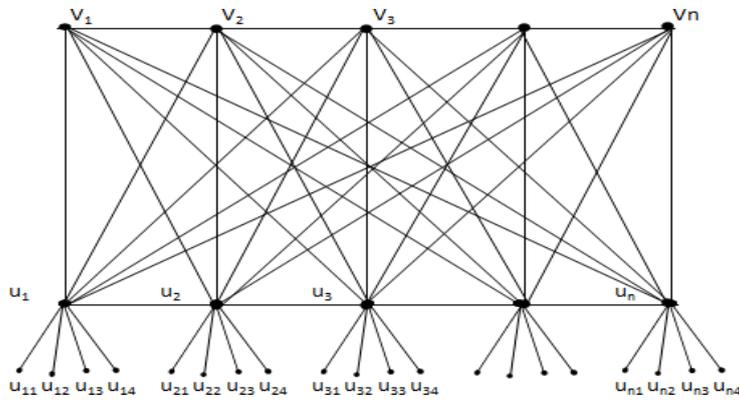


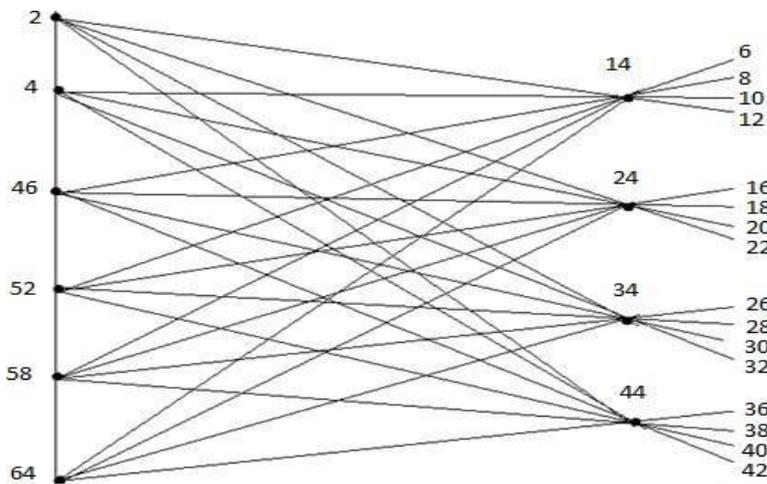
FIGURE 1

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the graph  $Q_m + k_n$  has vertex even mean labeling.

**ILLUSTRATION 3.2**

A graph obtained by adding 4 pendant edges to each vertex of the graph  $Q_6 + k_4$

FIGURE 2



This figure 2 shows the vertex even mean labeling of the graph  $Q_6 + k_4$ .

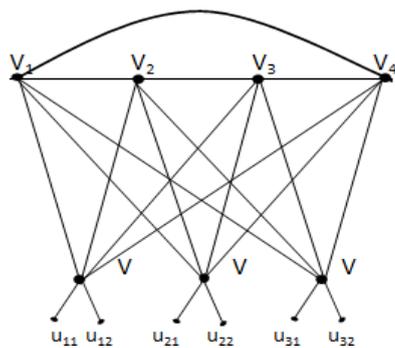
**THEOREM 3.2**

The graph  $k_3 + C_n$  has  $n+1$  vertex even mean labeling with 2 pendant edge

**PROOF:**

The graph  $k_3 + C_n$  has  $n+1$  vertices and  $2n$  edges

Let  $v$  be a vertices of  $k_3$  and  $v_1, v_2, \dots, v_n$  be the vertices of the cycle.



**FIGURE 3**

The ordinary labeling of  $k_3 + C_4$  is given in the above figure

Define a vertex labeling  $f: v(k_3 + C_n) \rightarrow \{2, 4, 6, \dots, 2q\}$  by follows as

$$f(u) = 2$$

$$f(v_j) = 4j, 1 \leq j \leq n$$

if  $n$  is odd

$$4j, j = 1$$

$$f(v_j) = \{2j + 4, j \text{ is even if } n \text{ is odd}$$

$$q + 2j + 2, j \text{ is odd}$$

$$10j + 6t$$

$$, t = 1, 1 \leq j \leq 2n$$

$$j t \quad f(u) = \{$$

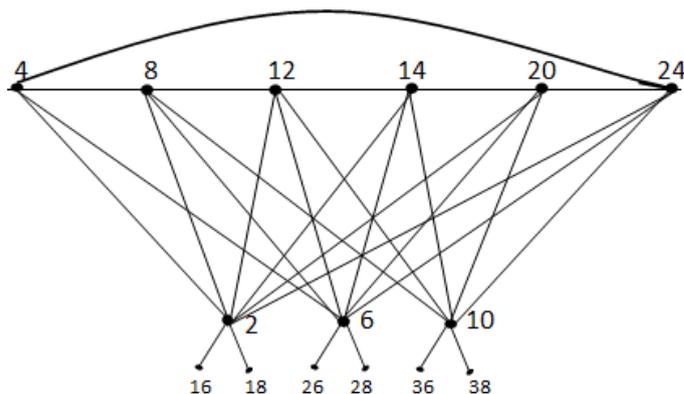
$$10j + 2t + 4, t = 2, 1 \leq j \leq 2n$$

clearly labels of the edges received by the mean of the labels on end vertices are all distinct.

Hence the graph  $k_3 + C_n$  has vertex even mean labeling with 2 pendant edges.

**ILLUSTRATION 3.4:**

A graph obtained by adding 2 pendant vertex edge to each vertex of the graph  $k_3 + C_6$  and its vertex even mean labeling is given in the below figure .



**FIGURE 4**

This figure 4 shows that vertex even mean labeling  $k_3 + C_6$  with 2 pendant edges.

## CONCLUSION:

In this paper, we have obtained some graphs that are vertex even mean labelling with pendant edges. We attempt Graph operations on cycles and complete graph with  $Q_m+K_n$  and  $K_3+C_n$ . In future, we prove yet another labelling on similar graphs.

## REFERENCES:

- [1]. F. Harary, Graph theory, Reading, MA: Addison- Wesley ,1994.
- [2]. J.A. Gallian, A dynamical survey of graph labeling, The Electronic journal of Combinatorics, 17 (2014).
- [3]. R. Ponraj and S. Somasundaram, Mean labeling of graphs, National Academy Science Letter 26(2003),210-213.
- [4]. K. Manickam and M. Marudai, odd mean labeling of graphs, Bulletin of pure and Applied Sciences 25 E (1) (2006).
- [5]. S. Arockiaraj and B. S. Mahadevaswamy, Even vertex odd mean labeling of graphs obtained from graph operations, Int, Journal of Advance Research in Edu, Tech. and Management 3(1) (2015), 192.
- [6]. A. Sasikala and P. Oviya, vertex odd mean labeling of some graphs with pendant edges.