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INITIAL BOUNDS FOR CERTAIN SUBCLASSES OF BI-UNIVALENT FUNCTION ASSOCIATED WITH THE HORADAM POLYNOMIAL

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ABSTRACT:

In the present research, we take into account new subclass of holomorphic bi-univalent characteristic defined through Haradam Polynomial. We achieve Co-efficient estimate for the defined elegance. Also, we debate Fekete-Szegö inequality for feature belongs to those subclasses.

1. Introduction and Preliminaries

Let $\mathcal{A}$ denote the class of analytic function $f(z)$ in the open unit disk Δ with a montel normalization $f(0) = 0 \quad and \quad f'(0) = 1$

A function $f \in \mathcal{A}$ has the Taylor series expansion of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \qquad (1.1)$$

In the Riemann mapping theorem, every simply connected domain Ω which is not the whole complex plane $\mathbb{C}$, can be mapped conformally onto the open unit disk.

$\Delta=\{z: z \in \mathbb{C}$ and $|z| < 1\}$

The Koebe one-quarter theorem [5] ensures that the range of every function $f \in \mathcal{S}$ contains the disk $w: |w| < \frac{1}{4}$

It is well known that every function $f \in \mathcal{S}$ has an inverse f' defined by

$$f^{-1}(f(z)) = z \qquad (z \in \Delta)$$

and

$$f(f^{-1}(w)) = w \left(|w| < r_0(f); \quad r_0(f) \geq \frac{1}{4}\right).$$

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3 f)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \ldots \qquad (1.2)$$

Let $f(z)$ and $g(z)$ be two analytic function in $\mathcal{A}$, then $f(z)$ is said to be subordinate to $g(z)$, denoted by $f(z) \prec g(z)$, if there exists a Schwarz function $w(z)$ which is analytic in Δ with $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$.

A function $f \in \mathcal{A}$ is said to be bi-univalent in Δ if both the function f and its f^{-1} are univalent in Δ.

Let $\sum$ denote the class of bi-univalent function in Δ given by(1.1)

The object of the present paper is to introduce two new subclasses of the function class $\sum$ employing the techniques used earlier by Srivastava et al.[10]. In order to derive our main results, the coefficient estimate problem involving the bound of $|a_n|(n \in N - \{1,2\}$,
$\mathcal{N} = \{1,2,3,..\})$ is still an open problem.

The Horadam polynomial $h_k(r)$ are defined by

$$h_k(r) = prh_{k-1}(r) + qh_{k-2}(r), \tag{1.3}$$

with $h_1(r)$=e, $h_2(r)$=br,$h_3(r) = pbr^2 + eq$ where e,b,p,and q are some real constants.

The generating function of the Horadam polynomials $h_k(r)$ is given by Horadam [6]

$$\psi(r,z) = \sum_{k=1}^{\infty} h_k(r) z^{k-1} = \frac{e+(b-ep)rz}{1-prz-qz^2}$$

Definition 1 : *Let* $\delta > -1, \vartheta \in \mathbb{C}/\{0\}, 0 \le \eta \le 1, 0 \le \rho \le 1 \quad and \quad 0 \le v \le 1$. *A function* $k \in \sum$ *is in the class* $\mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,v;\psi)$, *if it is satisfying the following subordination conditions:*

$$1 + \frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi)) + \eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}k(\xi)))}{(1-\rho)\xi + \rho(1-v)\mathcal{R}_q^{\delta}k(\xi) + v\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi)} - \frac{1}{[1+v(1-\rho)]}\right] \prec \psi(r,z) + 1 - e \tag{1.4}$$

and

$$1 + \frac{1}{\vartheta}\left[\frac{\omega\partial_q(\mathcal{R}_q^{\partial}\chi(w)) + \eta\omega^2\partial_q(\partial_q(\mathcal{R}_q^{\partial}\chi(w)))}{(1-\rho)\omega + \rho(1-v)\mathcal{R}_q^{\partial}\chi(w) + v\omega\partial_q\mathcal{R}_q^{\partial}\chi(w))} - \frac{1}{[1+v(1-\rho)]}\right] \prec \psi(r,w) + 1 - e \tag{1.5}$$

2. Main Results

Theorem 1: *Let* $k(\xi) \in \mathcal{S}_q^{\delta}(\vartheta,\eta,\rho,v;\psi)$ *be of the form in equation(1.1).Then*

$$|a_2| \le \frac{\sqrt{2}[2]_q|\vartheta||br|\sqrt{|br|}(\rho v - v - 1)^2}{\sqrt{|\vartheta[\delta+1]_q b^2 r^2(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q) - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|}} \tag{2.1}$$

and

$$|a_3| \le br|\vartheta|(\rho v - v - 1)^2\left(\frac{br|\vartheta|(\rho v - v - 1)^2}{\Theta^2(\eta,\rho,v,\delta,q)} + \frac{[2]_q}{|\Upsilon(\eta,\rho,v,\delta,q)|}\right) \tag{2.2}$$

and for some $v \in \mathcal{R}$,

$$|a_3 - va_2^2| \leq \begin{cases} \dfrac{|br|[2]_q v(\rho v - v - 1)^2}{2Y(\eta,\rho,v,\delta,q)} \\ if \quad |v-1| \leq \dfrac{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|}{2Y(\eta,\rho,v,\delta,q)2v^2(\rho v - v - 1)b^2r^2} \\ \dfrac{|br|^3(v-1)2v^2(\rho v - v - 1)^2}{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|} \\ if \quad |v-1| \geq \dfrac{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|}{2Y(\eta,\rho,v,\delta,q)2v^2(\rho v - v - 1)b^2r^2} \end{cases}$$

where

$$\Theta(\eta,\rho,v,\delta,q) = [\delta+1]_q([2]_q - q\rho v + [2]_q\eta v - [2]_q\rho\eta v - \rho + [2]_q\eta) \quad (2.3)$$

$$Y(\eta,\rho,v,\delta,q) = [\delta+1]_q[\delta+2]_q([3]_q - q[2]_q\rho v + [2]_q[3]_q\eta v - [2]_q[3]_q\rho\eta v\rho[2]_q[3]_q\eta) \quad (2.4)$$

and

$$\begin{aligned} \psi(\eta,\rho,v,\delta,q) = \; & -2[3]_q[\delta+2]_q + (4[3]_q[\delta+2]_q - 4[2]_q^2[\delta+1]_q)\rho v \\ & + (2[2]_q^3[\delta+1]_q - 4[2]_q[3]_q[\delta+2]_q)\eta v + (2q[2]_q[\delta+1]_q - 2q[2]_q[\delta+2]_q)\rho^2 v^2 \\ & + 2[2]_q^2][\delta+1]_q\rho\eta - 2[2]_q[\delta+1]_q\rho^2 - (2[2]_q(q-1)[\delta+1]_q + 2[\delta+2]_q)\rho^2 v \\ & + (2[2]_q^3[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q)\eta v^2 + (2[2]_q^2[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q)\rho^2\eta v^2 \\ & + (4[2]_q[3]_q[\delta+2]_q - 2[2]_q(1+[2]_q)[\delta+1]_q)\rho\eta v^2 + 4[2]_q[3]_q[\delta+2]_q\rho\eta v \\ & + (2[2]_q^3[\delta+1]_q - 2[3]_q[\delta+2]_q)v - 2[2]_q^2[\delta+1]_q\rho^2\eta v - 2[2]_q[3]_q[\delta+2]_q\eta \\ & + (2[2]_q^2[\delta+1]_q + 2[\delta+2]_q)\rho + (2q[2]_q^2[\delta+1]_q - 2q[2]_q[\delta+2]_q)\rho v^2 \end{aligned}$$

(2.5)

Proof: Let $f \in \mathcal{S}_q^\delta(\vartheta,\eta,\rho,v;\psi)$ and then there are two holomorphic functions $k, y: \Delta \to \Delta$ given by

$$|k(z)| = k_1 z + k_2 z^2 + k_3 z^3 + \ldots (z \in \Delta) \quad (2.6)$$

and

$$|y(w)| = y_1 w + y_2 w^2 + y_3 w^3 + \ldots (w \in \Delta) \quad (2.7)$$

with $k(0) = y(0) = 0, |k(z)| < 1, |y(w)| < 1$ and $z, w \in \Delta$ such that

$$1 + \frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^\delta k(\xi)) + \eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^\delta k(\xi)))}{(1-\rho)\xi + \rho(1-v)\mathcal{R}_q^\delta k(\xi) + v\xi\partial_q(\mathcal{R}_q^\delta k(\xi)} - \frac{1}{[1+v(1-\rho)]}\right] \prec \psi(r,k(z)) + 1 - e$$

and

$$1 + \frac{1}{\vartheta}\left[\frac{\omega\partial_q(\mathcal{R}_q^\partial \chi(w)) + \eta\omega^2\partial_q(\partial_q(\mathcal{R}_q^\partial \chi(w)))}{(1-\rho)\omega + \rho(1-v)\mathcal{R}_q^\partial \chi(w) + v\omega\partial_q\mathcal{R}_q^\partial \chi(w))} - \frac{1}{[1+v(1-\rho)]}\right] \prec \psi(r,y(w)) + 1 - e$$

Or,equivalently,

$$1+\frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi))+\eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}k(\xi)))}{(1-\rho)\xi+\rho(1-v)\mathcal{R}_q^{\delta}k(\xi)+v\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi)}-\frac{1}{[1+v(1-\rho)]}\right]$$
$$=1+h_1(r)-e+h_2(r)k(z)+h_3(r)[k(z)]^2+\ldots \tag{2.8}$$

$$1+\frac{1}{\vartheta}\left[\frac{\omega\partial_q(\mathcal{R}_q^{\partial}\chi(w))+\eta\omega^2\partial_q(\partial_q(\mathcal{R}_q^{\partial}\chi(w)))}{(1-\rho)\omega+\rho(1-v)\mathcal{R}_q^{\partial}\chi(w)+v\omega\partial_q\mathcal{R}_q^{\partial}\chi(w))}-\frac{1}{[1+v(1-\rho)]}\right]$$
$$=1+h_1(r)-e+h_2(r)y(w)+h_3(r)[y(w)]^2+\ldots \tag{2.9}$$

Combining (2.6),(2.7),(2.8) and (2.9) yields

$$1+\frac{1}{\vartheta}\left[\frac{\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi))+\eta\xi^2\partial_q(\partial_q(\mathcal{R}_q^{\delta}k(\xi)))}{(1-\rho)\xi+\rho(1-v)\mathcal{R}_q^{\delta}k(\xi)+v\xi\partial_q(\mathcal{R}_q^{\delta}k(\xi)}-\frac{1}{[1+v(1-\rho)]}\right]$$
$$=1+h_2(r)k_1z+[h_2(r)k_2+h_3(r)k_1^2]z^2+\ldots \tag{2.10}$$

$$1+\frac{1}{\vartheta}\left[\frac{\omega\partial_q(\mathcal{R}_q^{\partial}\chi(w))+\eta\omega^2\partial_q(\partial_q(\mathcal{R}_q^{\partial}\chi(w)))}{(1-\rho)\omega+\rho(1-v)\mathcal{R}_q^{\partial}\chi(w)+v\omega\partial_q\mathcal{R}_q^{\partial}\chi(w))}-\frac{1}{[1+v(1-\rho)]}\right]$$
$$=1+h_2(r)y_1w+[h_2(r)y_2+h_3(r)y_1^2]w^2+\ldots \tag{2.11}$$

It is clear that if $|k(z)|<1$ and $|y(w)|<1, z,w\in\Delta$, then

$$|k_i|\leq 1 \quad and \quad |y_i|\leq 1 \quad (i\in\mathcal{N}) \tag{2.12}$$

From (2.10) and (2.11), it follows that

$$\frac{[\delta+1]_q([2]_q-q\rho v+[2]_q\eta v-[2]_q\rho\eta v-\rho+[2]_q\eta)}{\vartheta(\rho v-v-1)^2}a_2=h_2(r)k_1 \tag{2.13}$$

$$\frac{[\delta+1]_q[\delta+2]_q([3]_q-q[2]_q\rho v+[2]_q[3]_q\eta v-[2]_q[3]_q\rho\eta v-\rho+[2]_q[3]_q\eta)}{\vartheta[2]_q(\rho v-v-1)^2}a_3$$
$$-\frac{[\delta+1]_q^2(\rho v-[2]_qv-\rho)([2]_q-q\rho v+[2]_q\eta v-[2]_q\rho\eta v-\rho+[2]_q\eta)}{\vartheta(\rho v-v-1)^3}a_2^2=h_2(r)k_2+h_3(r)k_1^2 \tag{2.14}$$

Moreover, we have

$$\frac{[\delta+1]_q([2]_q-q\rho v+[2]_q\eta v-[2]_q\rho\eta v-\rho+[2]_q\eta)}{\vartheta(\rho v-v-1)^2}a_2=h_2(r)y_1 \tag{2.15}$$

and

$$\frac{[\delta+1]_q[\delta+2]_q([3]_q-q[2]_q\rho v+[2]_q[3]_q\eta v-[2]_q[3]_q\rho\eta v-\rho+[2]_q[3]_q\eta)}{\vartheta[2]_q(\rho v-v-1)^2}(2a_2^2-a_3)$$
$$-\frac{[\delta+1]_q^2(\rho v-[2]_qv-\rho)([2]_q-q\rho v+[2]_q\eta v-[2]_q\rho\eta v-\rho+[2]_q\eta)}{\vartheta(\rho v-v-1)^3}a_2^2=h_2(r)y_2+h_3(r)y_1^2 \tag{2.16}$$

From(2.13) and (2.15), we get

$$k_1 = -y_1 \tag{2.17}$$

By adding Equation (2.14) and (2.16) and then using Equation (2.17), we obtain

$$\frac{[\delta+1]_q}{\vartheta[2]_q(\rho v - v - 1)^3}\{-2[3]_q[\delta+2]_q + (4[3]_q[\delta+2]_q - 4[2]_q^2[\delta+1]_q)\rho v$$
$$+(2[2]_q^3[\delta+1]_q - 4[2]_q[3]_q[\delta+2]_q)\eta v + (2q[2]_q[\delta+1]_q - 2q[2]_q[\delta+2]_q)\rho^2 v^2$$
$$+2[2]_q^2][\delta+1]_q\rho\eta - 2[2]_q[\delta+1]_q\rho^2 - (2[2]_q(q-1)[\delta+1]_q + 2[\delta+2]_q)\rho^2 v$$
$$+(2[2]_q^3[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q)\eta v^2 + (2[2]_q^2[\delta+1]_q - 2[2]_q[3]_q[\delta+2]_q)\rho^2\eta v^2$$
$$+(4[2]_q[3]_q[\delta+2]_q - 2[2]_q(1+[2]_q)[\delta+1]_q)\rho\eta v^2 + 4[2]_q[3]_q[\delta+2]_q\rho\eta v$$
$$+(2[2]_q^3[\delta+1]_q - 2[3]_q[\delta+2]_q)v - 2[2]_q^2[\delta+1]_q\rho^2\eta v - 2[2]_q[3]_q[\delta+2]_q\eta$$
$$+(2[2]_q^2[\delta+1]_q + 2[\delta+2]_q)\rho + (2q[2]_q^2[\delta+1]_q - 2q[2]_q[\delta+2]_q)\rho v^2\}a_2^2 = h_2(r)(k_2+y_2) + h_3(r)(k_1^2+y_1^2) \tag{2.18}$$

For the purpose of brevity, we will utilize the notations given in Equations (2.3)-(2.5). Now, making use of the notations defined above and combining equations (2.15) and (2.18), we get

$$a_2^2 = \frac{h_2(r)[h_2]^2 2v^2 2[2]_q(\rho v - v - 1)^4}{[\delta+1]_q v(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)[h_2(r)]^2 - 4[2]_q\Theta^2(\eta,\rho,v,\delta,q)h_3(r)} \tag{2.19}$$

$$a_2^2 \le \frac{br[br]^2|v|^2 2[2]_q(\rho v - v - 1)^4}{[\delta+1]_q||v|(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)b^2r^2 - 4[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+\epsilon q)} \tag{2.20}$$

So that

$$|a_2| \le \frac{\sqrt{2}[2]_q|\vartheta||br|\sqrt{|br|}(\rho v - v - 1)^2}{\sqrt{|\vartheta[\delta+1]_q br^2(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q) - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+\epsilon q)|}} \tag{2.21}$$

Where $\psi(\eta,\rho,v,\delta,q)$ and $\Theta(\eta,\rho,v,\delta,q)$ are given by Equation (2.5) and (2.3)respectively. Similarly, upon subtracting Equation (2.16) from Equation (2.14) and then using equation (2.18), we get

$$\frac{4Y(\eta,\rho,v,\delta,q)}{[2]_q v(\rho v - v - 1)^2}(a_3 - a_2^2) = h_2(r)(k_2+y_2) - h_3(r)(k_1^2+y_1^2) \tag{2.22}$$

Where $Y(\eta,\rho,v,\delta,q)$ is defined by Equation (2.4). It follows from Equation (2.15) and (2.22)

$$a_3 = \frac{(h_2(r))^2 v^2(\rho v - v - 1)^4}{4\Theta^2(\eta,\rho,v,\delta,q)} + \frac{h_2(r)[2]_q v(\rho v - v - 1)^2}{4Y(\eta,\rho,v,\delta,q)} \tag{2.23}$$

$$a_3 = \frac{(br)^2 v^2(\rho v - v - 1)^4}{4\Theta^2(\eta,\rho,v,\delta,q)} + \frac{br[2]_q v(\rho v - v - 1)^2}{4Y(\eta,\rho,v,\delta,q)} \tag{2.24}$$

$$|a_3| \leq br|\vartheta|(\rho v - v - 1)^2 \left(\frac{br|\vartheta|(\rho v - v - 1)^2}{\Theta^2(\eta,\rho,v,\delta,q)} + \frac{[2]_q}{|Y(\eta,\rho,v,\delta,q)|}\right) \tag{2.25}$$

Finally, for some $v \in R$, we obtain

$$\begin{aligned} a_3 - va_2^2 &= \frac{h_2(r)[2]_q v(\rho v - v - 1)^2}{4Y(\eta,\rho,v,\delta,q)}(k_2 - y_2) \\ &+ \frac{h_2(r)[h_2(r)]^2 2v^2[2]_q(\rho v - v - 1)^4(1-v)}{[\delta+1]_q v(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)[h_2(r)]^2 - 4[2]_q h_3 \Theta^2(\eta,\rho,v,\delta,q)}(k_2 + y_2) \end{aligned} \tag{2.26}$$

$$= \frac{h_2(r)[2]_q v(\rho v - v - 1)^2}{2}\left[\left(Q(v,r) + \frac{1}{2Y(\eta,\rho,v,\delta,q)}\right)k_2 + \left(Q(v,r) - \frac{1}{2Y(\eta,\rho,v,\delta,q)}\right)y_2\right]$$

where

$$Q(v,r) = \frac{[h_2(r)]^2 2v^2(\rho v - v - 1)^2(1-v)}{\frac{1}{2}[\delta+1]_q v(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)[h_2(r)]^2 - 2[2]_q h_3(r)\Theta^2(\eta,\rho,v,\delta,q)}$$

$$|a_3 - va_2^2| \leq \frac{|br|[2]_q v(\rho v - v - 1)^2}{2Y(\eta,\rho,v,\delta,q)} \quad if \quad \left(0 \leq |Q(v,r)| \leq \frac{1}{2Y(\eta,\rho,v,\delta,q)}\right)$$

$$\frac{|br|[2]_q v(\rho v - v - 1)^2}{|Q(v,r)|} \quad if \quad \left(|Q(v,r)| \geq \frac{1}{2Y(\eta,\rho,v,\delta,q)}\right)$$

We get,

$$|a_3 - va_2^2| \leq \begin{cases} \dfrac{|br|[2]_q v(\rho v - v - 1)^2}{2Y(\eta,\rho,v,\delta,q)} \\ if \quad |v-1| \leq \dfrac{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|}{2Y(\eta,\rho,v,\delta,q)2v^2(\rho v - v - 1)b^2r^2} \\ \dfrac{|br|^3(v-1)2v^2(\rho v - v - 1)^2}{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|} \\ if \quad |v-1| \geq \dfrac{|\frac{1}{2}\vartheta[\delta+1]_q(\rho v - v - 1)\psi(\eta,\rho,v,\delta,q)(br)^2 - 2[2]_q\Theta^2(\eta,\rho,v,\delta,q)(pbr^2+eq)|}{2Y(\eta,\rho,v,\delta,q)2v^2(\rho v - v - 1)b^2r^2} \end{cases}$$

Remark 1: *Setting* $\rho = v = \delta = 0 \quad and \quad q \to 1$ *in Theorem 1, we get the following corollary*

Corollary 1: *Let* $k(\xi) \in \sum (\vartheta, \eta; \psi)$ *be of the form in equation(1). Then*

$$|a_2| \leq \frac{\sqrt{2}[2]|\vartheta||br|\sqrt{|br|}}{\sqrt{|\vartheta b^2 r^2 3(1+2\eta) - 2[2](1+\eta)^2(pbr^2+eq)|}}$$

and

$$|a_3| \leq br|\vartheta|\frac{br|\vartheta|}{2(1+\eta)^2} + \frac{[2]}{3(1+2\eta)}$$

where

$$\Theta(\eta,\rho,v) = 2-\rho v+2\eta v-2\eta\rho v-\rho+2\eta,$$

$$Y(\eta,\rho,v) = 3-2\rho v+6\eta v-6\eta\rho v-\rho+6\eta,$$

and

$$\psi(\eta,\rho,v)= -3+2\rho v-8\eta v-\rho^2v^2+2\rho\eta-\rho 2-\rho^2 v-2\eta v^2+6\rho\eta v^2-4\rho 2\eta v^2+12\rho\eta v+v-2\rho^2\eta v+3\rho-6\eta.$$

3. CONCLUSION:

In this paper, Fekete-Szegö disparity for a specific subclasses of bi-univalent capacity connected with altered Horadam polynomial were introduced. The consequence of this paper assists different specialists with finding Fourth Hankel determinant.

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