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Mathematical Model of Planetary Gear Train for Geared Rotary Actuator

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Abstract

Planetary Gear Trains are basically of two types viz; Simple gear trains and Compound gear Train. In this paper we have discussed about Compound type Planetary Gear Trains. Compared to simple planetary gears, compound planetary gears have the advantages of larger reduction ratio, higher torque-to-weight ratio, and more flexible configurations. In spite of these advantages, vibration remains a major concern in planetary gear applications. Vibration creates undesirable noise, reduces fatigue life of the whole system, and decreases durability and reliability. Vibration reduction, therefore, is a key to the applications of compound planetary gears. Gears in the Planetary Gear Trains are one of the most critical components in the mechanical power transmission system in which failure of one gear will affect the whole transmission system, thus it is very necessary to determine the causes of failure in an attempt to reduce them. The different modes of failure of gears as bending failure (load failure), Pitting (contact stresses), scoring and abrasive wear, in any case it is related to the loads acting on the gear.

Index Terms: Planetary Gear Trains, Compound type gear train, Durability, Reliability, Vibration, Pitting and Bending Failure, Scoring and Abrasive wear

1. INTRODUCTION

1.1 What is GRA

Geared Rotary Actuators are mechanically operable devices widely used in the aerospace industry so that controlled motion can be provided to secondary flight control surfaces. Their usage can be seen in a variety of applications including powered hinges for aircraft structural movements, they support aerodynamic loads in addition to the surface hinge moment, as an actuator in a linked mechanism where they supply actuation torque. Mostly GRAs are situated along with the bay door drives, and also can be used for controlling leading or trailing edge movements of the aircraft. [1,2]

Geared Mechanical Actuator is a geared mechanism that is used to convert mechanical motion (often rotary) into rotary motion at a different combination of speeds and force. They are also used when required to change the direction of motion. Unlike hydraulic or electromechanical actuators, they receive their power from an external source [3,4,5]. Further GRA is classified as simple planetary actuators and compound differential planetary actuators. Simple planetary actuators are most commonly found in commercial leading-edge slat applications and are bolted to structure driving rack and pinions to translate the slat surface. Compound differential planetary actuators are most commonly used in trailing edge flap designs. It offers higher ratios for torque multiplication while driving rack and pinions to translate the flap surfaces [6,7].

Geared Rotary Actuators are initially used for the positional control of wing-mounted flight control Surfaces of an aircraft. Their usage ranges from their application in powered hinges, when directly supporting the aerodynamic load forces in addition to the surface hinge moment, to actuators in a linked mechanism where they provide the actuation torque and the aerodynamic load reaction and is delegated to other load-bearing equipment.

These actuators are typically part of a larger system which includes a power drive, mechanical interconnects, and feedback devices to control the motion of multiple devices [8,9,10].

A mechanism is termed a planetary mechanism if it contains at least one rigid body which is required to rotate about its own axis and at the same time to revolve about another axis. Points on this body will generate epicycloids or hypocycloids. Therefore planetary mechanism is often called an epicyclic or cyclic mechanism.

Planetary gears are widely used in all kinds of transmission systems, such as wind turbines, aircraft engines, automobiles, and machine tools, and they are classified into two categories: simple and compound planetary gears. Simple planetary gears have one sun, one ring, one carrier, and one planet set. Compound planetary gears involve one or more of the following three types of structures: meshed-planet (there are at least two or more planets in mesh with each other in each planet train), stepped-planet (there exists a shaft connection between two planets in each planet train), and multi-stage structures. Compared to simple planetary gears, compound planetary gears have the advantages of larger reduction ratio, higher torque-to-weight ratio, and more flexible configurations [11,12].

Here for our analysis purpose we have developed a compact carrierless compound type planetary gearbox model. To develop this model we have considered martensitic stainless steel viz; AISI 440C tempered at 316^0C. This model is developed to provide higher power to weight ratio, very high torque to speed ratio with good speed reduction capabilities. This type of gear arrangements generally used in aircraft flight control surfaces [13,14]. Hence, factor of safety, loads and stress generating in gears has to be analysed precisely such that performance of gear components will not be affected.

2. FAILURE OF GEARS:

In spite of various advantages, there are various factors that affect life of gear components. This planetary type of gear arrangements play vital role in transmission systems. In which failure of one of the gear components will affect the complete transmission system. Following are some of the factors that affect the life of gear components.

2.1 Vibration & Noise:

Vibration remains a major concern in planetary gear applications. Vibration creates undesirable noise, reduces fatigue life of the whole system, and decreases durability and reliability. Vibration reduction, therefore, is a key to the applications of compound planetary gears. This requires analytical study on compound planetary gear dynamics to provide fundamental understanding of the dynamics and guide vibration reduction.

2.2 Breakage of Gear Tooth:

The complete breakage of the tooth due to static and dynamic loads can be avoided by adjusting the parameters in the gear design, such as module and face width so that the beam strength of the gear tooth is more than the sum of static and dynamic loads.

2.3 Surface Destruction:

The surface destruction or tooth wear is classified according to the basis of their primary causes. The basic types of gear tooth wear are given below:

a) **Abrasive Wear:**

Foreign particles in the lubricant, such as dirt, rust, weld, spatter or metallic debris can scratch the tooth surface. Abrasive wear can be prevented by the provision of oil filters, increase surface hardness and use of high viscosity oils. A thick lubrication film will allow fine particles to pass without scratching.

b) **Corrosive Wear:**

Corrosive wear is due to the chemical action of the lubricating oil or the additives. The tooth is roughened due to wear and Chemical wear of flank of internal gear caused by the acidic lubricant.

c) **Adhesive Wear:**

Unlike scoring, adhesive wear is hard to detect. It occurs right from the start. Since the rate of wear is very low, it may take millions of cycles for noticeable wear. Prior to full load transmission, gears are run in at various fractions of full load for several cycles. The surface peaks are quashed over a long period of running and the surface gets the polished appearance.

d) **Scoring:**

Scoring is due to the combination of two distinct activities first, lubrication failure in the contact region and second, establishment of metal to metal contact. Later on, welding and tearing action resulting from metallic contact removes the metal rapidly and continuously so far, the load, speed and oil temperature remain at the same level. The scoring is classified into initial, moderate and destructive.

- **Initial Scoring:**

Initial scoring occurs at the high spots left by the previous machining. Lubrication failure at these spot leads to initial scoring or scuffing. Once these high spots are removed, the stress comes down as the load is distributed over a larger area. The scoring will then stop if the load, speed and temperature of oil remain unchanged or reduced. Initial scoring is non-progressive and has the corrective action associated with it.

- **Moderate Scoring:**

After initial scoring if the load, speed or oil temperature increases, the scoring will spread over to a larger area. The Scoring progresses at the tolerable rate. This is called moderate scoring.

- **Destructive Scoring:**

After the initial scoring, if the load, speed or oil temperature increases appreciably, then severe scoring sets in with heavy metal tore regions spreading quickly throughout. Scoring is normally predominant over the pitch line region since elasto hydrodynregion. In dry running, surfaces may seize.

2.4 Gear Failure due to Load:

2.4.1 Micropitting on Gear Teeth:

Micropitting can affect gears and failures due to micropitting are very common in gearboxes. Micropitting occurs when the lubricant film between contacting surfaces is not thick enough and the surfaces have high amounts of sliding action. Micropitting results in a frosted or matte finish surface in affected areas. Micropitting-related failures can be prevented by changing lubricant type or by reducing component surface roughness.

2.4.2 Macropitting on Gear Tooth:

Macropitting occurs when the contact stress in the gear exceeds the fatigue strength of the material. Macropitting that occurs before the end of the design life is an indication that one or more design assumptions, such as contact stress, material properties, lubricant condition or applied load, were not met. Macropitting results in craters on the gear tooth. Beach marks due to the presence of corrosion and lubricant in the crack are sometimes present and indicate a fatigue progression process. Macropitting failures can be prevented by reducing loads, improving gear profiles to reduce stress, using cleaner steel, or increasing material strength, through alloy selection or a heat treatment process.

2.4.3 Fretting Corrosion:

Fretting is a surface-wear phenomenon that occurs when two contacting surfaces have small oscillating relative motions, with no lubricant film between the surfaces. Fretting corrosion can be identified by the presence of ruts along the lines of contact, along with the presence of reddish-brown or black wear debris. Fretting corrosion can be prevented by minimizing the amount of time that a gearbox spends without rotating or by improving transportation conditions, depending on the cause of the fretting corrosion.

2.4.4 Axial Crack:

Axial cracking is a phenomenon that occurs in bearings, almost always on the bearing inner ring. Failures of this type have become very common in wind turbine gearboxes and were the subject of an article in the June 2013 issue of North American wind power. The cracks develop in the axial direction, perpendicular to the direction of rolling. Axial crack failures are most likely to occur in through-hardened bearings. Axial crack failures can be prevented by using case carburized bearings, ensuring that the appropriate amount of retained austenite is present, applying a black oxide coating, and ensuring the correct level of interference fit exists between the bearing inner ring and the shaft on which it is mounted.

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3. OBJECTIVE:

The main objective for this system is to make a trade-off between speed and torque i.e. to provide necessary high torque at relatively very low speeds. The torque provided by the actuator is not enough to accelerate the aircraft structure at the required rate and thus the torque needs to be increased at the expense of speed. This can be achieved by placing an efficient transmission system between motor and the differential. An arrangement of gears is an ideal system to achieve this goal (In this case our gearing model will provide the reduction ratio i.e. $1/r = 148:1$). Gears are used in almost every Transmission system to transmit power from one shaft to another and to regulate the speed and torque. Gears have a high efficiency and can be manufactured using basic machining processes. The different gear arrangements that can be used are a single stage gear reduction, a multistage reduction and a planetary or an epicyclic gear train. In single and multi-stage gear train, the gear axes are fixed, and the gears rotate about their respective axes. However, in a planetary gear train, the axes of some of the gears are not fixed but rotate about the axes of other gears. Planetary gear trains can transmit at very high transmission ratios and require smaller gears in a compact space.

4. METHODOLOGY:

In present era of manufacturing, gears are designed to obtain high efficiency solutions with minimum inputs. Wherein case of transmission systems or planetary gear trains it has to be designed and optimized with standard procedures. Above mentioned all the problems can be overcomed or minimized by performing standard mathematical calculations, CAD Modelling and then by analysing the mathematical and CAD model by Finite Element Approach.

First, to start designing the transmission system, the power obtained and the output parameters from the motor is needed. Based on the data available i.e. RPM of the motor available at the input, desired torque at the output of actuator. Performing permutations and combinations, different set of teeth values were determined. Based on the available data of gear teeth, using Willis's Equation value of Reduction Ratio is determined followed by determination of pitch circle diameter, pitch line velocity, beam strength, wear strength and factor of safety. Whole calculations are performed by considering that complete planetary gear transmission is taking place without carrier between planets. i.e. carrier-less planetary gear transmission. For proper alignment of Planet Gears around the periphery of sun gear, Support Rings are used. Using available data CAD model is designed using SOLIDWORKS 16.0.

5. DESIGN PARAMETERS:

As per the given data we need to design a device which will be capable of transmitting very high torque at very low speeds. As per the given data, device must be capable of providing 4500 N-m torque as output with output gear rotating at the speed of 20 RPM. So, here we have designed carrier less compound Planetary Gear Train which will be capable of providing high torque as an output. While designing gear set which will meet our requirements, we have performed number of iterations to find out suitable number of teeth values so that design will be compact and operable according to our requirements.

a) **Material to be used** = AISI 440C / ASTM S44004 / X105CrMo17
Ultimate tensile Strength (S_{ut}) = 2170 MPa
Yield Tensile Strength ($S_{yt)}$ = 2090 MPa
Brinell Hardness Number (BHN) = 610

b) Using Willi's formula for gear ratio we determined satisfactory value of reduction ratio. i.e. Reduction ratio = -148.2:1
c) Using this reduction ratio value and after satisfying all the tooth combination(selection) criteria's below are the obtained values of tooth combinations as:

- Number of teeth on Sun Gear, Z1 = 35
- Number of teeth on Sun Gear Spline, Z1 ′ = 34
- Number of teeth on Central Gear of planetary pinion, Z2 = 28
- Number of teeth on Outer Gear of planetary pinion, Z3 = 29
- Number of teeth on Moving Ring Gear, Z4 = 91
- Number of teeth on Fixed Ring Gear, Z5 = 92

Pitch Circle Diameter (D_p)

$D_{p1} = m * Z_1 = 1.75 * 35 = 61.25$ mm
$D_{p2} = m * Z_2 = 1.75 * 28 = 49$ mm
$D_{p3} = m * Z_3 = 1.75 * 29 = 50.75$ mm
$D_{p4} = m * Z_4 = 1.75 * 91 = 159.25$ mm
$D_{p5} = m * Z_5 = 1.75 * 92 = 161$ mm

Tangential force due to rated torque can be found as: (Considering 6 planet gears)

$$P_t = \frac{2 * M_t}{6D_{p1}} = \frac{2 * 30364.37}{6 * 61.25} = 165.2482721 \text{ N}$$

Where, M_t is torque available at input of sun gear in N-mm.
Effective load acting on gear tooth can be given as,

$$\mathbf{P_{eff}} = \frac{\mathbf{C_s}}{\mathbf{C_v}} * \mathbf{P_t}$$

$$\mathbf{P_{eff}} = \frac{1.75}{0.239891} * 165.248272 = \mathbf{1205.48226\ N}$$

Where C_s and C_v are Service factors and velocity factors respectively.

- **Design against Bending Failure:**

$$\begin{aligned}\mathbf{S_b} &= m * b * Y * \sigma_b \\ &= \mathbf{16525.45417\ N}\end{aligned}$$

Where, b – face width of sun gear,
Y – Lewis form factor,
σ_b – permissible bending stress ($\sigma_b = S_{ut}/3$)

FOS= $\frac{S_b}{P_{eff}} = \frac{16525.45417}{1205.48226} = 13.709$

- **Design against Pitting/Wear Failure:**

$$S_w = D_{p1} * b * Q * k$$

$$S_w = D_{p1} * b * Q * 0.16 * \left(\frac{BHN}{100}\right)^2$$

Where, Q is a Ratio factor,

$$\mathbf{S_w = 14181.144\ N}$$

FOS= $\frac{S_w}{P_{eff}} = \frac{14181.144}{1205.48226} = 11.764$

But, in this case factor of safety is very high. Hence, as per the standards for industrial purpose, here we have used Buckingham's equation for attaining required factor of safety (FOS).

Buckingham's equation for dynamic load ($\mathbf{P_d}$),

$$P_d = \frac{21v(c * e * b + P_t)}{21v + \sqrt{(c * e * b) + P_t}}$$

Where, c is deformation factor and e is sum of errors between two Meshing Teeth,

$$\mathbf{P_d} = \frac{21 * 9.5056(11400 * 0.0052 * 35 + 165.248272)}{21 * 9.5056 + \sqrt{(11400 * 0.0052 * 35) + 165.248272}}$$

$= \mathbf{1797.0296\ N}$

For Buckingham's Equation Effective load can be determined as,

$P_{eff} = (c_s * P_t + P_d)$

$= (1.75 * 165.248272) + 1797.0296$

$\mathbf{P_{eff} = 2086.2\ N}$

In industrial and heavy duty applications gears are mostly subjected to fatigue and repetitive type of stresses. Hence, for such applications spur gears are mostly designed with consideration of wear failure criterion. Hence, FOS can be found as:

FOS= $\frac{S_w}{P_{eff}} = \frac{4673.48224}{4535.542} = \mathbf{6.7975}$

So, here we got comparatively better values of factor of safety. But still there is scope of improvement by changing parameters like face width of gears (b) up to some extent or by changing material used for manufacturing of GRA.

6. RESULTS

As per the design requirements whole system must be compact and will provide less Weight to Torque ratio. As well as operation must be safe and efficient i.e. Factor of Safety of operation must be high. As per the available set of calculations, device must be capable of providing 4500 N-m torque as output with output gear rotating at the speed of 20 RPM. To attain the high value of factor of safety values of Beam strength and wear strength should be high compare to effective load. High value of beam or wear strength is achieved by selecting the material which is having high value of ultimate tensile strength. So that it will increase the beam or wear strength of gears without altering the other parameters. i.e. weight and size of the gear model.

d) Using Willi's formula for gear ratio we determined satisfactory value of reduction ratio. i.e. Reduction ratio = -148.2:1

e) Using this reduction ratio value and after satisfying all the tooth combination criteria's obtained values of tooth combinations are as:

Z_1=35, Z_1'=34, Z_2=28, Z_3=29, Z_4=91, Z_5=92

f) While satisfying all the dimensional criteria's of PGT, it's also required that torque must be transmitted efficiently without any failure of the component as well as without affecting the performance of device. Hence, factor of safety is determined based on wear strength failure criteria for each transmission pair of gears using buckingham's dynamic load equation. It's as follows:

- For 1-2 gear pair, determined value of factor of safety(FOS) is 6.7975
- For 2-4 gear pair, determined value of factor of safety(FOS) is 2.5292
- For 3-5 gear pair, determined value of factor of safety(FOS) is 2.0742

7. CONCLUSION

While considering various factors that affect the life and operation of compact gear systems, one of the prime objective of this work is to develop a system which is highly compact and light in weight. From the analytical and numerical results we are able to obtain high reduction ratio which made this system possible to provide low-speed high-torque as output. As well as, we obtained satisfying value of Factor of safety while taking into consideration the criteria of wear failure.

From the study it has been observed that Factor of Safety (FOS) can be further improved by altering various parameters. Basic parameters which also affect the FOS are Ultimate strength ($U_{ts)}$ well as yield strength ($Y_{ts)}$ of material, face width of gear, module of gear and service factor of gear.

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