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On Intuitionistic Fuzzy M-Closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract

In this paper, we introduce a new class of sets termed as $\mathcal{IF}\theta c$, $\mathcal{IF}\theta o$, $\mathcal{IF}\theta so$, $\mathcal{IF}\theta sc$, $\mathcal{IF}Mc$ and $\mathcal{IF}Mo$ sets with the help of $\mathcal{IF}\theta$ (resp. $\mathcal{IF}\delta$) interior and $\mathcal{IF}\theta$ (resp. $\mathcal{IF}\delta$) closure. Also using these sets we have introduce $\mathcal{IF}$- θclo, $\mathcal{IF}$-$\theta sclo$,$\mathcal{IF}$-δclo, $\mathcal{IF}$-$\delta pclo$, $\mathcal{IF}$-$Mclo$ sets. Furthermore, We study the topological properties and characterizations of these sets. Also we obtain the interrelations between these sets and already existing sets in the theory of intuitionistic fuzzy topological spaces, and we provide examples to illustrate the theory.

Keywords and phrases: intuitionistic fuzzy topological spaces, $\mathcal{IF}\theta c$, $\mathcal{IF}\theta o$, $\mathcal{IF}\theta so$, $\mathcal{IF}\theta sc$, $\mathcal{IF}Mc$, $\mathcal{IF}Mo$

AMS (2000) subject classification: 54A40, 54A99, 03E72, 03E99

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [16] in his classical paper. Fuzzy set have applications in many fields such as Information [12] and Control [13]. After the introduction of fuzzy sets, various authors introduced generalization of the notion of fuzzy set. Atanassov [3] generalized the fuzzy sets to intuitionistic fuzzy sets(in brief,$\mathcal{IFS}$). Some basic results on $\mathcal{IFS}$'s were published in [3, 4], and the book [4] provides a comprehensive coverage of virtually all results in the area of the theory and applications of $\mathcal{IFS}$'s. Coker and his colleague [6, 8, 7] defined intuitionistic fuzzy topology (in brief, $\mathcal{IFTS}$) in Chang's sense. After that the definition of $\mathcal{IFTS}$ in Samanta and Mondal [11, 10] ($\mathcal{IF}$ gradation of openness) was introduced and studied. In 2004, Caldas et al. [5], introduced some properties of θ open sets and in 2011, Maghrabi and Johany [9] introduced M open sets in topological spaces. In this paper, we study a new class of sets termed as $\mathcal{IF}\theta c$, $\mathcal{IF}\theta o$, $\mathcal{IF}\theta so$, $\mathcal{IF}\theta sc$, $\mathcal{IF}Mc$ and $\mathcal{IF}Mo$ sets with its topological properties and characterizations of these sets. Also we obtain the interrelations between these sets and already existing sets in the theory of intuitionistic fuzzy topological spaces, and we provide examples to illustrate the theory.

2 Preliminaries

Definition 2.1 ***[3]*** *Let Ω be a nonempty fixed set and I the closed interval* $[0,1]$. *An $\mathcal{IFS}$ μ is an object of the following form $\mu = \{\langle \varepsilon, \rho_\mu(\varepsilon), \varrho_\mu(\varepsilon)\rangle : \varepsilon \in \Omega\}$, where the mapping $\rho_\mu : X \to I$ and $\varrho_\mu : \Omega \to I$ denote the degree of membership (namely, $\rho_\mu(\varepsilon)$) and the degree of nonmembership (namely, $\varrho_\mu(\varepsilon)$) for each element $\varepsilon \in \Omega$ to the set μ, respectively, and $0 \le \rho_\mu(\varepsilon) + \varrho_\mu(\varepsilon) \le 1$ for each $\varepsilon \in \Omega$.*

Definition 2.2 *[1, 3] Let Ω be a nonempty set, and the $\mathcal{IFS}$'s μ and γ in Ω be the form $\mu = \{\langle \varepsilon, \rho_\mu(\varepsilon), \varrho_\mu(\varepsilon)\rangle : \varepsilon \in \Omega\}, \gamma = \{\langle \varepsilon, \rho_\gamma(\varepsilon), \varrho_\gamma(\varepsilon)\rangle : \varepsilon \in \Omega\}$ Furthermore, let $\{\mu_i : i \in J\}$ (J be an index set) be an arbitrary family of $\mathcal{IFS}$'s in Ω. Then*

1. $\mu \le \gamma$ if and only if $\rho_\mu(\varepsilon) \le \rho_\gamma(\varepsilon)$ and $\gamma_\mu(\varepsilon) \ge \gamma_\gamma(\varepsilon)$, for all $\varepsilon \in X$.
2. $\mu = \gamma$ if and only if $\mu \le \gamma$ and $\gamma \le \mu$.
3. $\mu \wedge \gamma = \{\langle \varepsilon, \rho_\mu(\varepsilon) \wedge \rho_\gamma(\varepsilon), \gamma_\mu(\varepsilon) \vee \gamma_\gamma(\varepsilon)\rangle : \varepsilon \in X\}$.
4. $\mu \vee \gamma = \{\langle \varepsilon, \rho_\mu(\varepsilon) \vee \rho_\gamma(\varepsilon), \gamma_\mu(\varepsilon) \wedge \gamma_\gamma(\varepsilon)\rangle : \varepsilon \in X\}$.
5. $\overline{\mu} = \{\langle \varepsilon, \gamma_\mu(\varepsilon), \rho_\mu(\varepsilon)\rangle : \varepsilon \in X\}$.
6. $\mu - \gamma = \mu \wedge \overline{\gamma}$.
7. $\wedge_{i\in\mathrm{N}} \mu_i = \{\langle \varepsilon, \wedge_{i\in\mathrm{N}} \rho_{\mu_i}(\varepsilon), \vee_{i\in\mathrm{N}} \gamma_{\mu_i}(\varepsilon)\rangle : \varepsilon \in X\}$.
8. $\vee_{i\in\mathrm{N}} \mu_i = \{\langle \varepsilon, \vee_{i\in\mathrm{N}} \rho_{\mu_i}(\varepsilon), \wedge_{i\in\mathrm{N}} \gamma_{\mu_i}(\varepsilon)\rangle : \varepsilon \in X\}$.
9. $\underline{0} = \{\langle \varepsilon, 0,1\rangle : \varepsilon \in X\}$ and $\underline{1} = \{\langle \varepsilon, 1,0\rangle : \varepsilon \in X\}$.

Definition 2.3 *[8] An $\mathcal{IFT}$ in Coker's sense on a nonempty set Ω is a family τ of $\mathcal{IFS}'s$ in Ω satisfying the following axioms*

1. $\underline{0}, \underline{1} \in \tau$.
2. $H_1 \wedge H_2 \in \tau$, for any $H_1, H_2 \in \tau$.
3. $\vee H_i \in \tau$ for any arbitrary family $\{H_i : i \in J\} \subseteq \tau$.

Each $\mathcal{IFS}$ μ which belongs to τ is called an $\mathcal{IF}$ open ($\mathcal{IF}o$) set in X. The complement $\overline{\mu}$ of an $\mathcal{IF}o$ set μ in Ω is called an $\mathcal{IF}$ closed ($\mathcal{IF}c$) set in Ω.

Definition 2.4 *[8] Let (Ω, τ) be an $\mathcal{IFTS}$ and $\mu = \{\langle \varepsilon, \mu_\mu, \nu_\mu \rangle : \varepsilon \in \Omega\}$ be an $\mathcal{IFS}$ in Ω. Then the $\mathcal{IF}$ closure (in brief, $\mathcal{IFC}$) and $\mathcal{IF}$ interior (in brief, $\mathcal{IFI}$) of μ are defined by*

1. $\mathcal{IFC}(\mu) = \wedge_{i \in \mathrm{N}} \{\iota : \iota$ is an IFcsin Ω and $\iota \geq \mu\}$.
2. $\mathcal{IFI}(\mu) = \vee_{i \in \mathrm{N}} \{\kappa : \kappa$ is an IFosin Ω and $\kappa \leq \mu\}$.

Definition 2.5 *[15] Let μ be $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (Ω, τ). μ is called an $\mathcal{IF}$*

1. regular open (in brief, $\mathcal{IF}ro$) set if $\mu = \mathcal{IFIIFC}(\mu)$.
2. regular closed (in brief, $\mathcal{IF}rc$) set if $\mu = \mathcal{IFCIFI}(\mu)$.

Definition 2.6 *[15] Let (Ω, τ) be an $\mathcal{IFTS}$ and $\mu =< \varepsilon, \mu_\mu(\varepsilon), \nu_\mu(\varepsilon) >$ be a $\mathcal{IFS}$ in Ω. Then the $\mathcal{IF}$ δ closure of μ are denoted and defined by $\mathcal{IF}\delta C(\mu) = \wedge$ $\{\iota : \iota$ is an $\mathcal{IF}rc$ set in Ω and $\mu \leq \iota\}$ and $\mathcal{IF}\delta I(\mu) = \vee$ $\{\kappa : \kappa$ is an $\mathcal{IF}ro$ set in Ω and $\kappa \leq \mu\}$.*

Definition 2.7 *[14] Let μ be an $\mathcal{IFS}$ in an $\mathcal{IFTS}$ (Ω, τ) then μ is called an $\mathcal{IF}$ [(i)]*

1. δ-preopen (briefly, $\mathcal{IF}\delta po$) set if $\mu \subseteq \mathcal{IF}int(\mathcal{IF}cl_\delta(\mu))$.
2. δ-semiopen (briefly, $\mathcal{IF}\delta so$) set if $\mu \subseteq \mathcal{IF}int(\mathcal{IF}cl_\delta(\mu))$.
3. e-open (briefly, $\mathcal{IF}eo$) set if $\mu \subseteq \mathcal{IF}cl\mathcal{IF}int_\delta(\mu) \cup \mathcal{IF}int\mathcal{IF}cl_\delta(\mu)$.
4. δ-preclosed (briefly, $\mathcal{IF}\delta pc$) set if $\mu \supseteq \mathcal{IF}cl(\mathcal{IF}int_\delta(\mu))$.
5. δ-semiclosed (briefly, $\mathcal{IF}\delta sc$) set if $\mu \supseteq \mathcal{IF}cl(\mathcal{IF}int_\delta(\mu))$.
6. e-closed (briefly, $\mathcal{IF}ec$) set if $\mu \supseteq \mathcal{IF}cl\mathcal{IF}int_\delta(\mu) \cap \mathcal{IF}int\mathcal{IF}cl_\delta(\mu)$.

3 Intuitionistic fuzzy M closed sets

Definition 3.1 *Let (Ω, τ) be a $\mathcal{IFTS}$, $\forall$ $\mathcal{IFS}$ γ, ν the operators $\mathcal{IF}$- θ interior and $\mathcal{IF}$- θ closure denoted by $(\mathcal{IF})\theta I$ and $\mathcal{IF}\theta C$ are defined as*

$$\mathcal{IF}\theta I(\gamma) = \bigvee_{i \in N} \{\nu \mid \nu \in \tau \ \& \ \mathcal{IFC}(\gamma) \leq \nu\}$$

and

$$\mathcal{IF}\theta C(\gamma) = \bigvee_{i \in N} \{\nu \mid \nu \in \tau \ \& \ \mathcal{IFI}(\gamma) \geq \nu\}.$$

Definition 3.2 *In an $\mathcal{IFTS}$ (Ω, τ) and $\mathcal{IFS}$ γ is called an*

1. $\mathcal{IF}$ -θ open (resp. $\mathcal{IF}$ -θ semi open) (briefly $\mathcal{IF}\theta o$ (resp. $\mathcal{IF}\theta so$)) set if $\gamma = \mathcal{IF}\theta I(\gamma)$. (resp. $\gamma \leq \mathcal{IFC}(\mathcal{IF}\theta I(\gamma))$).
2. $\mathcal{IF}$ -θ closed (resp. $\mathcal{IF}$-θ semi closed) (briefly $\mathcal{IF}\theta c$ (resp. $\mathcal{IF}\theta sc$)) set if $\overline{\gamma}$ is an $\mathcal{IF}$ θo (resp. $\mathcal{IF}\theta so$) set.

Definition 3.3 *In an $\mathcal{IFTS}$ (Ω, τ), and $\mathcal{IFS}$ γ is called an*

1. $\mathcal{IF}$-M closed (briefly $\mathcal{IFM}c$) set if $\gamma \geq \mathcal{IFI}(\mathcal{IF}\theta C(\gamma)) \wedge \mathcal{IFC}(\mathcal{IF}\delta I(\gamma))$.
2. $\mathcal{IF}$-M open (briefly $\mathcal{IFM}o$) set if $\overline{\gamma}$ is an $\mathcal{IFM}c$ set.

Definition 3.4 *Let (Ω, τ) be a $\mathcal{IFTS}$, then the*

1. union of all $\mathcal{IFM}o$ (resp. $\mathcal{IF}\theta so$) sets contained in γ is called the $\mathcal{IFM}$ (resp. $\mathcal{IF}\theta$ semi) interior of γ and is denoted by $\mathcal{IFMI}(\gamma)$ (resp. $\mathcal{IF}\theta sI(\gamma)$).
2. intersection of all $\mathcal{IFM}c$ (resp. $\mathcal{IF}\theta sc$) sets containing γ is called the $\mathcal{IFM}$ (resp. $\mathcal{IF}\theta$ semi) closure of γ and is denoted by $\mathcal{IFMC}(\gamma)$ (resp. $\mathcal{IF}\theta sC(\gamma)$).

Proposition 3.1 *In a* $\mathcal{IFTS}$ (Ω,τ) $\forall\gamma\nu\in I^X$,

1. $\mathcal{IFMI}(\underline{0})=\underline{0}$ and $\mathcal{IFMI}(\underline{1})=\underline{1}$.
2. $\mathcal{IFMC}(\underline{0})=\underline{0}$ and $\mathcal{IFMC}(\underline{1})=\underline{1}$.
3. $\mathcal{IFMI}(\gamma)=\mathcal{IFMC}(\overline{\gamma})$.
4. $\mathcal{IFMC}(\gamma)=\mathcal{IFMI}(\overline{\gamma})$.
5. If $\gamma<\nu$ then $\mathcal{IFMI}(\gamma)<\mathcal{IFMI}(\nu)$.
6. If $\gamma\leq\nu$ then $\mathcal{IFMC}(\gamma)\leq\mathcal{IFMC}(\nu)$.
7. $\mathcal{IFMI}(\gamma).\leq\gamma\leq\mathcal{IFMC}(\gamma)$.
8. $\mathcal{IFMC}(\gamma\vee\nu)\geq\mathcal{IFMC}(\gamma)\vee\mathcal{IFMC}(\nu)$.
9. $\mathcal{IFMC}(\gamma\wedge\nu)\leq\mathcal{IFMC}(\gamma)\wedge\mathcal{IFMC}(\nu)$.
10. $\mathcal{IFMC}(\mathcal{IFMC}(\gamma))=\mathcal{IFMC}(\gamma)$.
11. If γ is $\mathcal{IFMc}$ set then $\mathcal{IFMC}(\gamma)=\gamma$.
12. If ν is $\mathcal{IFMo}$ set then $\nu\ \ q\ \ \gamma$ iff $\nu\ \ q\ \ \mathcal{IFMC}(\gamma)$.
13. $\mathcal{IFMI}(\gamma\vee\nu)\geq\mathcal{IFMI}(\gamma)\vee\mathcal{IFMI}(\nu)$.
14. $\mathcal{IFMI}(\gamma\wedge\nu)\leq\mathcal{IFMI}(\gamma)\wedge\mathcal{IFMI}(\nu)$.
15. $\mathcal{IFMI}(\mathcal{IFMI}(\gamma),)=\mathcal{IFMI}(\gamma)$.
16. If γ is $\mathcal{IFMo}$ set then $\mathcal{IFMI}(\gamma)=\gamma$.
17. $\gamma\leq\mathcal{IFC}(\gamma)\leq\mathcal{IF\delta C}(\gamma)\leq\mathcal{IF\theta C}(\gamma)$.
18. $\mathcal{IF\theta I}(\gamma)\leq\mathcal{IF\delta I}(\gamma)\leq\mathcal{IFI}(\gamma)\leq\gamma$.

Proof. Straight Forward.

Theorem 3.1 *In any* $\mathcal{IFTS}$ (Ω,τ) *Ever*

1. $\mathcal{IF\theta sc}$ (resp. $\mathcal{IF\delta pc}$) set is an $\mathcal{IFMc}$ set.
2. $\mathcal{IF\theta c}$ set is an $\mathcal{IF\theta sc}$ set.
3. $\mathcal{IF\theta c}$ set is an $\mathcal{IFc}$ set.
4. $\mathcal{IFc}$ set is an $\mathcal{IF\delta pc}$ set.
5. $\mathcal{IFMc}$ set is an $\mathcal{IFec}$ set.

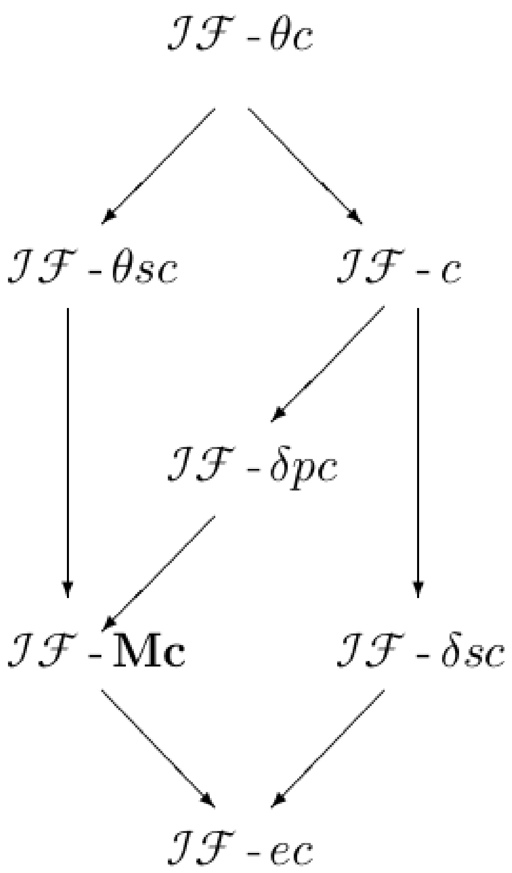
But not conversely.

Proof. Straight Forward.

Example 3.1 *Let* $\Omega=\{a,e,i,o\}$, $\upsilon=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0.2},\frac{o}{0}),(\frac{a}{0},\frac{e}{1},\frac{i}{0.7},\frac{o}{1})\right\rangle$, $\phi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{1},\frac{i}{0},\frac{o}{0}),(\frac{a}{1},\frac{e}{0},\frac{i}{1},\frac{o}{0.1})\right\rangle$, $\varphi=\left\langle\varepsilon,(\frac{a}{1},\frac{e}{0},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0},\frac{o}{0})\right\rangle$, $\psi=\left\langle\varepsilon,(\frac{a}{0},\frac{e}{0.8},\frac{i}{0},\frac{o}{1}),(\frac{a}{0},\frac{e}{0.2},\frac{i}{0.9},\frac{o}{0})\right\rangle$ *Then the family* $\tau=\{\underline{0},\underline{1},\upsilon,\phi,\upsilon\vee\phi\}$ *is an* $\mathcal{IFT}$ *on* Ω. $\overline{\varphi}$ *is* $\mathcal{IFec}$ *but not* $\mathcal{IF\delta sc}$ *and* $\mathcal{IFMc}$, $\overline{\psi}$ *is* $\mathcal{IFMc}$ *but not* $\mathcal{IF\theta sc}$ *and* $\mathcal{IF\delta pc}$, $\overline{\phi}$ *is* $\mathcal{IFc}$ *but not* $\mathcal{IF\theta c}$ *and* $\mathcal{IF\theta sc}$

Example 3.2 *Let* $\Omega=\{a,e\}$, $\upsilon=\left\langle\varepsilon,(\frac{a}{0.5},\frac{e}{0.5}),(\frac{a}{0.3},\frac{e}{0.5})\right\rangle$, $\phi=\left\langle\varepsilon,(\frac{a}{0.7},\frac{e}{0.2}),(\frac{a}{0.3},\frac{e}{0.2})\right\rangle$, $\varphi=\left\langle\varepsilon,(\frac{a}{0.3},\frac{e}{0.4}),(\frac{a}{0.5},\frac{e}{0.6})\right\rangle$, $\psi=\left\langle\varepsilon,(\frac{a}{0.5},\frac{e}{0.7}),(\frac{a}{0.3},\frac{e}{0.2})\right\rangle$. *Then the family* $\tau=\{\underline{0},\underline{1},\upsilon\}$ *is an* $\mathcal{IFT}$ *on* Ω. $\overline{\varphi}$ *is* $\mathcal{IF\delta pc}$ *but not* $\mathcal{IFc}$ *and* $\overline{\psi}$ *is* $\mathcal{IF\delta sc}$ *but not* $\mathcal{IFc}$.

From the Theorem 3.1, Examples 3.1, 3.2 the following



Note: $A \rightarrow B$ denotes A implies B, but not conversely.

Theorem 3.2 *Let* (Ω,τ) *be an* $\mathcal{IFTS}$,

1. $\underset{i\in \mathrm{N}}{\vee}\gamma_i$ is an $\mathcal{IF}Mo$ set if $\forall \quad i\in \mathrm{N},\ \gamma_i$ be an $\mathcal{IF}Mo$ set.
2. $\underset{i\in \mathrm{N}}{\wedge}\gamma_i$ is an $\mathcal{IF}Mc$ set if $\forall \quad i\in \mathrm{N},\ \gamma_i$ be an $\mathcal{IF}Mc$ set.

Proof. (i) Let γ_i be an $\mathcal{IF}Mo$ set, $\forall i\in \mathrm{N}$ then

$\gamma_i \le \mathcal{IFC}(\mathcal{IF}\theta I(\gamma_i)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\gamma_i)) \qquad \forall i\in \mathrm{N}.$

$\Rightarrow \underset{i\in \mathrm{N}}{\vee}\gamma_i \le \underset{i\in I}{\vee}(\mathcal{IFC}(\mathcal{IF}\theta I(\gamma_i)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\gamma_i)))$

$\le \mathcal{IFC}(\mathcal{IF}\theta I(\underset{i\in I}{\vee}\gamma_i)) \underset{i\in \mathrm{I}}{\vee} \mathcal{IFI}(\mathcal{IF}\delta C(\underset{i\in I}{\vee}\gamma_i)).$

Thus $\underset{i\in I}{\vee}\gamma_i$ is an $\mathcal{IF}Mo$ set.

(ii) Similar to the proof of (i).

Theorem 3.3 *In an* $\mathcal{IFTS}$ (Ω,τ) *let* γ , ν *be any* $\mathcal{IFS}$

1. $\gamma \wedge \nu$ is an $\mathcal{IF}Mo$ set if γ is an $\mathcal{IF}Mo$ set and $\nu \in \tau$.
2. $\gamma \vee \nu$ is an $\mathcal{IF}Mc$ set if γ is an $\mathcal{IF}Mc$ set and $\overline{\nu} \in \overline{\tau}$

Proof. (i) Let γ is an $\mathcal{IF}Mo$ set, and $\mathcal{IFS}, \nu \in \tau$ then

$\gamma \wedge \nu \le (\mathcal{IFC}(\mathcal{IF}\theta I(\gamma)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\gamma))) \wedge \nu$

$= (\mathcal{IFC}(\mathcal{IF}\theta I(\gamma)) \wedge \nu) \vee (\mathcal{IFI}(\mathcal{IF}\delta C(\gamma)) \wedge \nu)$

$\le (\mathcal{IFC}(\mathcal{IF}\theta I(\gamma) \wedge \nu)) \vee (\mathcal{IFI}(\mathcal{IF}\delta C(\gamma) \wedge \nu))$

$\leq (\mathcal{IFC}(\mathcal{IF}\theta I(\gamma \wedge \nu))) \vee (\mathcal{IFI}(\mathcal{IF}\delta C(\gamma \wedge \nu)))$

Hence $\gamma \wedge \nu$ is an $\mathcal{IFM}o$ set.

(ii) Similar to the proof of (i).

Theorem 3.4 *If $\mathcal{IFS}$ γ is both $\mathcal{IFM}o$ and $\mathcal{IF}c$ set in (Ω, τ) then γ is an $\mathcal{IF}\theta so$.*

Proof. Let γ be an $\mathcal{IFM}o$ then

$\gamma \leq \mathcal{IFC}(\mathcal{IF}\theta I(\gamma)) \vee \mathcal{IFI}(\mathcal{IF}\delta C(\gamma))$

$= \mathcal{IFC}(\mathcal{IF}\theta I(\gamma)) \vee \mathcal{IFI}(\gamma)$

$\leq \mathcal{IFC}(\mathcal{IF}\theta I(\gamma),).$

Hence γ is an $\mathcal{IF}\theta so$.

Theorem 3.5 *If $\gamma \in I^X$ is both $\mathcal{IFM}c$ and $\mathcal{IF}o$ set in (Ω, τ) then γ is an $\mathcal{IF}\theta sc$.*

Proof. Follows from Theorem 3.4.

Theorem 3.6 *In a $\mathcal{IFTS}$ (Ω, τ) $\forall$ $\mathcal{IFS}$ γ, [(i)]*

1. If $\gamma \in \tau$ then γ is an $\mathcal{IFM}o$ set.
2. $\mathcal{IFI}(\gamma)$ is an $\mathcal{IFM}o$ set.
3. $\mathcal{IFC}(\gamma)$ is an $\mathcal{IFM}c$ set.

Proof. Straight Forward.

4 Intuitionistic fuzzy M clopen sets

Definition 4.1 *In a $\mathcal{IFTS}$, (Ω, τ), an $\mathcal{IFS}$, ν is called an $\mathcal{IF}$- M clopen (resp. $\mathcal{IF}$- θ clopen, $\mathcal{IF}$- θ semiclopen, $\mathcal{IF}$- δ clopen and $\mathcal{IF}$- δ preclopen) (briefly $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$)) if ν is both $\mathcal{IFM}o$ (resp. $\mathcal{IF}\theta o$, $\mathcal{IF}\theta so$, $\mathcal{IF}\delta o$ and $\mathcal{IF}\delta po$) set and $\mathcal{IFM}c$ (resp. $\mathcal{IF}\theta c$, $\mathcal{IF}\theta sc$, $\mathcal{IF}\delta c$ and $\mathcal{IF}\delta pc$) set.*

Proposition 4.1 *In a $\mathcal{IFTS}$ (Ω, τ)*

1. $\underline{0}$ and $\underline{1}$ are $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) sets.
2. If $\mathcal{IFS}$ ν is $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) set then so is $(\overline{\nu})$.
3. If $\mathcal{IFS}$ ν, μ are $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) sets then $\nu \vee \mu$ and $\nu \wedge \mu$ are $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) set.
4. The set of all $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) sets may be used as a basis for a $\mathcal{IFTS}$, whereas the set of all $\mathcal{IFM}o$ sets do not form any basis.

Definition 4.2 *Let (Ω, τ) be a $\mathcal{IFTS}$, then the*

1. union of all $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) sets contained in γ is called the $\mathcal{IFM}$ clopen (resp. $\mathcal{IF}\theta$ clopen, $\mathcal{IF}\theta$ semiclopen, $\mathcal{IF}\delta$ clopen and $\mathcal{IF}\delta$ preclopen) interior of γ and is denoted by $\mathcal{IFMI}^{co}(\gamma)$ (resp. $\mathcal{IF}\theta I^{co}(\gamma)$, $\mathcal{IF}\theta sI^{co}(\gamma)$, $\mathcal{IF}\delta I^{co}(\gamma)$ and $\mathcal{IF}\delta pI^{co}(\gamma)$).
2. intersection of all $\mathcal{IFM}clo$ (resp. $\mathcal{IF}\theta clo$, $\mathcal{IF}\theta sclo$, $\mathcal{IF}\delta clo$ and $\mathcal{IF}\delta pclo$) sets containing γ is called the $\mathcal{IFM}$ clopen (resp. $\mathcal{IF}\theta$ clopen, $\mathcal{IF}\theta$ semiclopen, $\mathcal{IF}\delta$ clopen and $\mathcal{IF}\delta$ preclopen) closure of γ and is denoted by $\mathcal{IFMC}^{co}(\gamma)$ (resp. $\mathcal{IF}\theta C^{co}(\gamma)$, $\mathcal{IF}\theta sC^{co}(\gamma)$, $\mathcal{IF}\delta C^{co}(\gamma)$ and $\mathcal{IF}\delta pC^{co}(\gamma)$).

Proposition 4.2 *In a $\mathcal{IFTS}$ (Ω, τ) $\forall$ γ, ν be $\mathcal{IFS}$ [(i)]*

1. $\mathcal{IFMI}^{co}(\underline{0}) = \underline{0}$ and $\mathcal{IFMI}^{co}(\underline{1}) = \underline{1}$.
2. If $\gamma \leq \nu$ then $\mathcal{IFMI}^{co}(\gamma) \leq \mathcal{IFMI}^{co}(\nu)$.
3. $\mathcal{IFMI}^{co}(\gamma) \leq \mathcal{IFMI}(\gamma) \leq \gamma \leq \mathcal{IFMC}(\gamma) \leq \mathcal{IFMC}^{co}(\gamma)$.
4. $\mathcal{IFMI}^{co}(\mathcal{IFMI}^{co}(\gamma)) = \mathcal{IFMI}^{co}(\gamma)$.
5. $\mathcal{IFMI}^{co}(\gamma) = \mathcal{IFMC}^{co}(\overline{\gamma})$.
6. If γ is $\mathcal{IFM}clo$ set then $\mathcal{IFMC}^{co}(\gamma) = \gamma = \mathcal{IFMI}^{co}(\gamma)$.

The Proposition 4.2 holds for the operators $\mathcal{IF}\theta I^{co}(\gamma)$ (resp. $\mathcal{IF}\theta sI^{co}(\gamma)$, $\mathcal{IF}\delta I^{co}(\gamma)$, $\mathcal{IF}\delta pI^{co}(\gamma)$, $\mathcal{IF}MC^{co}(\gamma)$, $\mathcal{IF}\theta C^{co}(\gamma)$, $\mathcal{IF}\theta sC^{co}(\gamma)$, $\mathcal{IF}\delta C^{co}(\gamma$) and $\mathcal{IF}\delta pC^{co}(\gamma)$) with respect to their clopen sets.

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