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Several Types of Functions of Intuitionistic fuzzy M Open Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we introduce a new class of functions termed as intuitionistic fuzzy θ, θ semi, M continuous, θ open, θ closed, θ semiopen, θ semiclosed, M closed and M open mappings with the help of $\mathcal{IF}$-θc, $\mathcal{IF}$-θo, $\mathcal{IF}$-θso, $\mathcal{IF}$-θsc, $\mathcal{IF}$-δc, $\mathcal{IF}$-δo, $\mathcal{IF}$-δpo, $\mathcal{IF}$-δpc, $\mathcal{IF}$-Mo and $\mathcal{IF}$-Mc sets. Also, we study the topological properties and characterizations of these mappings. Furthermore we obtain the interrelations between these mappings and already existing mappings in the theory of intuitionistic fuzzy topological spaces, and we provide suitable examples to illustrate the theory.

Keywords and phrases: intuitionistic fuzzy topological spaces, $\mathcal{IF}$-θc, $\mathcal{IF}$-θo, $\mathcal{IF}$-θso, $\mathcal{IF}$-θsc, $\mathcal{IF}$-δc, $\mathcal{IF}$-δo, $\mathcal{IF}$-δpo, $\mathcal{IF}$-δpc, $\mathcal{IF}$-Mo, $\mathcal{IF}$-Mc

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1 Introduction

The concept of fuzzy sets was introduced by Zadeh [22] in his classical paper. Fuzzy set have applications in many fields such as Information [17] and Control [18]. After the introduction of fuzzy sets, various authors introduced generalization of the notion of fuzzy set. Atanassov [3] generalized the fuzzy sets to intuitionistic fuzzy sets(in brief,$\mathcal{IFS}$). Some basic results on $\mathcal{IFS}$'s were published in [3, 4], and the book [4] provides a comprehensive coverage of virtually all results in the area of the theory and applications of $\mathcal{IFS}$'s. Coker and his colleague [6, 8, 7] defined intuitionistic fuzzy topology (in brief, $\mathcal{IFTS}$) in Chang's sense. After that the definition of $\mathcal{IFTS}$ in Samanta and Mondal [16, 15] ($\mathcal{IF}$ gradation of openness) was introduced and studied. In 2004, Caldas et al. [5], introduced some properties of θ open sets and in 2011, Maghrabi and Johany [11] introduced M open sets in topological spaces. In 2013 and 2014, Maghrabi and Johany [12, 13, 14] introduced several mappings by using M open sets in topological spaces. In 2017, Fora [10] discussed some properties of fuzzy clopen sets in fuzzy topological spaces. In this paper, we introduce a new class of functions termed as intuitionistic fuzzy θ, θ semi, M continuous, θ open, θ closed, θ semiopen, θ semiclosed, M closed and M open mappings with the help of $\mathcal{IF}$-θc, $\mathcal{IF}$-θo, $\mathcal{IF}$-θso, $\mathcal{IF}$-θsc, $\mathcal{IF}$-δc, $\mathcal{IF}$-δo, $\mathcal{IF}$-δpo, $\mathcal{IF}$-δpc, $\mathcal{IF}$-Mo and $\mathcal{IF}$-Mc sets. Also, we study the topological properties and characterizations of these mappings. Furthermore we obtain the interrelations between these mappings and already existing mappings in the theory of intuitionistic fuzzy topological spaces, and we provide suitable examples to illustrate the theory.

2 Preliminaries

Definition 2.1 *[3] Let Ω be a nonempty fixed set and I the closed interval* $[0, 1]$. *An $\mathcal{IFS}$ μ is an object of the following form $\mu = \{\langle \varepsilon, \rho_\mu(\varepsilon), \varrho_\mu(\varepsilon)\rangle : \varepsilon \in \Omega\}$, where the mapping $\rho_\mu : \Omega \to I$ and $\varrho_\mu : \Omega \to I$ denote the degree of membership (namely, $\rho_\mu(\varepsilon)$) and the degree of nonmembership (namely, $\varrho_\mu(\varepsilon)$) $\forall$ element $\varepsilon \in \Omega$ to the set μ, respectively, and* $0 \le \rho_\mu(\varepsilon) + \varrho_\mu(\varepsilon) \le 1$ $\forall$ $\varepsilon \in \Omega$.

Definition 2.2 *[1, 3] Let Ω be a nonempty set, and the $\mathcal{IFS}$'s μ and γ in Ω be the form $\mu = \{\langle \varepsilon, \rho_\mu(\varepsilon), \varrho_\mu(\varepsilon)\rangle : \varepsilon \in \Omega\}, \gamma = \{\langle \varepsilon, \rho_\gamma(\varepsilon), \varrho_\gamma(\varepsilon)\rangle : \varepsilon \in \Omega\}$ Furthermore, let $\{\mu_i : i \in J\}$ (J be an index set) be an arbitrary family of $\mathcal{IFS}$'s in Ω. Then*

1. $\mu \le \gamma$ if and only if $\rho_\mu(\varepsilon) \le \rho_\gamma(\varepsilon)$ and $\gamma_\mu(\varepsilon) \ge \gamma_\gamma(\varepsilon)$, for all $\varepsilon \in \Omega$.
2. $\mu = \gamma$ if and only if $\mu \le \gamma$ and $\gamma \le \mu$.
3. $\mu \wedge \gamma = \{\langle \varepsilon, \rho_\mu(\varepsilon) \wedge \rho_\gamma(\varepsilon), \gamma_\mu(\varepsilon) \vee \gamma_\gamma(\varepsilon)\rangle : \varepsilon \in \Omega\}$.
4. $\mu \vee \gamma = \{\langle \varepsilon, \rho_\mu(\varepsilon) \vee \rho_\gamma(\varepsilon), \gamma_\mu(\varepsilon) \wedge \gamma_\gamma(\varepsilon)\rangle : \varepsilon \in \Omega\}$.

5. $\overline{\mu} = \{\langle \varepsilon, \gamma_\mu(\varepsilon), \rho_\mu(\varepsilon)\rangle : \varepsilon \in \quad \Omega\}$.
6. $\mu - \gamma = \mu \wedge \overline{\gamma}$.
7. $\wedge_{i\in N} \mu_i = \{\langle \varepsilon, \wedge_{i\in N} \rho_{\mu_i}(\varepsilon), \vee_{i\in N} \gamma_{\mu_i}(\varepsilon)\rangle : \varepsilon \in \Omega\}$.
8. $\vee_{i\in N} \ \mu_i = \{\langle \varepsilon, \vee_{i\in N} \rho_{\mu_i}(\varepsilon), \wedge_{i\in N} \gamma_{\mu_i}(\varepsilon)\rangle : \varepsilon \in \Omega\}$.
9. $\underline{0} = \{\langle \varepsilon, 0,1\rangle : \varepsilon \in \Omega\}$ and $\underline{1} = \{\langle \varepsilon, 1,0\rangle : \varepsilon \in \Omega\}$.

Definition 2.3 *[8] An $\mathcal{IFT}$ in Coker's sense on a nonempty set Ω is a family τ of $\mathcal{IFS}'s$ in Ω satisfying the following axioms*

1. $\underline{0}, \underline{1} \in \tau$.
2. $H_1 \wedge H_2 \in \tau$, for any $H_1, H_2 \in \tau$.
3. $\vee H_i \in \tau$ for any arbitrary family $\{H_i : i \in J\} \subseteq \tau$.

Each $\mathcal{IFS}$ μ which belongs to τ is called an $\mathcal{IF}$ open ($\mathcal{IF}o$) set in Ω. The complement $\overline{\mu}$ of an $\mathcal{IF}o$ set μ in Ω is called an $\mathcal{IF}$ closed ($\mathcal{IF}c$) set in Ω.

Definition 2.4 *[8] Let (Ω, τ) be an $\mathcal{IFTS}$ and $\mu = \{\langle \varepsilon, \mu_\mu, \nu_\mu\rangle : \varepsilon \in \Omega\}$ be an $\mathcal{IFS}$ in Ω. Then the $\mathcal{IF}$ closure (in brief, $\mathcal{IFC}$) and $\mathcal{IF}$ interior (in brief, $\mathcal{IFI}$) of μ are defined by*

1. $\mathcal{IFC}(\mu) = \wedge_{i\in N} \{\iota : \iota$ isanIFcsin Ω and $\iota \geq \mu\}$.
2. $\mathcal{IFI}(\mu) = \vee_{i\in N} \{\kappa : \kappa$ isanIFosin Ω and $\kappa \leq \mu\}$.

Definition 2.5 *[21] Let μ be $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (Ω, τ). μ is called an $\mathcal{IF}$*

1. regular open (in brief, $\mathcal{IF}ro$) set if $\mu = \mathcal{IFIIFC}(\mu)$.
2. regular closed (in brief, $\mathcal{IF}rc$) set if $\mu = \mathcal{IFCIFI}(\mu)$.

Definition 2.6 *[21] Let (Ω, τ) be an $\mathcal{IFTS}$ and $\mu = < \varepsilon, \mu_\mu(\varepsilon), \nu_\mu(\varepsilon) >$ be a $\mathcal{IFS}$ in Ω. Then the $\mathcal{IF}$ δ closure of μ are denoted and defined by $\mathcal{IF}\delta C(\mu) = \wedge \ \{\iota : \iota$ is an $\mathcal{IF}rc$ set in Ω and $\mu \leq \iota\}$ and $\mathcal{IF}\delta I(\mu) = \vee \ \{\kappa : \kappa$ is an $\mathcal{IF}ro$ set in Ω and $\kappa \leq \mu\}$.*

Definition 2.7 *[19] Let μ be an $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (Ω, τ) then μ is called an $\mathcal{IF}$ [(i)]*

1. δ-preopen (briefly, $\mathcal{IF}\delta po$) set if $\mu \subseteq \mathcal{IF}int(\mathcal{IF}cl_\delta(\mu))$.
2. δ-semiopen (briefly, $\mathcal{IF}\delta so$) set if $\mu \subseteq \mathcal{IF}int(\mathcal{IF}cl_\delta(\mu))$.
3. e-open (briefly, $\mathcal{IF}eo$) set if $\mu \subseteq \mathcal{IF}cl\mathcal{IF}int_\delta(\mu) \cup \mathcal{IF}int\mathcal{IF}cl_\delta(\mu)$.
4. δ-preclosed (briefly, $\mathcal{IF}\delta pc$) set if $\mu \supseteq \mathcal{IF}cl(\mathcal{IF}int_\delta(\mu))$.
5. δ-semiclosed (briefly, $\mathcal{IF}\delta sc$) set if $\mu \supseteq \mathcal{IF}cl(\mathcal{IF}int_\delta(\mu))$.
6. e-closed (briefly, $\mathcal{IF}ec$) set if $\mu \supseteq \mathcal{IF}cl\mathcal{IF}int_\delta(\mu) \cap \mathcal{IF}int\mathcal{IF}cl_\delta(\mu)$.

Definition 2.8 *[8, 19] A function ι from a $\mathcal{IFTS}$ (Ω, τ) to a $\mathcal{IFTS}$ (ω, σ) is called as $\mathcal{IF}$ (resp. δ pre, and e) continuous (briefly $\mathcal{IFC}ts$, (resp. $\mathcal{IF}\delta pCts$, and $\mathcal{IF}eCts$)) function if $\iota^{-1}(\mu)$ is an $\mathcal{IF}c$ (resp. $\mathcal{IF}\delta pc$, and $\mathcal{IF}ec$) set in τ $\forall$ $\mathcal{IF}c$ set $\mu \in \sigma$.*

Definition 2.9 *[9] A $\mathcal{IFS}$ λ in a $\mathcal{IFTS}$ (Ω, τ) is called an $\mathcal{IF}$ dense (resp.$\mathcal{IF}$ nowhere dense) if there exists no $\mathcal{IF}o$ (resp. non-zero $\mathcal{IF}o$) set μ in (Ω, τ) such that $\lambda < \mu < \underline{1}$ (resp. $\mu < \mathcal{IFC}(\lambda)$).*

Lemma 2.1 *[19] For a $\mathcal{IFTS}$ (Ω, τ), every $\mathcal{IF}$ dense set is $\mathcal{IF}\delta po$.*

Definition 2.10 *[8, 19] A function ι from a $\mathcal{IFTS}$ (Ω, τ) to a $\mathcal{IFTS}$ (ω, σ), is called as a $\mathcal{IF}$ open (resp. $\mathcal{IF}$ θ semiopen, $\mathcal{IF}$ δ preopen, $\mathcal{IF}$ M open and $\mathcal{IF}$ e open) (briefly $\mathcal{IFO}$, (resp. $\mathcal{IF}\theta sO$, $\mathcal{IF}\delta pO$, $\mathcal{IFMO}$ and $\mathcal{IF}eO$)) function if $\iota(\mu)$ is an $\mathcal{IF}o$ (resp. $\mathcal{IF}\theta o$, $\mathcal{IF}\theta so$, $\mathcal{IF}\delta po$, $\mathcal{IFM}o$ and $\mathcal{IF}eo$) set in σ $\forall$ $\mathcal{IF}o$ set $\mu \in \tau$*

Theorem 2.1 *[19] Let $\iota:(\Omega,\tau)\to(\omega,\sigma)$ be a mapping. Every $\mathcal{IFO}$ (resp. $\mathcal{IFC}$) is $\mathcal{IF}\delta p O$ (resp. $\mathcal{IF}\delta pC$) mapping. But not conversely.*

Definition 2.11 *[20] Let (Ω,τ) be a $\mathcal{IFTS}$, $\forall$ $\mathcal{IFS}$ γ,ν the operators $\mathcal{IF}$- θ interior and $\mathcal{IF}$- θ closure denoted by $(\mathcal{IF})\theta I$ and $\mathcal{IF}\theta C$ are defined as*

$$\mathcal{IF}\theta I(\gamma)=\bigvee_{i\in N}\{\nu \mid \nu\in\tau \ \& \ \mathcal{IFC}(\gamma)\le\nu\}$$

$$and$$

$$\mathcal{IF}\theta C(\gamma)=\bigvee_{i\in N}\{\nu \mid \nu\in\tau \ \& \ \mathcal{IFI}(\gamma)\ge\nu\}.$$

Definition 2.12 *[20] In an $\mathcal{IFTS}$ (Ω,τ) and $\mathcal{IFS}$ γ is called an [(i)]*

1. $\mathcal{IF}$ -θ open (resp. $\mathcal{IF}$ -θ semi open) (briefly $\mathcal{IF}\theta o$ (resp. $\mathcal{IF}\theta so$)) set if $\gamma=\mathcal{IF}\theta I(\gamma)$. (resp. $\gamma\le\mathcal{IFC}(\mathcal{IF}\theta I(\gamma))$).
2. $\mathcal{IF}$ -θ closed (resp. $\mathcal{IF}$-θ semi closed) (briefly $\mathcal{IF}\theta c$ (resp. $\mathcal{IF}\theta sc$)) set if $\overline{\gamma}$ is an $\mathcal{IF}$ θo (resp. $\mathcal{IF}\theta so$) set.

Definition 2.13 *[20] In an $\mathcal{IFTS}$ (Ω,τ), and $\mathcal{IFS}$ γ is called an*

1. $\mathcal{IF}$-M closed (briefly $\mathcal{IFM}c$) set if $\gamma\ge\mathcal{IFI}(\mathcal{IF}\theta C(\gamma))\wedge\mathcal{IFC}(\mathcal{IF}\delta I(\gamma))$.
2. $\mathcal{IF}$-M open (briefly $\mathcal{IFM}o$) set if $\overline{\gamma}$ is an $\mathcal{IFM}c$ set.

Definition 2.14 *[20] Let (Ω,τ) be a $\mathcal{IFTS}$, then the [(i)]*

1. union of all $\mathcal{IFM}o$ (resp. $\mathcal{IF}\theta so$) sets contained in γ is called the $\mathcal{IFM}$ (resp. $\mathcal{IF}\theta$ semi) interior of γ and is denoted by $\mathcal{IFMI}(\gamma)$ (resp. $\mathcal{IF}\theta sI(\gamma)$).
2. intersection of all $\mathcal{IFM}c$ (resp. $\mathcal{IF}\theta sc$) sets containing γ is called the $\mathcal{IFM}$ (resp. $\mathcal{IF}\theta$ semi) closure of γ and is denoted by $\mathcal{IFMC}(\gamma)$ (resp. $\mathcal{IF}\theta sC(\gamma)$).

3 Intuitionistic fuzzy M continuous functions

Definition 3.1 *A function ι from a $\mathcal{IFTS}$ (Ω,τ) to a $\mathcal{IFTS}$ (ω,σ) is called as $\mathcal{IF}\theta$ (resp. θ semi, and M) continuous (briefly $\mathcal{IF}\theta Cts$ (resp. $\mathcal{IF}\theta sCts$, and $\mathcal{IFM}Cts$)) function if $\iota^{-1}(\mu)$ is an $\mathcal{IF}\theta c$, (resp. $\mathcal{IF}\theta sc$ and $\mathcal{IFM}c$) set in τ $\forall$ $\mathcal{IF}c$ set $\mu\in\sigma$.*

Theorem 3.1 *Let $\iota:(\Omega,\tau)\to(\omega,\sigma)$ be a mapping. Every*

1. $\mathcal{IF}\theta sCts$ (resp. $\mathcal{IF}\delta pCts$) is $\mathcal{IFM}Cts$
2. $\mathcal{IF}\theta Cts$ is $\mathcal{IF}\theta sCts$
3. $\mathcal{IF}\theta Cts$ is $\mathcal{IF}Cts$
4. $\mathcal{IF}Cts$ is $\mathcal{IF}\delta pCts$
5. $\mathcal{IFM}Cts$ is $\mathcal{IF}eCts$

function. But not conversely.

Example 3.1 *Let $\Omega=\omega=\{a,e,i,o\}$, $\upsilon=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0.2},\frac{o}{0}),(\frac{a}{0},\frac{e}{1},\frac{i}{0.7},\frac{o}{1})\right\rangle$, $\phi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{1},\frac{i}{0},\frac{o}{0}),(\frac{a}{1},\frac{e}{0},\frac{i}{1},\frac{o}{0.1})\right\rangle$, $\varphi=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0},\frac{o}{0})\right\rangle$, $\psi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{0.8},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0.9},\frac{o}{0})\right\rangle$ Then the families $\tau=\{\underline{0},\underline{1},\upsilon,\phi,\upsilon\vee\phi\}$ is an $\mathcal{IFT}$ on Ω and $\sigma=\{\underline{0},\underline{1},\upsilon,\varphi\}$ is an $\mathcal{IFT}$ on ω. Let us consider the function $\iota:(\Omega,\tau)\to(\omega,\sigma)$ then φ is $\mathcal{IF}eCts$ but not $\mathcal{IF}\delta sCts$ and $\mathcal{IF}\delta MCts$.*

Example 3.2 *Let $\Omega=\omega=\{a,e,i,o\}$, $\upsilon=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0.2},\frac{o}{0}),(\frac{a}{0},\frac{e}{1},\frac{i}{0.7},\frac{o}{1})\right\rangle$, $\phi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{1},\frac{i}{0},\frac{o}{0}),(\frac{a}{1},\frac{e}{0},\frac{i}{1},\frac{o}{0.1})\right\rangle$, $\varphi=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0},\frac{o}{0})\right\rangle$, $\psi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{0.8},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0.9},\frac{o}{0})\right\rangle$ Then the families $\tau=\{\underline{0},\underline{1},\upsilon,\phi,\upsilon\vee\phi\}$ is an $\mathcal{IFT}$ on Ω and*

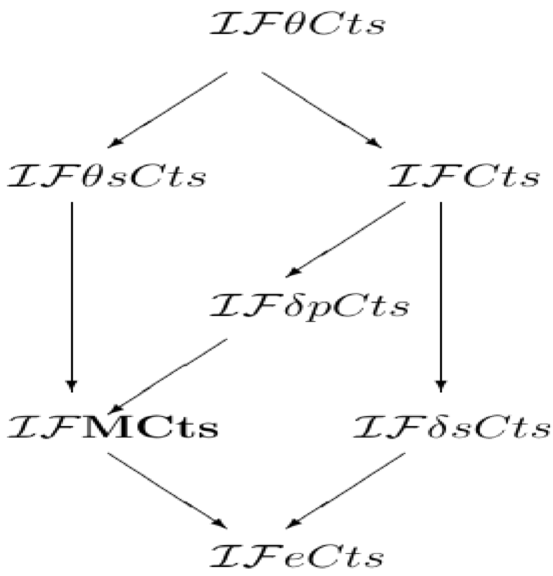
$\sigma = \{\underline{0}, \underline{1}, \upsilon, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *then* ψ *is* $\mathcal{IFMC}ts$ *but not* $\mathcal{IF}\theta sCts$ *and* $\mathcal{IF}\delta pCts$.

Example 3.3 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \phi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *then* ψ *is* $\mathcal{IFC}ts$ *but not* $\mathcal{IF}\theta Cts$ *and* $\mathcal{IF}\theta sCts$.

Example 3.4 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \varphi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *then* φ *is* $\mathcal{IF}\delta pCts$ *but not* $\mathcal{IFC}ts$

Example 3.5 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *then* ψ *is* $\mathcal{IF}\delta pCts$ *but not* $\mathcal{IFC}ts$

From the Theorem 3.1 and Examples 3.1, 3.2, 3.3, 3.4 and 3.5 the following implications are hold.



Note: $A \to B$ denotes A implies B, but not conversely.

Definition 3.2 *Let* (Ω, τ) *be a* $\mathcal{IFTS}$, $\mu \in \tau$, $x_{t,s}$ *is a* $\mathcal{IF}$ *point then* μ *is called* $\mathcal{IFQ}$ *[?] (resp.* $\mathcal{IFMQ}$*) -neighborhood of* $x_{t,s}$ *if* $\mu \in \tau$ *(resp.* $\mathcal{IFM}o$*) and* $x_{t,s} q \mu$.

Definition 3.3 *A mapping* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *is called* $\mathcal{IFMC}ts$ *at a* $\mathcal{IF}$ *point* $x_{t,s}$ *if the inverse image of each* $\mathcal{IFQ}$ *neighbourhood of* $\iota(x_{t,s})$ *is an* $\mathcal{IFMQ}$ *neighbourhood of* $x_{t,s} \in \tau$.

Theorem 3.2 *A mapping* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *is* $\mathcal{IFMC}ts$ *iff it is* $\mathcal{IFMC}ts$ *at every* $\mathcal{IF}$ *point* $x_{t,s} \in \tau$.

Theorem 3.3 *Let* (Ω, τ) *and* (ω, σ) *be* $\mathcal{IFTS}$*'s and* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *be a mapping. Then*

1. ι is $\mathcal{IFMC}ts$ function.
2. $\iota^{-1}(\lambda) \in \tau$ is an $\mathcal{IFM}o$, $\forall$ $\mathcal{IF}o$ set $\lambda \in \sigma$.
3. $\iota^{-1}(\lambda) \in \tau$ is an $\mathcal{IFM}c$, $\forall$ $\mathcal{IF}c$ set $\lambda \in \sigma$.
4. $\iota(\mathcal{IFMC}(\lambda)) \leq \mathcal{IFC}(\iota(\lambda))$, $\forall$ $\lambda \in \tau$.
5. $\mathcal{IFMC}(\iota^{-1}(\lambda)) \leq \iota^{-1}(\mathcal{IFC}(\lambda)), \forall$ $\lambda \in \sigma$.
6. $\mathcal{IFI}(\mathcal{IF}\theta C(\iota^{-1}(\lambda))) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\iota^{-1}(\lambda))) \leq \iota^{-1}(\mathcal{IFC}(\lambda))$, $\forall$ $\lambda \in \sigma$.
7. $\iota^{-1}(\mathcal{IFI}(\lambda)) \leq \mathcal{IFMI}(\iota^{-1}(\lambda))$, $\forall$ $\lambda \in \sigma$.
8. $\iota^{-1}(\mathcal{IFI}(\mu)) \leq \mathcal{IFC}(\mathcal{IF}\theta I(\iota^{-1}(\mu))) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\iota^{-1}(\mu),),)$ $\forall$ $\mu \in I^Y$

are equivalent.

Proof. (ii)⇒(iii), (v)⇒(vii), (vi)⇒(viii), (viii)⇒(iii) are direct to prove, other results are provided here.

(i)⇒(ii): Let λ be an $\mathcal{IF}o$ set in (ω,σ), ι is a $\mathcal{IFMC}ts$ function, then we have $\iota^{-1}(\overline{\lambda})$ is an $\mathcal{IFM}c$ set of (Ω,τ). Therefore $\iota^{-1}(\lambda)$ is an $\mathcal{IFM}o$ set of (Ω,τ).

(iii)⇒(iv): Let $\lambda \in \tau$, since $\mathcal{IFI}(\iota(\lambda)) \in \sigma$ Then by (iii), $\iota^{-1}(\mathcal{IFC}(\iota(\lambda)))$ is an $\mathcal{IFM}c$ set of (Ω,τ). Since $\lambda \leq \iota^{-1}(\iota(\lambda)) \leq \iota^{-1}(\mathcal{IFC}(\iota(\lambda)))$, we have $\mathcal{IFMC}(\lambda) \leq \iota^{-1}(\mathcal{IFC}(\iota(\lambda)))$. Hence $\iota(\mathcal{IFMC}(\lambda)) \leq \mathcal{IFC}(\iota(\lambda))$.

(iv)⇒(v): For all $\lambda \in \sigma$, let $\iota^{-1}(\lambda)$ instead of λ in (iv), we have

$$\iota(\mathcal{IFMC}(\iota^{-1}(\lambda),)) \leq \mathcal{IFC}(\iota(\iota^{-1}(\lambda))) \leq \mathcal{IFC}(\lambda).$$

It implies that

$$\mathcal{IFMC}(\iota^{-1}(\lambda)) \leq \iota^{-1}(\mathcal{IFC}(\lambda)).$$

(vii)⇒(i): Let λ be an $\mathcal{IF}c$ set in (ω,σ). Then $\overline{\lambda} = I(\overline{\lambda})$. By (vii), $\iota^{-1}(\overline{\lambda}) \leq \mathcal{IFMI}(\iota^{-1}(\overline{\lambda}))$. But we know that $\iota^{-1}(\overline{\lambda}) \geq \mathcal{IFMI}(\iota^{-1}(\overline{\lambda}))$. Thus, $\iota^{-1}(\overline{\lambda}) = \mathcal{IFMI}(\iota^{-1}(\overline{\lambda}))$, that is, $\iota^{-1}(\overline{\lambda})$ is $\mathcal{IFM}o$ set. Since, $\iota^{-1}(\lambda)$ is $\mathcal{IFM}c$ set. Therefore ι is $\mathcal{IFMC}ts$ function.

(iii)⇒(vi): For all $\lambda \in \sigma$, since $\mathcal{IFC}(\lambda)$ is an $\mathcal{IF}c$ set in (ω,σ), by (iii), we have that $\iota^{-1}(\mathcal{IFC}(\lambda))$ is an $\mathcal{IFM}c$ set in (Ω,τ). Hence $\iota^{-1}(\mathcal{IFC}(\lambda)) \geq \mathcal{IFI}(\mathcal{IF}\theta C(\iota^{-1}(C(\lambda)))) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\iota^{-1}(C\ (\lambda)))) \geq \mathcal{IFI}(\mathcal{IF}\theta C(\iota^{-1}(\lambda))) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\iota^{-1}(\lambda)))$.

(vi)⇒(iii): For all $\lambda \in \sigma$, since $\mathcal{IFC}(\lambda)$ is an $\mathcal{IF}c$ set in (ω,σ), and let $\mathcal{IFC}(\lambda)$ instead of λ in (vi), we have that

$$\begin{aligned} &\mathcal{IFI}(\mathcal{IF}\theta C(\iota^{-1}(\mathcal{IFC}(\lambda)))) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\iota^{-1}(\mathcal{IFC}(\lambda)))) \\ &\leq \iota^{-1}(\mathcal{IFC}(\mathcal{IFC}(\lambda))) \\ &= \iota^{-1}(\mathcal{IFC}(\lambda)). \end{aligned}$$

Hence $\iota^{-1}(\mathcal{IFC}(\lambda))$ is an $\mathcal{IFM}c$ set in (Ω,τ).

Proposition 3.1 *Let $\iota: (\Omega,\tau) \to (\omega,\sigma))$ $\mathcal{IFMC}ts$ mapping and if for any $\mathcal{IFS}$ λ of Ω is $\mathcal{IF}$ nowhere dense then ι is $\mathcal{IF}\delta pCts$.*

Proof. Let $\mu \in \sigma$ Since ι is an $\mathcal{IFMC}ts$ mapping, then $\iota^{-1}(\mu)$ is an $\mathcal{IFM}o$ set in (Ω,τ). Put $\iota^{-1}(\mu) = \lambda$ is an $\mathcal{IFM}o$ set in Ω. Hence

$$\lambda \leq \mathcal{IFC}(\mathcal{IF}\theta I(\lambda)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\lambda)).$$

But $\mathcal{IF}\theta I(\lambda) \leq \mathcal{IFI}(\lambda) \leq \mathcal{IFC}(\lambda)$, then

$$\mathcal{IF}\theta I(\lambda) \leq \mathcal{IFI}(\mathcal{IFC}(\lambda)).$$

Since λ is $\mathcal{IF}$ nowhere dense and Lemma ??, we have $\mathcal{IF}\theta I(\lambda) = \underline{0}$. Therefore ι is $\mathcal{IF}\delta pCts$.

Definition 3.4 *A mapping $\iota: (\Omega,\tau) \to (\omega,\sigma)$ is called $\mathcal{IF}$ θ-open map (briefly $\mathcal{IF}\theta O$) if the image of every $\mathcal{IF}o$ set of (Ω,τ) is $\mathcal{IF}\theta o$ set in (ω,σ).*

Definition 3.5 *A mapping $\iota: (\Omega,\tau) \to (\omega,\sigma)$ is called $\mathcal{IF}$ θ-bicontinuous (briefly, $\mathcal{IF}\theta biCts$) if ι is $\mathcal{IF}\theta O$ map and $\mathcal{IF}\theta Cts$ map.*

Theorem 3.4 *If $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a $\mathcal{IF}\theta biCts$ mapping then the inverse image of each $\mathcal{IFM}o$ set in (ω,σ) under ι is $\mathcal{IFM}o$ set in (Ω,τ).*

Proof. Let ι be a $\mathcal{IF}\theta biCts$ and μ be a $\mathcal{IFM}o$ set in (ω,σ). Then

$\mu \leq \mathcal{IFC}(\mathcal{IF}\theta I(\mu)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\mu))$.

$\iota^{-1}(\mu) \leq \iota^{-1}(\mathcal{IFC}(\mathcal{IF}\theta I(\mu))) \vee \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\delta C(\mu)))$.

$\leq \mathcal{IFC}(\iota^{-1}(\mathcal{IF}\theta I(\mu))) \vee \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\delta C(\mu)))$.

Since ι is an $\mathcal{IF}\theta biCts$ mapping, then ι is $\mathcal{IF}\theta O$ map and $\mathcal{IF}\theta Cts$ map. Then ι is $\mathcal{IF}\theta sCts$ map and $\mathcal{IF}\delta pCts$ map. Hence

$\iota^{-1}(\mu) \leq \mathcal{IFC}(\mathcal{IF}\theta I(\iota^{-1}(\mu))) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\iota^{-1}(\mu)))$.

This shows that $\iota^{-1}(\mu)$ is $\mathcal{IFM}o$ set in (Ω, τ).

Remark 3.1 *If $\iota: (\Omega, \tau) \rightarrow (\omega, \sigma)$ be a $\mathcal{IF}\theta biCts$ mapping. Then the inverse image of each $\mathcal{IF}\delta po$ (resp. $\mathcal{IF}\theta so$) set in Y under ι is $\mathcal{IFM}o$ set in Ω.*

The next theorem gives the conditions under which the composition of $\mathcal{IFM}Cts$ mapping is $\mathcal{IFM}Cts$.

Theorem 3.5 *Let (Ω, τ), (ω, σ) and (Z, γ) be $\mathcal{IFTS}$'s. If $\iota: (\Omega, \tau) \rightarrow (\omega, \sigma)$ and $\jmath: (\omega, \sigma) \rightarrow (Z, \gamma)$ are mappings, then $\jmath \circ \iota$ is $\mathcal{IFM}Cts$ mapping if*

1. ι is $\mathcal{IFM}Cts$ and $\jmath$ is $\mathcal{IF}Cts$.
2. ι is $\mathcal{IF}\theta biCts$ and $\jmath$ is $\mathcal{IFM}Cts$ mapping.

Proof. (i) Let $\mu \in \gamma$ and $\tau_3^*(\mu) \leq \kappa$. Since $\jmath$ is $\mathcal{IF}Cts$ then $\jmath^{-1}(\mu) \in \sigma$. Since ι is $\mathcal{IFM}Cts$, then $\iota^{-1}(\jmath^{-1}(\mu)) = (\jmath \circ \iota)^{-1}(\mu)$ is $\mathcal{IFM}o$ set in (Ω, τ). Hence $\jmath \circ \iota$ is $\mathcal{IFM}Cts$.

(ii) Let $\mu \in \gamma$. Since $\jmath$ is $\mathcal{IFM}Cts$, then $\jmath^{-1}(\mu)$ is an $\mathcal{IFM}o$ set in (ω, σ). Since ι is $\mathcal{IF}\theta biCts$, by Theorem 3.4, $(\jmath \circ \iota)^{-1}(\mu)$ is $\mathcal{IFM}o$ set in (Ω, τ). Hence $\jmath \circ \iota$ is $\mathcal{IFM}Cts$.

4 Intuitionistic fuzzy M open mappings

Definition 4.1 *A function ι from a $\mathcal{IFTS}$ (Ω, τ) to a $\mathcal{IFTS}$ (ω, σ), is called as a $\mathcal{IF}$ θ open (resp. $\mathcal{IF}$ θ semiopen, and $\mathcal{IF}$ M open) (briefly $\mathcal{IF}\theta O$ (resp. $\mathcal{IF}\theta sO$ and $\mathcal{IFM}O$)) function if $\iota(\mu)$ is an $\mathcal{IF}\theta o$ (resp. $\mathcal{IF}\theta so$ and $\mathcal{IFM}o$) set in σ $\forall$ $\mathcal{IF}o$ set $\mu \in \tau$*

Definition 4.2 *A function ι from a $\mathcal{IFTS}$ (Ω, τ) to a $\mathcal{IFTS}$ (ω, σ), is called as a $\mathcal{IF}$ θ closed (resp. $\mathcal{IF}$ θ semiclosed, and $\mathcal{IF}$ M closed) (briefly $\mathcal{IF}\theta C$ (resp. $\mathcal{IF}\theta sC$ and $\mathcal{IFM}C$)) function if $\iota(\mu)$ is an $\mathcal{IF}\theta c$ (resp. $\mathcal{IF}\theta sc$ and $\mathcal{IFM}c$) set in σ $\forall$ $\mathcal{IF}c$ set $\mu \in \overline{\tau}$*

Theorem 4.1 *Let $\iota: (\Omega, \tau) \rightarrow (\omega, \sigma)$ be a mapping. Every*

1. $\mathcal{IF}\theta sO$ (resp. $\mathcal{IF}\delta pO$) is $\mathcal{IFM}O$
2. $\mathcal{IF}\theta sC$ (resp. $\mathcal{IF}\delta pC$) is $\mathcal{IFM}C$
3. $\mathcal{IF}\theta O$ (resp. $\mathcal{IF}\theta C$) is $\mathcal{IF}\theta sO$ (resp. $\mathcal{IF}\theta sC$)
4. $\mathcal{IF}\theta O$ (resp. $\mathcal{IF}\theta C$) is $\mathcal{IF}O$ (resp. $\mathcal{IF}C$)
5. $\mathcal{IF}O$ (resp. $\mathcal{IF}C$) is $\mathcal{IF}\delta pO$ (resp. $\mathcal{IF}\delta pC$)
6. $\mathcal{IFM}O$ (resp. $\mathcal{IFM}C$) is $\mathcal{IF}eO$ (resp. $\mathcal{IF}eC$)

mapping. But not conversely.

Example 4.1 *Let $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ Then the families $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ is an $\mathcal{IFT}$ on Ω and $\sigma = \{\underline{0}, \underline{1}, \upsilon, \varphi\}$ is an $\mathcal{IFT}$ on ω. Let us consider the function $\iota: (\omega, \sigma) \rightarrow (\Omega, \tau)$ then φ is $\mathcal{IF}eO$ but not $\mathcal{IF}\delta sO$ and $\mathcal{IF}\delta MO$.*

Example 4.2 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* ψ *is* $\mathcal{IF}MO$ *but not* $\mathcal{IF}\theta sO$ *and* $\mathcal{IF}\delta pO$.

Example 4.3 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \phi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* ψ *is* $\mathcal{IF}O$ *but not* $\mathcal{IF}\theta O$ *and* $\mathcal{IF}\theta sO$.

Example 4.4 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \varphi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* φ *is* $\mathcal{IF}\delta pO$ *but not* $\mathcal{IF}O$

Example 4.5 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* ψ *is* $\mathcal{IF}\delta pO$ *but not* $\mathcal{IF}O$

Example 4.6 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \varphi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* $\overline{\varphi}$ *is* $\mathcal{IF}eC$ *but not* $\mathcal{IF}\delta sC$ *and* $\mathcal{IF}\delta MC$.

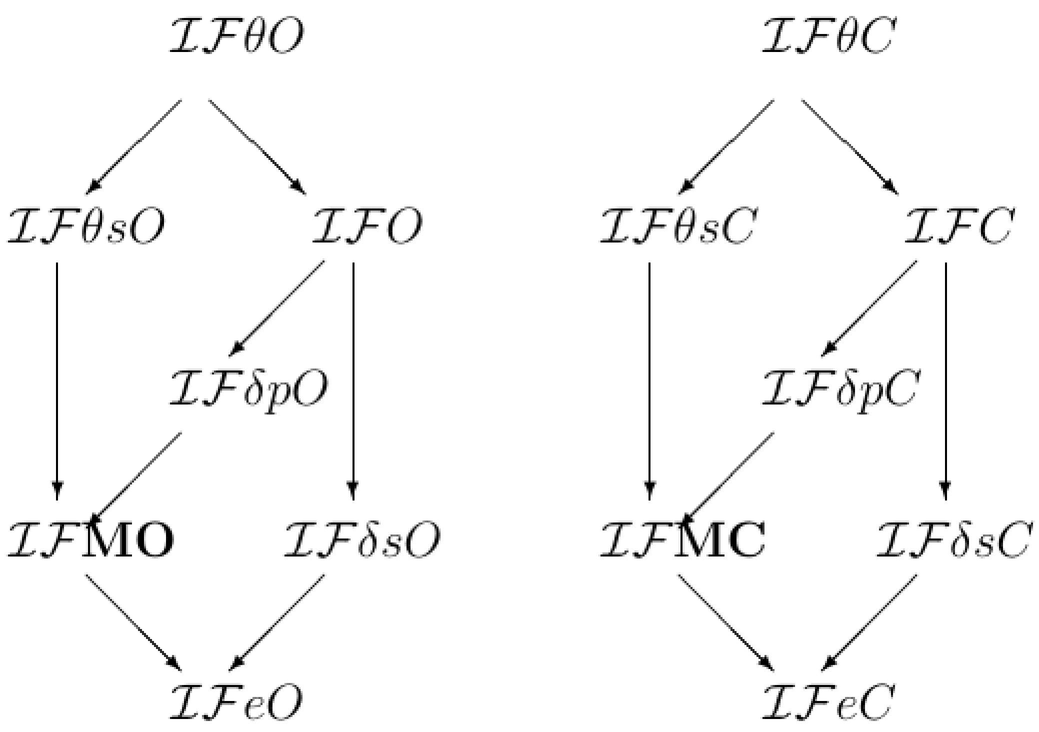
Example 4.7 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* $\overline{\psi}$ *is* $\mathcal{IF}MC$ *but not* $\mathcal{IF}\theta sC$ *and* $\mathcal{IF}\delta pC$.

Example 4.8 *Let* $\Omega = \omega = \{a, e, i, o\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0.2}, \frac{o}{0}), (\frac{a}{0}, \frac{e}{1}, \frac{i}{0.7}, \frac{o}{1}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{1}, \frac{i}{0}, \frac{o}{0}), (\frac{a}{1}, \frac{e}{0}, \frac{i}{1}, \frac{o}{0.1}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{1}, \frac{e}{0}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0}, \frac{o}{0}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0}, \frac{e}{0.8}, \frac{i}{0}, \frac{o}{1}), (\frac{a}{0}, \frac{e}{0.2}, \frac{i}{0.9}, \frac{o}{0}) \right\rangle$ *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon, \phi, \upsilon \vee \phi\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \upsilon, \phi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* $\overline{\psi}$ *is* $\mathcal{IF}C$ *but not* $\mathcal{IF}\theta C$ *and* $\mathcal{IF}\theta sC$.

Example 4.9 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \varphi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* $\overline{\varphi}$ *is* $\mathcal{IF}\delta pC$ *but not* $\mathcal{IF}C$

Example 4.10 *Let* $\Omega = \omega = \{a, e\}$, $\upsilon = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.5}), (\frac{a}{0.3}, \frac{e}{0.5}) \right\rangle$, $\phi = \left\langle \varepsilon, (\frac{a}{0.7}, \frac{e}{0.2}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$, $\varphi = \left\langle \varepsilon, (\frac{a}{0.3}, \frac{e}{0.4}), (\frac{a}{0.5}, \frac{e}{0.6}) \right\rangle$, $\psi = \left\langle \varepsilon, (\frac{a}{0.5}, \frac{e}{0.7}), (\frac{a}{0.3}, \frac{e}{0.2}) \right\rangle$. *Then the families* $\tau = \{\underline{0}, \underline{1}, \upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω *and* $\sigma = \{\underline{0}, \underline{1}, \psi\}$ *is an* $\mathcal{IFT}$ *on* ω. *Let us consider the function* $\iota: (\omega, \sigma) \to (\Omega, \tau)$ *then* $\overline{\psi}$ *is* $\mathcal{IF}\delta pC$ *but not* $\mathcal{IF}C$

From the above Theorem 2.1 and 4.1 Examples 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9 and 4.10 the following implications are hold.



Note: $A \to B$ denotes A implies B, but not conversely.

Definition 4.3 *A mapping* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *is called* $\mathcal{IFMO}$ *at a* $\mathcal{IF}$ *point* $x_{t,s}$ *if the image of each* $\mathcal{IF}$*-*Q *neighbourhood of* $x_{t,s}$ *is an* $\mathcal{IF}$*-*MQ *neighbourhood of* $\iota(x_{t,s}) \in \sigma$.

Theorem 4.2 *A mapping* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *is* $\mathcal{IFMO}$ *iff it is* $\mathcal{IFMO}$ *at every* $\mathcal{IF}$ *point* $x_{t,s} \in \tau$.

Theorem 4.3 *Let* (Ω, τ) *and* (ω, σ) *be* $\mathcal{IFTS}$*'s and* $\iota: (\Omega, \tau) \to (\omega, \sigma)$ *be a mapping. Then*

1. ι is $\mathcal{IFMO}$ function.
2. $\iota(\lambda)$ is an $\mathcal{IFM}o$ set in (ω, σ) $\forall$ $\mathcal{IF}o$ set λ in (Ω, τ).
3. ι is $\mathcal{IFMC}$ function.
4. $\iota(\lambda)$ is an $\mathcal{IFM}c$ set in (ω, σ) $\forall$ $\mathcal{IF}c$ set λ in (Ω, τ).
5. $\mathcal{IFMC}(\iota(\lambda),) \leq \iota(\mathcal{IFC}(\lambda)) \forall \lambda \in \tau$.
6. $\mathcal{IFI}(\mathcal{IF}\theta C(\iota(\lambda))) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\iota(\lambda))) \leq \iota(\mathcal{IFC}(\lambda))$ $\forall$ $\lambda \in \tau$.
7. $\iota(\mathcal{IFI}(\lambda)) \leq \mathcal{IFC}(\mathcal{IF}\theta I(\iota(\lambda))) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\iota(\lambda)))$ $\forall$ $\lambda \in I^{\Omega}$.
8. $\iota(\mathcal{IFI}(\lambda,)) \leq \mathcal{IFMI}(\iota(\lambda))$ $\forall$ $\lambda \in \tau$.
9. $\mathcal{IFI}(\iota^{-1}(\lambda)) \leq \iota^{-1}(\mathcal{IFMI}(\lambda))$ $\forall$ $\lambda \in \sigma$

are equivalent.

Proof. (i)⇒(ii), (iii)⇒(iv), (v)⇒(vi), (vii)⇒(viii), are direct to prove, other results are provided here.

(ii)⇒(iii): Let $\overline{\varpi}$ be an $\mathcal{IF}o$ set in (Ω, τ), by (ii), we have $\iota(\overline{\varpi})$ is an $\mathcal{IFM}o$ set of (ω, σ). Therefore $\iota(\lambda)$ is an $\mathcal{IFM}c$ set of (ω, σ) $\forall \lambda$ $\in (\Omega, \tau), \mathcal{IF}c$ set.

(iv) ⇒ (v): Since $\mathcal{IFC}(\lambda)$ is an $\mathcal{IF}c$ set, then $\iota(\mathcal{IFC}(\lambda))$ is an $\mathcal{IFM}c$ set in Y. Hence

$$\mathcal{IFMC}(\iota(\lambda)) \leq \mathcal{IFMC}(\iota(\mathcal{IFC}(\lambda))) = \iota(\mathcal{IFC}(\lambda)).$$

(vi) ⇒ (vii): Let $\overline{\varpi}$ instead of λ in (vi), then, (vii) will follows directly.

(viii)⇒(ix) Let $\lambda \in \sigma$, by (viii) we have

$\iota(\mathcal{IFI}(\iota^{-1}(\lambda))) \leq \mathcal{IFMI}(ff^{-1}(\lambda)) \leq \mathcal{IFMI}(\lambda)$,

then $I(\iota^{-1}(\lambda))) \leq \iota^{-1}(\mathcal{IFMI}(\lambda))$.

(ix)⇒(i): For each $\lambda \in \tau$, since $\mathcal{IFI}(\lambda) = \lambda$, $\iota(\lambda) \leq \mathcal{IFMI}(\iota(\lambda)) \leq \iota(\lambda)$. Thus $\iota(\lambda) = \mathcal{IFMI}(\iota(\lambda))$. $\iota(\lambda)$ is $\mathcal{IFMo}$ in ω.

Theorem 4.4 *Let (Ω,τ) and (ω,σ) be $\mathcal{IFTS}$'s. Let $\iota: \Omega \to \omega$ be a $\mathcal{IFMC}$ mapping iff ι is surjective, then $\forall$ subset μ of ω and each $\mathcal{IFo}$ set α in Ω containing $\iota^{-1}(\mu)$, there exists an $\mathcal{IFMo}$ set β of ω containing μ such that $\iota^{-1}(\beta) \leq \alpha$.*

Proof. Suppose that α is an $\mathcal{IFo}$ set of Ω containing $\iota^{-1}(\mu)$. Then by hypothesis, β is $\mathcal{IFMo}$ in ω. But $\iota^{-1}(\mu) \leq \alpha$, then $\mu \leq \iota(\alpha)$ and $\mu \leq \beta$, $\iota^{-1}(\beta) \leq \alpha$.

Conversely, let δ be a $\mathcal{IFc}$ set and $y_{t,s}$ be any $\mathcal{IF}$ point of $\iota(\overline{\delta})$. Then $\iota^{-1}(y_{t,s}) \in \overline{\delta}$ which is $\mathcal{IFo}$ set in Ω. Hence by hypothesis, $\exists$ $\mathcal{IFMo}$ set β containing $y_{t,s}$ such that $\iota^{-1}(\beta) \leq \overline{\delta}$. But ι is surjective, then $y_{t,s} \in \beta \leq \iota(\overline{\delta})$ and $\iota(\overline{\delta})$ is the union of $\mathcal{IFMo}$ sets and hence $\iota(\delta)$ is $\mathcal{IFMc}$ set in ω. Therefore, ι is $\mathcal{IFMC}$ map. t

Theorem 4.5 *Let (Ω,τ) and (ω,σ) be $\mathcal{IFTS}$'s and $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a $\mathcal{IFMO}$ (resp. $\mathcal{IF}\delta sO$, $\mathcal{IF}\delta pO$) mapping. If $\mu \in \sigma$ and $\lambda \in \tau$, such that $\iota^{-1}(\mu) \leq \lambda$, then there exists an $\mathcal{IFMc}$ (resp. $\mathcal{IF}\delta sc$, $\mathcal{IF}\delta pc$) set ν of ω such that $\mu \leq \nu, \iota^{-1}(\nu) \leq \lambda$.*

Proof. Since $\iota^{-1}(\mu) \leq \lambda$, we have $\iota(\overline{\lambda}) \leq \overline{\mu}$. Since ι is $\mathcal{IFMO}$ map, then ν is $\mathcal{IFMc}$ in Y and $\iota^{-1}(\nu) = \lambda$. The other cases of the theorem can be proved in a same manner.

Theorem 4.6 *If $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a $\mathcal{IFMO}$ mapping. Then $\forall$ $\mu \in \sigma$, $\iota^{-1}(\mathcal{IFC}(\mathcal{IF}\theta I(\mu))) \wedge \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\delta C(\mu))) \leq \mathcal{IFC}(\iota^{-1}(\mu))$.*

Proof. Since $\mu \in \omega, \mathcal{IF}(\iota^{-1}(\mu)) \in \Omega$ and $\iota^{-1}(\mu) \leq \mathcal{IFC}(\iota^{-1}(\mu))$ $\forall$ $\mu \in \sigma$, it follows from Theorem 4.5, that there exists an $\mathcal{IFMc}$ set λ of ω, $\mu \leq \lambda$ such that $\iota^{-1}(\lambda) \leq \mathcal{IFC}(\iota^{-1}(\mu))$. So $\lambda \geq \mathcal{IFC}(\mathcal{IF}\delta I(\lambda)) \wedge \mathcal{IFI}(\mathcal{IF}\theta C(\lambda))$, hence

$$\iota^{-1}(\lambda) \geq \iota^{-1}(\mathcal{IFC}(\mathcal{IF}\delta I(\lambda))) \wedge \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\theta C(\lambda)))$$

$$\geq \iota^{-1}(\mathcal{IFC}(\mathcal{IF}\delta I(\mu))) \wedge \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\theta C(\mu))).$$

Thus it concludes the proof.

Theorem 4.7 *If $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a bijective mapping such that $\iota^{-1}(\mathcal{IFC}(\mathcal{IF}\delta I(\mu))) \wedge \iota^{-1}(\mathcal{IFI}(\theta C(\mu))) \leq \mathcal{IFC}(\iota^{-1}(\mu))$, $\forall$ $\mu \in \sigma$, then ι is $\mathcal{IFMO}$ map.*

Proof. Let $\lambda \in \tau$ Then, hypothesis, $\iota^{-1}(\mathcal{IFC}(\mathcal{IF}\delta I(\iota(\overline{\lambda})))) \wedge \iota^{-1}(\mathcal{IFI}(\mathcal{IF}\delta C(\iota(\overline{\lambda})))) \leq \mathcal{IFC}(\iota^{-1}(\iota(\overline{\lambda}))) = \mathcal{IFC}(\overline{\lambda}) = \overline{\lambda}$ and so $\mathcal{IFC}(\mathcal{IF}\delta I(\iota(\overline{\lambda}))) \wedge \mathcal{IFI}(\mathcal{IF}\delta C(\iota(\overline{\lambda}))) \leq \iota(\overline{\lambda})$, which shows that $\iota(\overline{\lambda})$ is an $\mathcal{IFMc}$ set of ω. Since ι is bijective, then $\iota(\lambda)$ is an $\mathcal{IFMo}$ set of ω, therefore ι is $\mathcal{IFMO}$ map.

Theorem 4.8 *Let (Ω,τ) and (ω,σ) be $\mathcal{IFTS}$'s. Let $\iota: \Omega \to \omega$ be a $\mathcal{IFMC}$ mapping. Then the following statements hold.*

1. If ι is a surjective map and $\iota^{-1}(\alpha)\overline{q}\iota^{-1}(\beta)$ in Ω, then there exists α, $\beta \in \sigma$ such that $\alpha\overline{q}\beta$.
2. $\mathcal{IFMI}(\mathcal{IFMC}(\iota(\lambda))) \leq \iota(\mathcal{IFC}(\lambda))$, $\forall$ $\lambda \in \Omega$.

Proof. (i) Let γ_1, $\gamma_2 \in \Omega$ such that $\iota^{-1}(\alpha) \leq \gamma_1$ and $\iota^{-1}(\beta) \leq \gamma_2$ such that $\gamma_1\overline{q}\gamma_2$. Then there exists two $\mathcal{IFMo}$ sets μ_1 and μ_2 such that $\iota^{-1}(\alpha) \leq \mu_1 \leq \gamma_1$, $\iota^{-1}(\beta) \leq \mu_2 \leq \gamma_2$. But ι is a surjective map, then $ff^{-1}(\alpha) = \alpha \leq \iota(\mu_1) \leq \iota(\gamma_1)$ and $ff^{-1}(\beta) = \beta \leq \iota(\mu_2) \leq \iota(\gamma_2)$. Since $\gamma_1\overline{q}\gamma_2$, then also $\iota(\gamma_1 \wedge \gamma_2) = \underline{0}$. Hence $\alpha \wedge \beta \leq \iota(\mu_1 \wedge \mu_2) \leq \iota(\gamma_1 \wedge \gamma_2) = \underline{0}$. Therefore, $\alpha\overline{q}\beta$ in ω. that is $\alpha \wedge \beta = \underline{0}$.

(ii) Since $\lambda \leq \mathcal{IFC}(\lambda) \leq \underline{1}$ and ι is a $\mathcal{IFMC}$ mapping, then $\iota(\mathcal{IFC}(\lambda))$ is $\mathcal{IFMc}$ set in ω. Hence $\iota(\lambda) \leq \mathcal{IFMC}(\lambda) \leq \iota(\mathcal{IFC}(\lambda))$. So $\mathcal{IFMI}(\mathcal{IFMC}\ (\iota(\lambda))) \leq \iota(\mathcal{IFC}(\lambda))$.

Proposition 4.1 *Let $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a $\mathcal{IFMO}$ mapping and if for any $\mathcal{IFS}$ λ of ω is $\mathcal{IF}$ nowhere dense then ι is $\mathcal{IF}\delta pO$ map.*

Proof. Let $\mu \in \Omega$. Since ι is an $\mathcal{IFMO}$ mapping, then $\iota(\mu)$ is an $\mathcal{IFMo}$ set in (ω,σ). Put $\iota(\mu) = \lambda$ is an $\mathcal{IFMo}$ set in ω. Hence $\lambda \leq \mathcal{IFC}(\mathcal{IF}\theta I(\lambda)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\lambda))$. But $\mathcal{IF}\theta I(\lambda) \leq \mathcal{IFI}(\lambda) \leq \mathcal{IFC}(\lambda)$, and since λ is $\mathcal{IF}$ nowhere dense, then

$$\mathcal{IF}\theta I(\lambda) \leq \mathcal{IFI}(\mathcal{IFC}(\lambda))$$

we have $\mathcal{IF}\theta I(\lambda) = \underline{0}$. Using Lemma ??, ι is $\mathcal{IF}\delta pO$ map.

Theorem 4.9 *If $\iota: (\Omega,\tau) \to (\omega,\sigma)$ be a $\mathcal{IF}\theta biCts$ mapping then the image of each $\mathcal{IFMo}$ set in (Ω,τ) under ι is $\mathcal{IFMo}$ set* *in* (ω,σ).

Proof. Let ι be a $\mathcal{IF}\theta biCts$ and μ be a $\mathcal{IFM}o$ set in (Ω,τ)). Then

$\mu \leq \mathcal{IFC}((\mathcal{IF}\theta I(\mu)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\mu))$.

This implies that

$\iota(\mu) \leq \iota(\mathcal{IFC}(\mathcal{IF}\theta I(\mu))) \vee \iota(\mathcal{IFI}(\mathcal{IF}\delta C(\mu)))$

$\leq \mathcal{IFC}(\iota(\mathcal{IF}\theta I(\mu))) \vee \iota(\mathcal{IFI}(\mathcal{IF}\delta C(\mu)))$.

Since ι is an $\mathcal{IF}\theta biCts$ mapping, then ι is $\mathcal{IF}\theta O$ map and $\mathcal{IF}\theta Cts$ map. Then ι is $\mathcal{IF}\theta sCts$ map and $\mathcal{IF}\theta pCts$ map. Hence $\iota(\mu) \leq \mathcal{IFC}(\mathcal{IF}\theta I(\iota(\mu))) \vee \mathcal{IF}(\mathcal{IF}\delta C(\iota(\mu)))$. This shows that $\iota(\mu)$ is $\mathcal{IFM}o$ set in (ω,σ).

Theorem 4.10 *Let* (Ω,τ), (ω,σ) *and* (Z,γ) *be* $\mathcal{IFTS}$*'s. If* $\iota:(\Omega,\tau)\to(\omega,\sigma)$ *and* $\jmath:(\omega,\sigma)\to(Z,\gamma)$ *are mappings, then* $\jmath\circ\iota$ *is* $\mathcal{IFMO}$ *mapping if*

1. ι is $\mathcal{IFO}$ and $\jmath$ is $\mathcal{IFMO}$.
2. ι is $\mathcal{IFMO}$ and $\jmath$ is $\mathcal{IF}\theta biCts$ mapping.

Proof. (i) Let $\mu \in \Omega$. Since ι is $\mathcal{IFO}$ then $\iota(\mu) \in \omega$. Since $\jmath$ is $\mathcal{IFMO}$, then $\jmath(\iota(\mu)) = (\jmath\circ\iota)(\mu)$ is $\mathcal{IFM}o$ set in (Z,γ). Hence $\jmath\circ\iota$ is $\mathcal{IFMO}$.

(ii) Let $\mu \in \Omega$. Since ι is $\mathcal{IFMO}$, then $\iota(\mu)$ is an $\mathcal{IFM}o$ set in (ω,σ). Since $\jmath$ is $\mathcal{IF}\theta biCts$, by Theorem 4.9, $(\jmath\circ\iota)(\mu)$ is $\mathcal{IFM}o$ set in (Z,γ). Hence $\jmath\circ\iota$ is $\mathcal{IFMO}$.

Theorem 4.11 *Let* (Ω,τ), (ω,σ) *and* (Z,γ) *be* $\mathcal{IFTS}$*'s. If* $\iota:(\Omega,\tau)\to(\omega,\sigma)$ *and* $\jmath:(\omega,\sigma)\to(Z,\gamma)$ *are mappings, then*

1. If $\jmath\circ\iota$ is $\mathcal{IFMO}$ mapping and ι is a surjective $\mathcal{IFC}ts$ map, then $\jmath$ is $\mathcal{IFMO}$ map.
2. If $\jmath\circ\iota$ is $\mathcal{IFO}$ mapping and $\jmath$ is an injective $\mathcal{IFMC}ts$ map, then ι is $\mathcal{IFMO}$ map.

Proof. (i) Let $\mu \in \omega$. Since ι is $\mathcal{IFC}ts$, then $\iota^{-1}(\mu)$ is an $\mathcal{IF}o$ set in (Ω,τ). But $\jmath\circ\iota$ is $\mathcal{IFMO}$ map, then $(\jmath\circ\iota)(\iota^{-1}(\mu))$ is $\mathcal{IFM}o$ set in (Z,γ). Hence by surjective of ι, we have $\jmath(\mu)$ is $\mathcal{IFM}o$ set of (Z,γ). Hence, $\jmath$ is $\mathcal{IFMO}$ map.

(ii) Let μ is an $\mathcal{IF}o$ set in (Ω,τ). and $\jmath\circ\iota$ be an $\mathcal{IFO}$. Then $(\jmath\circ\iota)(\mu) = \jmath(\iota(\mu))$ is an $\mathcal{IF}o$ set in (Z,γ). Since $\jmath$ is an injective $\mathcal{IFMC}ts$ map, hence $\iota(\mu)$ is fMo set in (ω,σ). Therefore ι is $\mathcal{IFMO}$.

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