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MAGDM PROBLEMS USING TRIANGULAR FUZZY SETS AND ITS APPLICATION

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ABSTRACT:

Solving Multiple Attribute Group Decision Making (MAGDM) problem has become one of the most important research in recent days. In this situations the information or the data is of the form of Triangular Interval Type 2 Fuzzy Numbers (TIT2FNs). These methods are applied in decision making problems. The Triangular Interval Type 2 Fuzzy Frank Weighted Averaging (TIT2FFWA) operator and Triangular Interval Type 2 Fuzzy Frank Weighted Geometric (TIT2FFWG) operator are applied in Fuzzy numbers.The ranking function is utilized to rank the best alternatives. Numerical illustration is proposed to point out the effectiveness of the tactic.

KEY WORDS: MAGDM, Triangular interval type 2 fuzzy numbers (TIT2FN), TIT2FFWA, TIT2FFWG.

1.1. INTRODUCTION:

The multiple attribute group deciding problem is to seek out a desirable solution from a finite number of feasible alternatives assessed on multiple attributes quantitative and qualitative. In the process of MAGDM problems with triangular interval type 2 fuzzy information, in the attribute values in the form of triangular interval type 2 fuzzy number. Attnassov [1] introduced the concept of Intuitionistic Fuzzy Set (IFS). Di.Li & Yang [2] provided fuzzy linear programming technique for Decision making in fuzzy environments. Guha & Chakraborty [3] proposed the fuzzy multi attribute group deciding method to understand consensus under the degrees of confidence of experts, opinions, computers and industrial engineering. Karnick & Mendal [4,5] provided the operations on Type 2 fuzzy sets. Mendal [6] defined type 2 fuzzy sets.Mitchell &Tizhoosh [7,8] are recognition using type 2 fuzzy sets. Mendal, John &Liu[9] are investigated the interval type 2 fuzzy logic systems. Sarkoci [10] in the families of frank and Hamacher t-norms.

Wang & Liu [11-13] refered this paper Intuitionistic fuzzy information aggregation operator using Einstein operations. Calvo, Baets and Foder [14] provided by the functional equations of frank and Alsina for uninorms and nullnorms. Dereli & Altan [15-17] derived by the interval type 2 fuzzy sets and systems. Wang & He [18] is provided on flexible probability logic operator based on frank t-norms. Chen [19] provided A linerar assignment method for multiple criteria decision analysis with interval type 2 fuzzy sets.

Chen, Yang, Lee & yang [20] are introduced Fuzzy multiple attribute group decision making based on ranking interval type 2 fuzzy sets. Choi & Chung-Hoon Rhee [21] proposed Interval type 2 fuzzy membership function generation methods for pattern recognition. Wang & Lee [22] are analyze the TOPSIS for fuzzy multiple criteria group decision making for pattern recognition. Qiu, Xiao, Yu, Han & Iqbal [23] are provided by A modified interval type 2 fuzzy C means algorithm with application in MR image segmentation.

In this paper the basic concept of T2Fs, ITFN and Triangular interval type 2 fuzzy sets are presented. Some aggregation operator discussed the procedure for decision making using ranking function for TIT2FNs is also presented. A numerical example is provided to illustrate of our method.

1.2. BASIC CONCEPTS OF TRIANGULAR INTERVAL TYPE 2 FUZZY NUMBERS:

1.2.1 Definition:A **Type 2 Fuzzy Set(T2FS)** A in the universe of discourse X can be represented by a type 2 membership function $\mu_A(x,u)$ as follows:

$$A=\{(x,\mu),\mu_A(x,u))\,|\,\forall x\in X,\forall u\in J_x\subseteq[0,1]\}$$

whereJ_x denotes an interval in [0,1]. Then, type 2 fuzzy set can be expressed as the form:

$$A=\int_{x\in X}\int_{u\in J_x}\frac{\mu_A(x,u)}{(x,u)}=\int_{x\in X}\frac{\left(\int_{u\in J_x}\mu_A(x,u)/u\right)}{x}$$

Where $J_x \subseteq [0,1]$ is the primary membership at x, and $\int_{u \in J_x} \mu_A(x,u)/u$ indicates the second membership at x. For discreat situations, $\int$ is replaced by $\sum$.

1.2.2 Definition: Let A be a type-2 fuzzy set in the universe of discourse X represented by a type-2 membership function $\mu_A(x,u)$. If all $\mu_A(x,u)$ =1 , then A is called an **Interval Type-2 Fuzzy Set (IT2FS)**. An interval type-2 fuzzy set can be regarded as a special case of the type-2 fuzzy set, which is defined as,

$$A = \int_{x \in X} \int_{u \in J_x} \frac{\mu_A(x,u)}{(x,u)} = \int_{x \in X} \frac{\left(\int_{u \in J_x} \mu_A(x,u)/u \right)}{x}$$

1.2.3 Definition: Let A=($[a_j,b_j],c_j,[d_j,e_j]$) be a **triangular interval type-2 fuzzy set** on X, where a_j,b_j,c_j,d_j,e_j are the reference points of the triangular interval type-2 fuzzy set, satisfying the condition $0 \leq a_j \leq b_j \leq c_j \leq d_j \leq e_j \leq 1$. The upper and lower membership functions of are defined as,

$$UMF_A(x) = \begin{cases} \dfrac{x-a_j}{c_j-a_j} & a_j \leq x \text{ p } c_j \\ 1 & x = c_j \\ \dfrac{x-e_j}{c_j-e_j} & c_j \leq x \text{ p } e_j \end{cases} \qquad LMF_A(x) = \begin{cases} \dfrac{x-b_j}{c_j-b_j} & b_j \leq x \text{ p } c_j \\ 1 & x = c_j \\ \dfrac{x-d_j}{c_j-d_j} & c_j \leq x \text{ p } d_j \end{cases}$$

1.2.4Definition: Let A=($[a_j,b_j],c_j,[d_j,e_j]$) be a triangular interval type 2 fuzzy number. TheRanking value Rank(A) of the triangular interval type 2 fuzzy number A is defined as,

$$Rank(A) = \left(\frac{a_j + e_j}{2} + 1 \right) \times \frac{a_j + b_j + d_j + e_j + 4c_j}{8}$$

1.3. Frank Triangular Norms:

Frank operations include the Frank product and Frank sum, which are examples of triangularnorms and triangular conorms, respectively. Frank product $\otimes_F$ is a -norm and Frank sum $\oplus_F$ is a -conorm, and the mathematical forms are defined as,

$$a \underset{F}{\oplus} b = 1 - \log_\lambda \left(1 + \frac{(\lambda^{1-a}-1)(\lambda^{1-b}-1)}{\lambda - 1} \right) \lambda \text{ f } 1 \forall (a,b) \in [0,1]^2$$

$$a \underset{F}{\otimes} b = \log_\lambda \left(1 + \frac{(\lambda^a - 1)(\lambda^b - 1)}{\lambda - 1}\right) \lambda \text{ f } 1, \forall (a,b) \in [0,1]^2.$$

1.3.1. Frank Operations of Triangular Interval Type-2 Fuzzy Numbers: The basic operation laws of triangular interval type-2 fuzzy numbers,

Let A,A_1 and A_2 be three TIT2FNs, and k,k_1, k_2>0; then,

(1) $A_1 \bigoplus_F A_2 = A_2 \bigoplus_F A_1$

(2) $k \bullet_F (A_1 \bigoplus_F A_2) = k \bullet_F A_1 \bigoplus_F k \bullet_F A_2$

(3) $k_1 \bullet_F A \bigoplus_F k_2 \bullet_F A = (k_1 + k_2) \bullet_F A$

(4) $(k_1 k_2)\cdot_F A = k_1 \cdot_F (k_2 \cdot_F A)$.

1.4. TRIANGULAR INTERVAL TYPE 2 FUZZY FRANK WEIGHTED AVERAGING OPERATOR FOR GROUP :

1.4.1 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs, and let TIT2FFWA is denoted by,

$$TIT2FFWA_w(A_1,A_2,\ldots\ldots.A_n) = w_1 \cdot_F A_1 \underset{F}{\oplus} w_2 \cdot_F A_2 \underset{F}{\oplus} w_3 \cdot_F A_3 \underset{F}{\oplus} \ldots\ldots.. \underset{F}{\oplus} w_j \cdot_F A_n$$

then the function TIT2FFWA is called a Triangular Interval Type-2 Fuzzy Frank WeightedAveraging(TIT2FFWA) operator, where $w=(w_1,w_2,\ldots\ldots..w_n)^T$ is the weight vectorof $A_j(j=1,2,\ldots\ldots.n)$, $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$. In particular, if $w=(1/n,1/n,\ldots\ldots1/n)^T$ then the TIT2FFWA operator is reduced to a Triangular Interval Type 2 Fuzzy Frank Arithmetic Averaging (TIT2FFAA) operator of dimension , which is defined as follows:

$$TIT2FFAA_w(A_1,A_2,\ldots\ldots A_n) = \frac{1}{n} \cdot_F (A_1 \bigoplus_F A_2 \bigoplus_F \ldots\ldots\ldots \bigoplus_F A_n)$$

1.4.2 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs,then their aggregated value by TIT2FFWA operator is still a TIT2FN,and

$$TIT2FFWA_w(A_1,A_2,\ldots\ldots.A_n) =$$

$$([1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-a_j}-1)^{w_j}),1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-b_j}-1)^{w_j})],1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-c_j}-1)^{w_j}),[1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-d_j}-1)^{w_j}),1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-e_j}-1)^{w_j})])$$

1.4.3 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs, and assume that $\lambda>1$, as $\lambda\rightarrow1$ the TIT2FFWA operator approach the limit

$$\lim_{\lambda\to1} TIT2FFWA_w(A_1,A_2,\ldots\ldots.A_n) =$$

$$([1-\prod_{j=1}^{n}(1-a_j)^{w_j},1-\prod_{j=1}^{n}(1-b_j)^{w_j}],1-\prod_{j=1}^{n}(1-c_j)^{w_j},[1-\prod_{j=1}^{n}(1-d_j)^{w_j},1-\prod_{j=1}^{n}(1-e_j)^{w_j}])$$

1.4.4 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs, and assume that $\lambda>1$, as $\lambda\rightarrow\infty$ the TIT2FFWA operator approach the limit $\lim_{\lambda\to\infty} TIT2FFWA_w(A_1,A_2,\ldots..A_n) = ([\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{n} w_j b_j], \sum_{j=1}^{n} w_j c_j, [\sum_{j=1}^{n} w_j d_j, \sum_{j=1}^{n} w_j e_j])$

1.5. TRIANGULAR INTERVAL TYPE 2 FUZZY FRANK WEIGHTED GEOMETRIC OPERATOR:

1.5.1 Definition: Let $A=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs, and let TIT2FFWG is denoted by,

$TIT2FFWG_w(A_1,A_2,\ldots\ldots A_n) = A_1^{\cdot_F w_1} \underset{F}{\otimes} A_2^{\cdot_F w_2} \underset{F}{\otimes} \ldots\ldots\ldots\ldots \underset{F}{\otimes} A_n^{\cdot_F w_n}$,then the function TIT2FFWG is called a Triangular Interval Type 2 Fuzzy Frank Weighted Geometric(TIT2FFWG) operator, where $w=(w_1,w_2,\ldots\ldots..w_n)^T$ is the weight vectorof $A_j(j=1,2,\ldots\ldots.n)$, $w_j \geq 0$, and $\sum_{j=1}^{n} w_j = 1$. In particular, if $w=(1/n,1/n,\ldots\ldots1/n)^T$ then the TIT2FFWG operator is reduced to a Triangular Interval Type 2 Fuzzy Frank Geometric Averaging(TIT2FFGA) operator of dimension , which is defined as,

$$TIT2FFGA_w(A_1,A_2,\ldots\ldots.A_n) = (A_1 \underset{F}{\otimes} A_2 \underset{F}{\otimes} \ldots\ldots\ldots \underset{F}{\otimes} A_n)^{\cdot_F 1/n}$$

1.5.2 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs,then their aggregated value by TIT2FFWG operator is still a TIT2FN,and

$$TIT2FFWG_w(A_1,A_2,\ldots\ldots.A_n) =$$

$$([\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{a_j}-1)^{w_j}),\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{b_j}-1)^{w_j})],\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{c_j}-1)^{w_j}),[\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{d_j}-1)^{w_j}),\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{e_j}-1)^{w_j})])$$

1.5.3 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots.n)$ be a collection of TIT2FNs, and assume that $\lambda>1$, as $\lambda\rightarrow1$ the TIT2FFWG operator approach the limit

$$\lim_{\lambda\to1} TIT2FFWG_w(A_1,A_2,\ldots..A_n) =$$

$$([\prod_{j=1}^{n}(a_j)^{w_j},\prod_{j=1}^{n}(b_j)^{w_j}],\prod_{j=1}^{n}(c_j)^{w_j},[\prod_{j=1}^{n}(d_j)^{w_j},\prod_{j=1}^{n}(e_j)^{w_j}])$$

1.5.4 Definition: Let $A_j=([a_j,b_j],c_j,[d_j,e_j])(j=1,2,3,\ldots\ldots n)$ be a collection of TIT2FNs, and assume that $\lambda>1$, as $\lambda\to\infty$ the TIT2FFWA operator approach the limit

$$\lim_{\lambda\to\infty} TIT2FFWG_w(A_1,A_2,\ldots..A_n)=([\sum_{j=1}^{n}w_j a_j,\sum_{j=1}^{n}w_j b_j],\sum_{j=1}^{n}w_j c_j,[\sum_{j=1}^{n}w_j d_j,\sum_{j=1}^{n}w_j e_j])$$

1.6. An approach to group decision making with Triangular interval type 2 Fuzzy information:

Let $A=\{A_1,A_2,\ldots\ldots.A_n\}$ be a discrete set of alternatives, and $C=\{C_1,C_2,\ldots\ldots C_n\}$ be the attributes $w=\{w_1,w_2,\ldots\ldots w_n\}$is the weighting vector of the attribute. Where,w_j€[0,1], $\sum_{j=1}^{n}w_j=1$. Let $D=\{D_1,D_2,\ldots\ldots D_n\}$ be the set of decision makers. Let $R_k=(a_{ij})=(r_{ij})_{m\times n}=([a_j,b_j],c_j,[d_j,e_j])$, be a triangular interval type 2 fuzzy decision matrix, they satisfying the condition

$0\le a_j\le b_j\le c_j\le d_j\le e_j\le 1$. Which is provided by the decision maker D_k for the alternative A_i with respect to C_j.

In the following we will use TIT2FFWA (or TIT2FFWG) operator to develop an approach to MAGDM problem with triangular interval type 2 fuzzy information.

STEP 1: Normalize the decision making information matrix $R^{(k)}=(a_{ij})_{m\times n}$.

STEP 2: Using TIT2FFWA$(r_{ij}^{(1)},r_{ij}^{(2)},\ldots\ldots r_{ij}^{(n)})$ Where i=1,2,…..m ; j=1,2,……n

(Or) TIT2FFWG$(r_{ij}^{(1)},r_{ij}^{(2)},\ldots\ldots r_{ij}^{(n)})$ Where i=1,2,…..m ; j=1,2,……n

To aggregate the individual triangular interval type 2 fuzzy decision matrix $R^{(k)}=(r_{ij}^{(k)})_{m\times n}$ into the triangular interval type 2 fuzzy decision matrix $R^{(k)}=(r_{ij})_{m\times n}$.

STEP 3: Aggregate the triangular interval type 2 fuzzy values r_{ij} for each alternatives by using TIT2FFWA (or TIT2FFWG) operator:

r_i=TIT2FFWA$(r_{i1},r_{i2},\ldots\ldots r_{in})$ i=1,2,…..n (OR)

r_i=TIT2FFWG$(r_{i1},r_{i2},\ldots\ldots r_{in})$ i=1,2,…..n

STEP 4: Calculate the ranking values $R(r_i)$ (i=1,2…..n) of the overall preference values r_i(i=1,2,….n)

STEP 5: Rank all the alternative A_j and select the best one(s) in accordance with $R(r_i)$

1.7NUMERICAL EXAMPLE:

An Numerical example of the new approach in a decision making problem. Let us consider the investor start a new business wants to invest, in order to get high profits. They manufacturing four products in that company.Initially an investor considers the products into a four possible alternatives:

A_1- Toys

A_2 - Baby cradle and Bassinet

A_3 -Baby Care Products

A_4 - Medicines

Most of the companies are sales our products to develop into only in the advertisements. So, The investor planned sales our products into advertisement. The advertisements is denoted by C_n. These are called attributes. The four possible attributes:

C_1 - Online advertising

C_2 -Newspaper

C_3 - Radio

C_4 - Magazine

The four possible alternatives (A_1,A_2,A_3,A_4) are to be evaluated using the interval type 2 fuzzy numbers under by weighting vector, $\omega=(0.30,0.30,0.25,0.15)^T$under the above four attributes is the weight vector of them. The decision committee contains three decision makers D_k(k = 1,2,3)including Manufacturing expert, financial expert and quality expert whose weight vector is e = (0.25,0.35,0.40). The decision makers D_k(k = 1,2,3) the attribute values of the sales A_i(i=1,2,3,4)with respect to C_i(i=1,2,3,4)by TIT2FNs,are listed in Tables 1,2,3.

1.7.1. Algorithm for TIT2FFWAOperator using the decision making problems:

STEP 1: Consider the all attributes are benefit type therefore the decision matrices do not need normalization hence therefore $R^{(k)}=(a_{ij})^{(k)}=(r_{ij}^{(k)})_{4\times 4}$

The result is shown in (Table 1,2,3)

TABLE: 1

	C_1	C_2	C_3	C_4
A_1	([0.2,0.3],0.4,[0.5,0.6])	([0.4,0.5],0.6,[0.7,0.8])	([0.2,0.3],0.4,[0.5,0.6])	([0.3,0.4],0.5,[0.6,0.7])
A_2	([0.5,0.6],0.7,[0.8,0.9])	([0.2,0.3],0.4,[0.5,0.6])	([0.1,0.2],0.3,[0.4,0.5])	([0.4,0.5],0.6,[0.7,0.8])
A_3	([0.4,0.5],0.6,[0.7,0.8])	([0.5,0.6],0.7,[0.8,0.9])	([0.4,0.5],0.6,[0.7,0.8])	([0.6,0.7],0.8,[0.9,1])
A_4	([0.6,0.7],0.8,[0.9,1])	([0.2,0.3],0.4,[0.5,0.6])	([0.3,0.4],0.5,[0.6,0.7])	([0.2,0.3],0.4,[0.5,0.6])

TABLE:2

	C_1	C_2	C_3	C_4
A_1	([0.4,0.5],0.6,[0.7,0.8])	([0.5,0.6],0.7,[0.8,0.9])	([0.4,0.5],0.6,[0.7,0.8])	([0.3,0.4],0.5,[0.6,0.7])
A_2	([0.1,0.2],0.3,[0.4,0.5])	([0.2,0.3],0.4,[0.5,0.6])	([0.6,0.7],0.8,[0.9,1])	([0.5,0.6],0.7,[0.8,0.9])
A_3	([0.6,0.7],0.8,[0.9,1])	([0.4,0.5],0.6,[0.7,0.8])	([0.2,0.3],0.4,[0.5,0.6])	([0.3,0.4],0.5,[0.6,0.7])
A_4	([0.3,0.4],0.5,[0.6,0.7])	([0.6,0.7],0.8,[0.9,1])	([0.1,0.2],0.3,[0.4,0.5])	([0.2,0.3],0.4,[0.5,0.6])

TABLE:3

	C_1	C_2	C_3	C_4
A_1	([0.3,0.4],0.5,[0.6,0.7])	([0.2,0.3],0.4,[0.5,0.6])	([0.5,0.6],0.7,[0.8,0.9])	([0.1,0.2],0.3,[0.4,0.5])
A_2	([0.1,0.2],0.3,[0.4,0.5])	([0.6,0.7],0.8,[0.9,1])	([0.4,0.5],0.6,[0.7,0.8])	([0.3,0.4],0.5,[0.6,0.7])
A_3	([0.5,0.6],0.7,[0.8,0.9])	([0.6,0.7],0.8,[0.9,1])	([0.3,0.4],0.5,[0.6,0.7])	([0.5,0.6],0.7,[0.8,0.9])
A_4	([0.6,0.7],0.8,[0.9,1])	([0.4,0.5],0.6,[0.7,0.8])	([0.2,0.3],0.4,[0.5,0.6])	([0.6,0.7],0.8,[0.9,1])

STEP 2: Using TIT2FFWA Operator to aggregate all the individual value into the collective overall triangular interval type 2 fuzzy decision matrix R = $(r_{ij}^{(k)})_{4\times4}$. Let λ=2 The result is shown in (table 4).

$$TIT2FFWA_w(A_1,A_2,.......A_n)=$$

$$([1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-a_j}-1)^{w_j}),1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-b_j}-1)^{w_j})],1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-c_j}-1)^{w_j}),[1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-d_j}-1)^{w_j}),1-\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{1-e_j}-1)^{w_j})])$$

Letλ=2

log 1=0

$\log_2 2=1$

This equation implies,

$$1-\log_\lambda 1-\prod_{j=1}^{n}\log_\lambda(\lambda^{1-a_j}-1)^{w_j}\Rightarrow 1-\prod_{j=1}^{n}\log_\lambda(\lambda^{1-a_j}-1)^{w_j}$$

$$\Rightarrow 1-\prod_{j=1}^{n}(1-a_j)^{w_j}$$

$$\Rightarrow$$

Weget, $$([1-\prod_{j=1}^{n}(1-a_j)^{w_j},1-\prod_{j=1}^{n}(1-b_j)^{w_j}],1-\prod_{j=1}^{n}(1-c_j)^{w_j},[1-\prod_{j=1}^{n}(1-d_j)^{w_j},1-\prod_{j=1}^{n}(1-e_j)^{w_j}])$$

A_{11}=([1-{$(1-0.2)^{0.25}(1-0.4)^{0.35}(1-0.3)^{0.40}$},1-{$(1-0.3)^{0.25}(1-0.5)^{0.35}(1-0.4)^{0.40}$}],

$1-\{(1-0.4)^{0.25}(1-0.6)^{0.35}(1-0.5)^{0.40},[1-\{(1-0.5)^{0.25}(1-0.7)^{0.35}(1-0.6)^{0.40}\},1-\{(1-0.6)^{0.25}(1-0.8)^{0.35}(1-0.7)^{0.40}\}])$

$=([1-\{(0.8)^{0.25}(0.6)^{0.35}(0.7)^{0.40}\},1-\{(0.7)^{0.25}(0.5)^{0.35}(0.6)^{0.40}\}],1-\{(0.6)^{0.25}(0.4)^{0.35}(0.5)^{0.40}\},$

$[1-\{(0.5)^{0.25}(0.3)^{0.35}(0.4)^{0.40}\},1-\{(0.4)^{0.25}(0.2)^{0.35}(0.3)^{0.40}\}])$

=([1-{(0.9457)(0.8363)(0.8670)},1-{(0.9147)(0.7846)(0.8152)}],1-{(0.8801)(0.7256)(0.7579)},[1-{(0.8409)(0.6561)(0.6931)},1-{(0.7953)(0.5693)(0.6178)}])

=([1-0.6857,1-0.5850],1-0.4840,[1-0.3824,1-0.2797])

A_{11}=([0.3143,0.4149],0.5160,[0.6176,0.7203])

Similarly,

A_{12}=([0.3684,0.4709],0.5746,[0.6807,0.7929])

A_{13}=([0.4006,0.5026],0.6054,[0.7102,0.8197])

A_{14}=([0.2260,0.3268],0.4280,[0.5296,0.6320])

A_{21}=([0.2230,0.3273],0.4336,[0.5440,0.6656])

A_{22}=([0.3937,0.5012],0.6134,[0.7373,1])

A_{23}=([0.4238,0.5297],0.6390,[0.7571,1])

A_{24}=([0.4013,0.5026],0.6045,[0.7079,0.8155])

A_{31}=([0.5160,0.6176],0.7203,[0.8263,1])

A_{32}=([0.5126,0.6145],0.7179,[0.8253,1])

A_{33}=([0.2942,0.3950],0.4960,[0.5975,0.7002])

A_{34}=([0.4680,0.5710],0.6759,[0.7856,1])

A_{41}=([0.5135,0.6176],0.7244,[0.8375,1])

A_{42}=([0..4406,0.5452],0.6527,[0.7679,1])

A_{43}=([0.1937,0.2942],0.3950,[0.4960,0.5975])

A_{44}=([0.3937,0.5012],0.6134,[0.7373,1])

TABLE:4

	C_1	C_2	C_3	C_4
A_1	([0.3143,0.4149],0.5160, [0.6176,0.7203])	([0.3684,0.4709],0.5746, [0.6807,0.7929])	([0.4006,0.5026],0.6054, [0.7102,0.8197])	([0.2260,0.3268],0.4280, [0.5296,0.6320])
A_2	([0.2230,0.3273],0.4336, [0.5440,0.6656])	([0.3937,0.5012],0.6134, [0.7373,1])	([0.4238,0.5297],0.6390, [0.7571,1])	([0.4013,0.5026],0.6045, [0.7079,0.8155])
A_3	([0.5160,0.6176],0.7203, [0.8263,1])	([0.5126,0.6145],0.7179, [0.8253,1])	([0.2942,0.3950],0.4960, [0.5975,0.7002])	([0.4680,0.5710],0.6759, [0.7856,1])
A_4	([0.5135,0.6176],0.7244, [0.8375,1])	([0.4406,0.5452],0.6527, [0.7679,1])	([0.1937,0.2942],0.3950, [0.4960,0.5975])	([0.3937,0.5012],0.6134, [0.7373,1])

STEP 3: Using TIT2FFWA Operator to aggregate all thepreference values r_{ij} (1,2,3,4) in theith line of R, and the we get overall preference values r_i (1,2,3,4) as follows:

Let weight vector = $\{0.30,0.30,0.25,0.15\}^T$

$$\lim_{\lambda\to 1} TIT2FFWA_w(A_1, A_2, ...A_n) =$$

$$([1-\prod_{j=1}^{n}(1-a_j)^{w_j}, 1-\prod_{j=1}^{n}(1-b_j)^{w_j}], 1-\prod_{j=1}^{n}(1-c_j)^{w_j}, [1-\prod_{j=1}^{n}(1-d_j)^{w_j}, 1-\prod_{j=1}^{n}(1-e_j)^{w_j}])$$

$r_1=([1-\{(1-0.3143)^{0.30}(1-0.3684)^{0.30}(1-0.4006)^{0.25}(1-0.2260)^{0.15}\},1-\{(1-0.4149)^{0.30}(1-0.4709)^{0.30}(1-0.5026)^{0.25}(1-0.3268)^{0.15}\}],1-\{(1-0.5160)^{0.30}(1-0.5746)^{0.30}(1-0.6054)^{0.25}(1-0.4280)^{0.15}\},[1-\{(1-0.6176)^{0.30}(1-0.6807)^{0.30}(1-0.7102)^{0.25}(1-0.5296)^{0.15}\},1-\{(1-0.7203)^{0.30}(1-0.7929)^{0.30}(1-0.8197)^{0.25}(1-0.6320)^{0.15}\}])$

=([1-{(0.8930)(0.8712)(0.8799)(0.9623)},1-{(0.8515)(0.8262)(0.8398)(0.9424)}],1-{(0.8044)(0.7738)(0.7926)(0.9424)},[1-{(0.7495)(0.7100)(0.7337)(0.8930)},1-{(0.6824)(0.6235)(0.6516)(0.8608)}])

= ([1-0.6587,1-0.5568],1-0.4649,[1-0.3487,1-0.2386])

r_1 = ([0.3413,0.4432],0.5351,[0.6513,0.7614])

Similarly,

r_2= ([0.3563,0.4626],0.5724,[0.6912,1])

r_3 = ([0.4593,0.5626],0.6678,[0.7784,1])

r_4= ([0.4050,0.5114],0.6218,[0.7421,1])

STEP 4: Calculate the rank values $R(r_i)(i = 1,2,3,4)$ of the overall preference values $r_i(i = 1,2,3,4)$respectively,

$$Rank(A) = \left(\frac{a_j + e_j}{2} + 1\right) \times \frac{a_j + b_j + d_j + e_j + 4c_j}{8}$$

$$R(r_1)=\left(\frac{0.3413+0.7614}{2}+1\right)\left(\frac{0.3413+0.4432+0.6513+0.7614+4(0.5351)}{8}\right)$$

=(1.5514)(0.5422)

$R(r_1)$ = 0.8412

Similarly,

$R(r_2)$ = 1.0068

$R(r_3)$ = 1.1829

$R(r_4)$ = 1.0950

STEP 5: Rank all the alternatives $A_i(i = 1,2,3,4)$and select the best one in accordance with $R(r_i)$

$A_3 > A_4 > A_2 > A_1$

Hence, the best one is A_3.

1.7.2. Algorithm for TIT2FFWG Operator using the decision making problems:

STEP 1: Consider the all attributes are benefit type therefore the decision matrices do not need normalization hence therefore $R^{(k)} = (a_{ij})^{(k)} = (r_{ij}^{(k)})_{4\times4}$

The result same is shown in (table 1,2,3)

STEP 2: Using TIT2FFWG Operator to aggregate all the individual value into the collective overall trianglualr interval type 2 fuzzy decision matrix $R = (r_{ij}^{(k)})_{4\times4}$. Let λ=2 The result is shown in (Table 5).

$$TIT2FFWG_w(A_1, A_2, ...A_n) =$$

$$([\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{a_j}-1)^{w_j}), \log_\lambda(1+\prod_{j=1}^{n}(\lambda^{b_j}-1)^{w_j})], \log_\lambda(1+\prod_{j=1}^{n}(\lambda^{c_j}-1)^{w_j}), [\log_\lambda(1+\prod_{j=1}^{n}(\lambda^{d_j}-1)^{w_j}), \log_\lambda(1+\prod_{j=1}^{n}(\lambda^{e_j}-1)^{w_j})])$$

Let λ=2

A_{11}=([$\log_2(1+\{(2^{0.2}-1)^{0.25}(2^{0.4}-1)^{0.35}(2^{0.3}-1)^{0.40}\}$,$\log_2(1+\{(2^{0.3}-1)^{0.25}(2^{0.5}-1)^{0.35}(2^{0.4}-1)^{0.40}\}$],$\log_2(1+\{(2^{0.4}-1)^{0.25}(2^{0.6}-1)^{0.35}(2^{0.5}-1)^{0.40}\}$,[$\log_2(1+\{(2^{0.5}-1)^{0.25}(2^{0.7}-1)^{0.35}(2^{0.6}-1)^{0.40}\}$,$\log_2(1+\{(2^{0.6}-1)^{0.25}(2^{0.8}-1)^{0.35}(2^{0.7}-1)^{0.40}\}$]

=([$\log_2$(1+{(0.6210)(0.6708)(0.5566)}),$\log_2$(1+{(0.6934)(0.7346)(0.6336)})],$\log_2$(1+{(0.7518)(0.7931)(0.7029)}),[$\log_2$(1+{(0.8022)(0.8481)(0.7673)}),$\log_2$(1+{(0.8474)(0.9004)(0.8283)}])

=([$\log_2$(1+0.2318),$\log_2$(1+0.3227)],$\log_2$(1+0.4191),[$\log_2$(1+0.5220),$\log_2$(1+0.6320)])

=([$\log_2$(1.2318),$\log_2$(1.3227)],$\log_2$(1.4191),[$\log_2$(1.5220),$\log_2$(1.6320)

A_{11}=([0.3008,0.4035],0.5050,[0.6060,0.7066])

Similarly,

A_{12}=([0.3308,0.4373],0.5413,[0.6439,0.7459])

A_{13}=([0.3705,0.4759],0.5791,[0.6812,0.7827])

A_{14}=([0.1951,0.3049],0.4093,[0.5118,0.6134])

A_{21}=([0.1521,0.2665],0.3744,[0.4795,0.5831])

A_{22}=([0.3156,0.4265],0.5334,[0.6382,0.7417])

A_{23}=([0.3335,0.4544],0.5647,[0.6710,0.7753])

A_{24}=([0.3867,0.4886],0.5899,[0.6908,0.7914])

A_{31}=([0.5050,0.6060],0.7067,[0.8072,0.9077])

A_{32}=([0.4987,0.5999],0.7008,[0.8015,0.9020])

A_{33}=([0.2807,0.3834],0.4849,[0.5859,0.6867])

A_{34}=([0.4401,0.5435],0.6458,[0.7473,0.8486])

A_{41}=([0.4745,0.5791],0.6822,[0.7844,0.8860])

A_{42}=([0.3918,0.4991],0.6036,[0.7066,0.8088])

A_{43}=([0.1747,0.2807],0.3834,[0.4849,0.5859])

A_{44}=([0.3156,0.4265],0.5334,[0.6382,0.7417]

TABLE:5

	C_1	C_2	C_3	C_4
A_1	([0.3008,0.4035],0.5050, [0.6060,0.7066])	([0.3308,0.4374],0.5413, [0.6439,0.7459])	([0.3705,0.4759],0.5791, [0.6812,0.7827])	([0.1951,0.3049],0.4093, [0.5118,0.6134])
A_2	([0.1521,0.2665],0.3744, [0.4795,0.5831])	([0.3156,0.4265],0.5334, [0.6382,0.7417])	([0.3335,0.4544],0.5647, [0.6710,0.7753])	([0.3867,0.4886],0.5899, [0.6908,0.7914])
A_3	([0.5050,0.6060],0.7067, [0.8072,0.9077])	([0.4987,0.5999],0.7008, [0.8015,0.9020])	([0.2807,0.3834],0.4849, [0.5859,0.6867])	([0.4401,0.5435],0.6458, [0.7473,0.8486])
A_4	([0.4745,0.5791],0.6822, [0.7844,0.8860])	([0.3918,0.4991],0.6036, [0.7066,0.8088])	([0.1747,0.2807],0.3834, [0.4849,0.5859])	([0.3156,0.4265],0.5334, [0.6382,0.7417])

STEP 3: Using TIT2FFWG Operator to aggregate all thepreference values r_{ij} (1,2,3,4) in theith line of R, and the we get overall preference values r_i (1,2,3,4) as follows:

$$\lim_{\lambda\to 1} TIT2FFWG_w(A_1, A_2, ...A_n) =$$

$$([\prod_{j=1}^{n}(a_j)^{w_j}, \prod_{j=1}^{n}(b_j)^{w_j}], \prod_{j=1}^{n}(c_j)^{w_j}, [\prod_{j=1}^{n}(d_j)^{w_j}, \prod_{j=1}^{n}(e_j)^{w_j}])$$

r_1=([{$(0.3008)^{0.30}(0.3308)^{0.30}(0.3705)^{0.25}(0.1951)^{0.15}$},{$(0.4035)^{0.30}(0.4374)^{0.30}(0.4759)^{0.25}(0.3049)^{0.15}$}],{$(0.5050)^{0.30}(0.5413)^{0.30}(0.5791)^{0.25}(0.4093)^{0.15}$},[{$(0.6060)^{0.30}(0.6439)^{0.30}(0.6812)^{0.25}(0.5118)^{0.15}$},{$(0.7066)^{0.30}(0.7459)^{0.30}(0.7827)^{0.25}(0.6134)^{0.15}$}])

=([{(0.6974)(0.7176)(0.7802)(0.7826)},{(0.7616)(0.7803)(0.8306)(0.8368)}],{(0.8147)(0.8192)(0.8723)(0.8746)},[{(0.8605)(0.8763)(0.9085)(0.9044)},{(0.9011)(0.9158)(0.9406)(0.9293)}])

r_1=([0.3056,0.4131],0.5092,[0.6195,0.7213])

Similarly,

r_2 = ([0.2650,0.3840],0.4940,[0.6002,0.7046])

r_3 = ([0.4255,0.5301],0.6330,[0.7349,0.8365])

r_4 = ([0.3283,0.4414],0.5487,[0.6535,0.7570])

STEP 4: Calculate the rank values $R(r_i)$(i = 1,2,3,4) of the overall preference values r_i(i=1,2,3,4)respectively,

$$Rank(A) = \left(\frac{a_j + e_j}{2} + 1\right) \times \frac{a_j + b_j + d_j + e_j + 4c_j}{8}$$

$R(r_1)$=$\left(\frac{0.3056+0.7213}{2}+1\right)\left(\frac{0.3056+0.4131+0.6195+0.7213+4(0.5092)}{8}\right)$

=(1.5135)(0.5120)

$R(r_1)$=0.7749

Similarly,

$R(r_2)$ = 0.7293

$R(r_3)$ = 1.0314

$R(r_4) = 0.8437$.

STEP 5: Rank all the alternatives $A_i(i = 1,2,3,4)$and select the best one in accordance with $R(r_i)$

$A_3 > A_4 > A_1 > A_2$

Hence, the best one is A_3.

1.7.3. COMPARISON ANALYSIS:

TABLE 6:

Aggregation operator	Computational complexity	Order of alternatives
TIT2FFWA	High	$A_3 > A_4 > A_2 > A_1$
TIT2FFWG	High	$A_3 > A_4 > A_1 > A_2$

1.8. APPLICATIONS OF FUZZY SET:

The fuzzy pure mathematics and related branches are widely applied within the models of optimal control, decision-making under uncertainty, processing vague econometric or demographic data, behavioral studies, and methods of AI .

For example, there already exists a functional model of a helicopter controlled from the bottom by simple "fuzzy" commands in tongue , like "up", "slowly down" "turn moderately left", "high speed", etc. "Fuzzy" wash-machines, cameras or shavers are common commercial products. Fuzzy sets can also be applied in sociology, politics , and anthropology, also as in any field of inquiry handling complex patterns of causation.

The TIT2FS are used in different fields in day to day life.

(i) Agriculture:

Agriculture department not only depond upon only in climate. Most of the process in the sector by the decion maker. Fuzzy setsare able to manage to increase the production in the agricultural domain, helping frarmers to take a right decisions for their cultivated.

(ii) Medical field:

The fuzzy set theory in the field of medical diagnosis. The fuzzy inference machines to improve the quality of the day by day life. Real executed by an interdisciplinary research term comprising the doctors and IT engineers. The proposed approach also allows the remote monitoring of patients clinical conditions and hence can help to reduce hospitalizations.

(iii) Financial:

The impact of the Fuzziness is to assessment made by a investors. Due to the fluctuation of the financial market from time to time. So the investor consider the fact that the fuzzy factors. Therefore the fuzzy set theory may be useful tool for modeling this kind of imprecise problem.

(iv) Business:

Bussiness man to calculate our data in the type of fuzzy numbers and then investment to the business. So this is main helpful to move our business without any loss or problems.

(v) Production Management:

The fuzzy set theory has used to model system that are precisely. Fuzzy set theory represents an attractive and effective tool to aid in research in the production management. It helps to the manufacturing the product to best quality and quantity. So the product sales very quickly and get high profit to the management.

1.9. CONCLUSION:

This paper explained our new method to solve MAGDM problems with the triangular interval type 2 fuzzy set. We defined frank operation laws of interval type 2 fuzzy set, and using frank triangular norms and aggregation operator including the TIT2FFWA & TIT2FFWG operators are developed. MAGDM problems are solved using TIT2FFWA & TIT2FFWG under triangular interval type 2 fuzzy set. Finally, this research give the best advice of the investor to invest mostly, **A_3 - Baby Care Products**becausetogive the **high profit** of this business.

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