

# Squeeze Film Lubrication between Secant Circular Plates with Rabinowitsch Fluid Model

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## Abstract:

The theoretical investigation of MHD squeeze film lubrication of secant circular plates with Rabinowitsch fluid model has been studied. In view of the stoke theory modified Reynolds-type equation is derived. The closed form of solution is obtained for pressure, load carrying capacity and squeezing time to study the influence of rabinowitsch fluid. The Non-dimensional pressure, load and squeeze time is obtained analytically and graphically. From this analysis we obtained that the pressure, load support and squeeze film time is increases for increasing the lower curvature where as the reverse case found in upper curvature parameter.

**Keywords :** Squeeze film, Rabinowitsch fluid model, Secant circular plates, Load carrying capacity.

## 1. Introduction:

Magneto-hydrodynamic (MHD) is the study of the magnetic properties and behavior electrically conducting fluids. Examples of such magneto fluids include plasmas, liquid metals, salt water and electrolytes. Bearing ensure high efficiency and reliability depending on their size, type of functioning, Materials used in Manufacturing and use of fluids for lubrication. Many researcher studied effect of Magnetic field on the performance of bearing by Hughes [1] presented magneto hydrodynamic lubrication flow between parallel rotating disks, The Magneto-hydrodynamic finite journal bearing by Kuzma [2], Analysis of finite magneto-hydrodynamic journal bearings by Malik and Singh [3], Lin [4] analyzed the magneto-hydrodynamic lubrication of finite slider bearings. They realized that the application of Magnetic field which increases the load carrying capacity and decreases the frictional forces. Kamiyama [5] studied Magneto-hydrodynamic journal bearing (report-I). Snyder [6] presented magneto-hydrodynamic slider bearing and it is observed that load carrying capacity of the bearing depends on the electromagnetic boundary condition passing through the conductivity of the surface. All these studies are belongs to classical hydrodynamic lubrication in which lubricant assume to behave as the Newtonian fluid. This is not much satisfactory prediction for practical application in engineering fields.

The researchers found the desirable lubricant by adding some polymers to Newtonian fluid known as non-Newtonian fluid and they started to study the characteristics of bearings lubricated with non-Newtonian fluids. Lin [7] studied the effects on MHD steady and dynamic characteristics wide tapered-land slider bearings; author reported that the MHD bearing provides higher values of load carrying capacity. Lin and Lu [8] presented MHD study and dynamic characteristics wide tapered land slider bearings; they found that the presence of applied magnetic fields signifies an enhancement of film pressure. In this result the use of applied magnetic field effects of the characterized by the Hartmann number gives increase in the values of load carrying capacity. Stochastic Reynolds equation for diverse shaped slider bearing when lubricated with couple stress fluid and by applying MHD studied by Fatima et al [9] are found that the load carrying capacity of Parabolic slider is more significant and which improves the normal functioning of the bearings.

Effect of magneto-hydrodynamics and rabinowitsch fluid was analyzed by many researchers Haung et al [10] studied the hydrostatic thrust bearing by rabinowitsch fluid it is found that the application of MHD and rabinowitsch lubricant which improves the pressure and load support in the thrust bearings. Squeeze film between rough circular stepped plates by Naduvinamani [11]. Hiremath et al [12-13] was analyzed effect of MHD couple stress on pivoted curved slider bearings and curved circular and flat plates, from this study shows the influence of MHD couple stress provides increment of squeeze film attributes. Boubendir S et al [14] analyzed the self lubricating journal by rabinowitsch fluids, It is shown that the dimensionless permeability has significant effects on the static characteristics of the bearings. The pseudo-plastic lubricant coefficient decreases the bearing characteristics (load capacity, pressure) compared to Newtonian lubricant cases. The use of the pseudo-plastic fluids reduces the performance of the bearings and this reduction grows with the porous bearing properties.

Udaya.P.Singh et.al [15-16] steady on the performance of hydrostatic thrust bearing: Rabinowitsch Fluid Model.

Hence from the present time there is no investigation done on Squeeze Film Lubrication between Secant Circular Plates with Rabinowitsch Fluid Model. Hence in this paper an investigation made to study the influence of Rabinowitsch Fluid Model on a secant curved circular plates.

## 2. Mathematical Formulation and Solution

An upper and lower plate is curved circular in nature having radius  $a$  considered is displayed in figure 1. The thickness between the plates is  $h$  and central thickness be the  $h_m$ . Let magnetic field  $B_0$  is applied perpendicular to the plate and non-Newtonian lubricant is filled between the plates.

The MHD momentum equations and continuity equation is defined for the flow following the basic Stokes theory is given by

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial p}{\partial r} = \frac{\partial \mathfrak{S}_{xy}}{\partial y} \quad (2)$$

$$\frac{\partial p}{\partial y} = 0 \quad (3)$$

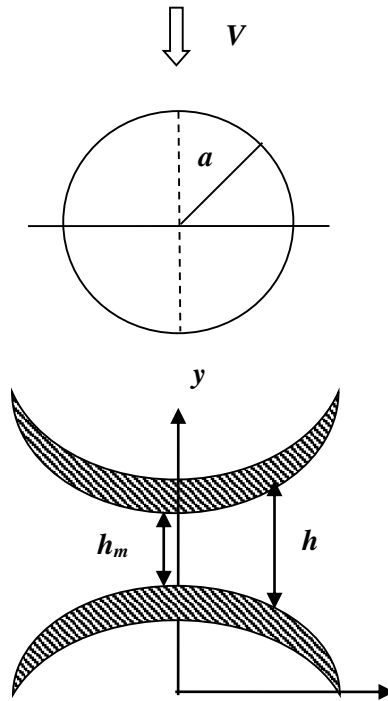


Figure 1. Geometry of system bearing

The relevant boundary Conditions are

i) For upper plate  $y = h$ :

$$u = 0, v = -V = \frac{\partial h}{\partial t} \quad (4a)$$

ii) For Lower Plate  $y = 0$ :

$$u = 0, v = 0 \quad (4b)$$

The plate lying along Upper surface determined by the relation

$$z_u = h_m \sec(\beta r^2); \quad 0 \leq r \leq a$$

The plate lying along the Lower surface determined by the relation

$$z_l = h_m \left\{ \sec(\gamma r^2) - 1 \right\}; \quad 0 \leq r \leq a$$

Here, the curvature parameters are  $\beta$  and  $\gamma$  of the corresponding plates, The film thickness  $h(r)$  and  $h_m$  is the central film thickness. Then,

$$h(r) = h_m \left\{ \sec(\beta r^2) - \sec(\gamma r^2) + 1 \right\}; \quad 0 \leq r \leq a$$

Integrating equation (2) with respect to  $y$

$$\int \frac{\partial p}{\partial r} dy = \int \frac{\partial \mathfrak{F}_{xy}}{\partial y}$$

$$\Rightarrow \mathfrak{F}_{xy} = fy + I_1 \quad (5)$$

Where  $f = \frac{\partial p}{\partial r}$  the pressure gradient and  $I_1$  is constant of integration.

#### Rabinowitsch Fluid Model

$$\mathfrak{F}_{xy} + k \mathfrak{F}_{xy}^3 = \mu \frac{\partial u}{\partial y} \quad (6)$$

Where,  $\mu$  is Newtonian fluid of initial viscosity and nonlinear factor of lubricants means  $k$ ,

Substituting equation (5) in the equation (6) we have

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \left[ (fy + I_1) + k(fy + I_1)^3 \right] \quad (7)$$

Integrating equation (7) with respect to  $y$  and using the boundary conditions (4a) and (4b) we get an velocity equation which is mentioned below

$$u = \frac{1}{\mu} \left[ \frac{1}{2} f F_1 + k f^3 F_2 \right] \quad (8)$$

$$\text{Where, } F_1 = y(y-h), F_2 = \frac{y^4}{4} - \frac{y^3 h}{2} + \frac{3}{8} y^2 h^2 - \frac{y h^3}{8}$$

Using equation (8) in equation (1) integrating with respect to  $r$  between the limits 0 to  $h$  which gives the Reynolds equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left\{ h^3 \frac{\partial p}{\partial r} + \frac{3kh^5}{20} \left( \frac{\partial p}{\partial r} \right)^3 \right\} \right] = 12\mu \frac{dh}{dt} \quad (10)$$

Introducing the non-dimensional parameters

$$r^* = \frac{r}{a}; p^* = \frac{ph_0^2}{\mu a \left( -\frac{dh}{dt} \right)}; \beta = k \frac{\mu^2 a^2}{h_0^2 \left( -\frac{dH^*}{dt} \right)^2}; h^* = \frac{h}{h_0}$$

Using above parameters in equation (10) we get Dimensionless Reynolds equation

$$\frac{\partial}{\partial r^*} \left[ r^* \left\{ h^{*3} \frac{\partial p^*}{\partial r^*} + \frac{3\beta}{20} h^{*5} \left( \frac{\partial p^*}{\partial r^*} \right)^3 \right\} \right] = -12r^* \quad (11)$$

Where,

$$h^* = \sec(Kr^2) - \sec(Jr^2) + 1; 0 \leq r \leq a$$

#### Perturbation Technique for evaluating non-dimensional pressure

$$p^* = p_0^* + \beta p_1^* \quad (12)$$

$$\frac{\partial}{\partial r^*} \left[ r^* \left\{ h^{*3} \left( \frac{\partial p_0^*}{\partial r^*} + \beta \frac{\partial p_1^*}{\partial r^*} \right) + \frac{3\beta}{20} h^{*7} \left( \frac{\partial p_0^*}{\partial r^*} + \beta \frac{\partial p_1^*}{\partial r^*} \right)^3 \right\} \right] = -12r^* \quad (13)$$

Equating the powers of  $\beta^0$  and  $\beta$  and neglecting the others

$$\frac{\partial}{\partial r^*} \left[ r^* \left\{ h^{*3} \frac{\partial p_1^*}{\partial r^*} + \frac{3}{20} h^{*5} \left( \frac{\partial p_0^*}{\partial r^*} \right)^3 \right\} \right] = 0 \quad (14)$$

Integrating equation (13) and solving the boundary condition  $\frac{dp^*}{dr^*} = 0$  at  $r^* = 0$  and  $p^* = 0$  at  $r^* = 1$

$$p^* = -\int_1^{r^*} \frac{6r^*}{h^{*3}} dr^* + \frac{162\beta}{5} \int_1^{r^*} \frac{r^{*3}}{h^{*7}} dr^* \quad (15)$$

Integrating equation (15) with respect to  $r^*$  between the limit 0 to 1 we obtain the dimensionless load carrying capacity

$$W^* = 2\pi \int_0^1 r^* p^* dr^* \quad W^* = -2\pi \int_0^1 \left( \int_1^{r^*} \frac{6r^*}{h^{*3}} dr^* \right) r^* dr^* + \frac{324\pi\beta}{5} \int_0^1 \left( \int_1^{r^*} \frac{r^{*3}}{h^{*7}} dr^* \right) r^* dr^* \quad (16)$$

Integrating above equation over the minimum film thickness from this we got squeezing time which is shown below

$$T^* = -2\pi \int_{h_f}^1 \left\{ \int_0^1 \left( \int_1^{r^*} \frac{6r^*}{h^{*3}} dr^* \right) r^* dr^* \right\} dh_m^* + \frac{324\pi\beta}{5} \int_{h_f}^1 \left\{ \int_0^1 \left( \int_1^{r^*} \frac{r^{*3}}{h^{*7}} dr^* \right) r^* dr^* \right\} dh_m^* \quad (28)$$

### 3. Results and Discussions:

Finally the investigation on the squeeze film lubrication between secant circular plates with Rabinowitsch fluid model has been done. All the results have been discussed by graphically with various non-dimensional parameters like upper and lower curved circular plates  $K$  &  $J$  respectively,  $\beta$  curvature parameter. To compute results for load carrying capacity  $W$ , pressure  $P$  and squeeze film time  $T$ , different parametric values has been chosen like;  $K = 0, 0.2, 0.4$        $J = 0, 0.2, 0.4$  and  $\beta = -0.004, 0, 0.004$ .

#### 3.1. Non-dimensional Pressure:

The dimensionless pressure  $P^*$  is plotted with  $r^*$  for various values of  $\beta$  is as shown in Fig.2 for  $\beta = -0.004, 0, 0.004$  keeping curvature parameters held fixed  $K = 0.6$  and  $J = 0.4$ . By increasing the nonlinear parameter  $\beta$  it decreases the pressure. Fig.3. expresses the variations of non-dimensional pressure  $P^*$  with  $r^*$  for different values of  $K = 0, 0.2, 0.4$  with  $\beta = 0.004$  &  $J = 0.4$ . The pressure decreases by increasing the upper curvature parameter  $K$ . Similarly Fig.4 depicts that by increasing the lower curvature parameter it increases the pressure  $P^*$  with  $r^*$  for  $K = 0.6$  &  $\beta = 0.004$ .

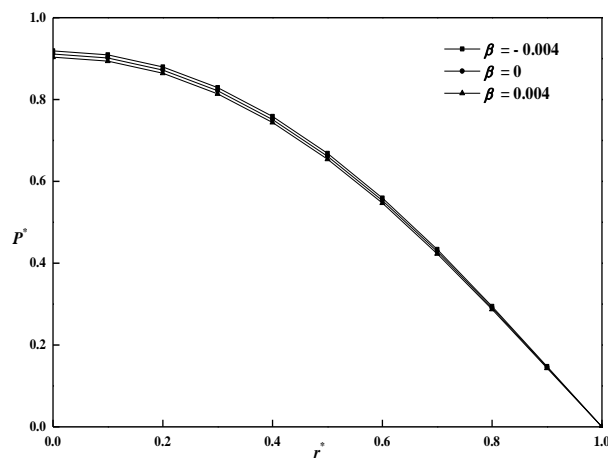


Figure 2: Variation of non-dimensional pressure  $P^*$  with  $r^*$  for different values of  $\beta$  with  $K = 0.6$  and  $J = 0.4$

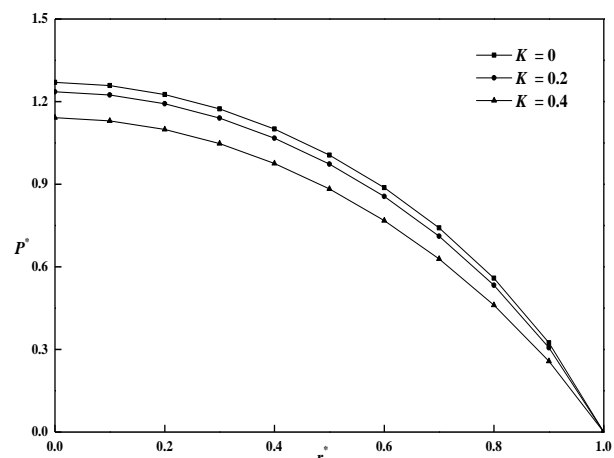


Figure 3: Variation of non-dimensional Pressure  $P^*$  with  $r^*$  for different values of  $K$  with  $\beta = 0.004$  and  $J = 0.4$

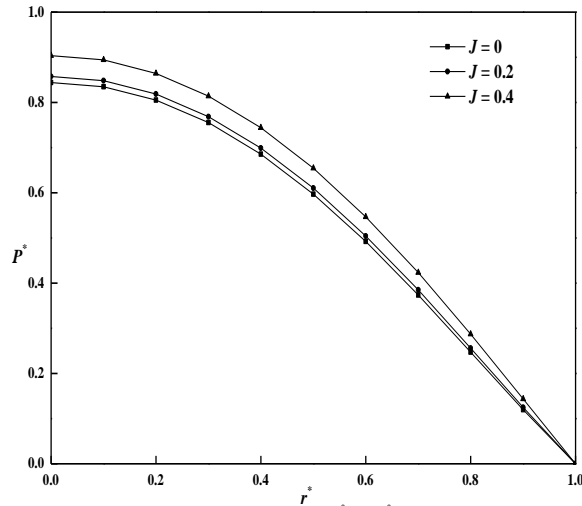


Figure 4: Variation of non-dimensional Pressure  $P^*$  with  $r^*$  for different values of  $J$  with  $K = 0.6$  and  $\beta = 0.004$

### 3.2. Non-dimensional load carrying capacity:

The non-dimensional load carrying capacity  $W^*$  is plotted with upper & lower curved circular plates,  $K^*$  &  $J^*$  respectively. Fig.5. defines that the load carrying support  $W^*$  with  $K^*$  is decreases by increasing the non-linear parameter and  $\beta = -0.004, 0, 0.004$  with  $J = 0.4$ . Fig.6. expresses that load carrying support  $W^*$  with  $K^*$  is increases by increasing the value of lower curvature parameter  $J = 0, 0.2, 0.4$  with  $\beta = 0.004$ . Fig.7. shows that the load carrying capacity  $W^*$  with  $J^*$  is decreases by increasing the non-linear parameter  $\beta = -0.004, 0, 0.004$  with  $K = 0.6$  similarly Fig.8. Predicts that the load carrying capacity  $W^*$  with  $J^*$  is decreases by increase in the value of upper curvature parameter  $K = 0, 0.2, 0.4$  with  $\beta = 0.004$ .

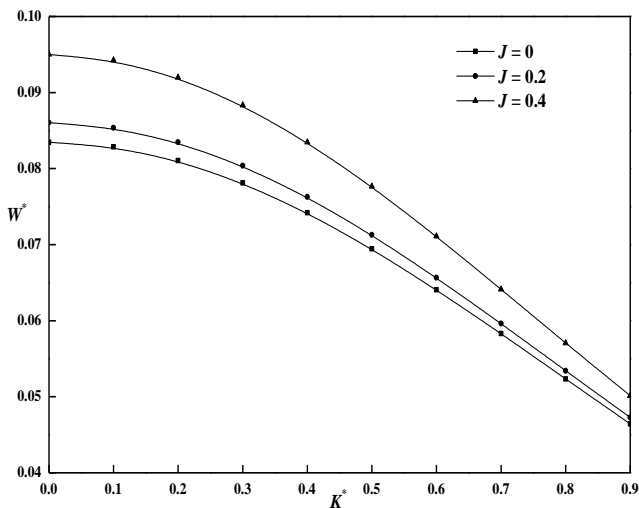


Figure 6: Variation of non-dimensional Pressure  $W^*$  with  $K^*$  for different values of  $J$  with  $\beta = 0.004$

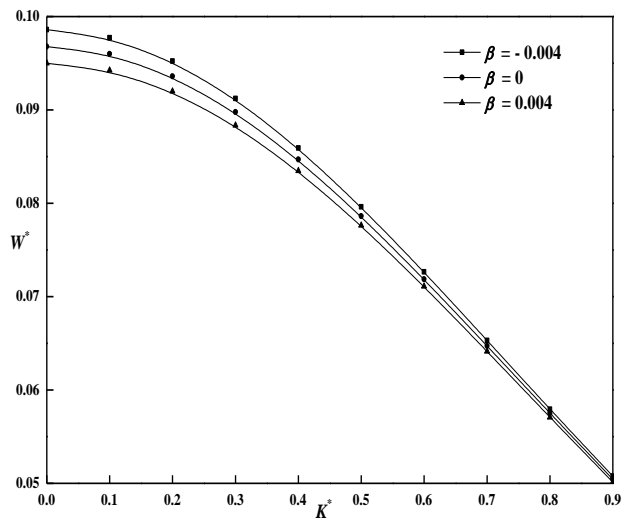


Figure 5: Variation of non-dimensional Load Carrying Capacity  $W^*$  with  $K^*$  for different values of  $\beta$  with  $J = 0.4$

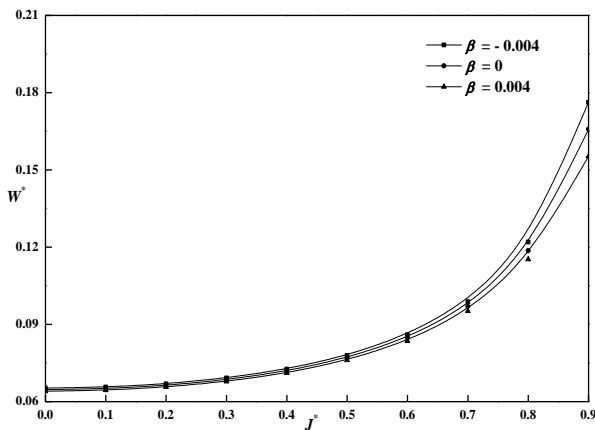


Figure 7: Variation of non-dimensional Pressure  $W^*$  with  $J^*$  for different values of  $\beta$  with  $K = 0.6$

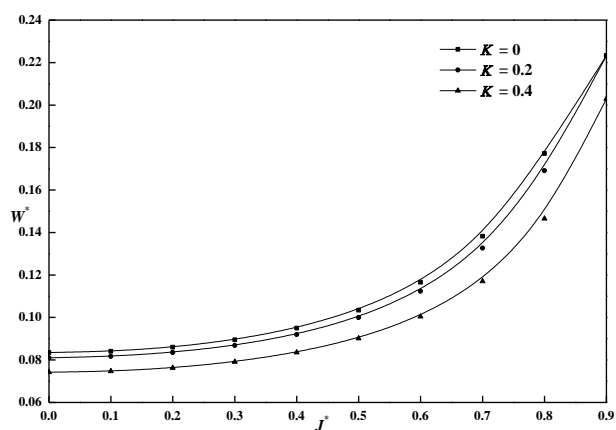


Figure 8: Variation of non-dimensional Pressure  $W^*$  with  $J^*$  for different values of  $K$  with  $\beta = 0.004$

### 3.3 Non-dimensional squeeze film time:

The non-dimensional squeeze film time  $T^*$  is plotted with  $h_m^*$  for different values of  $\beta$ ,  $K$ ,  $J$ . Fig.9. Defines the variations of non-dimensional squeeze film time with  $h_m^*$  for different values of  $\beta = -0.004, 0, 0.004$ . By increasing the value of curvature parameter  $\beta$  it decreases the squeeze film time  $T^*$ . Fig.10. shows that the fluctuation of  $T^*$  with  $h_m^*$  by increasing the  $K$  value it decreases the time. Similarly Fig.11 depicts by increasing the lower curvature parameter it increases the squeeze film time  $T^*$ .

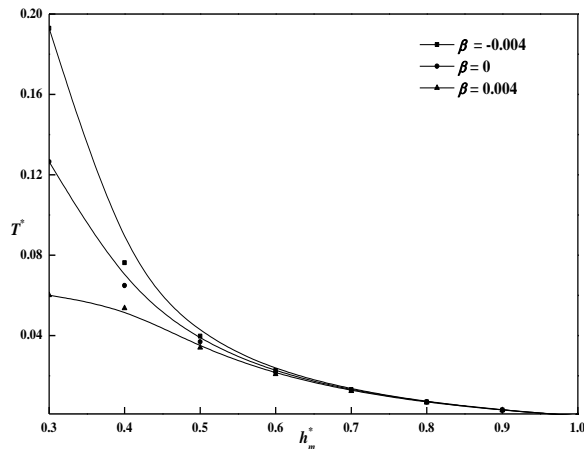


Figure 9: Variation of non-dimensional Pressure  $T^*$  with  $h_m^*$  for different values of  $\beta$  with  $K = 0.6$  and  $J = 0.4$

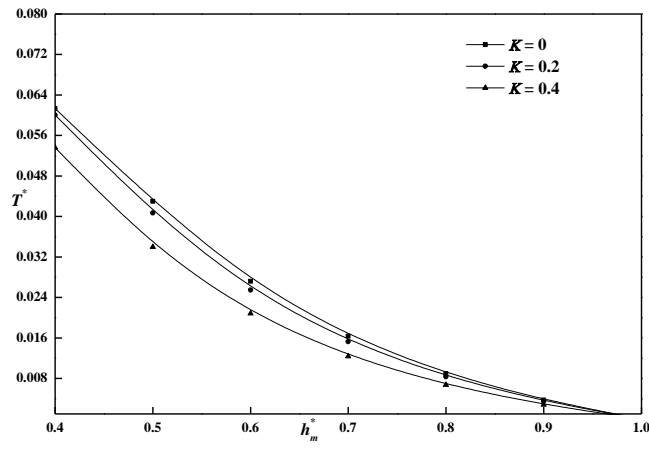


Figure 10: Variation of non-dimensional Pressure  $T^*$  with  $h_m^*$  for different values of  $K$  with  $\beta = 0.004$  and  $J = 0.6$

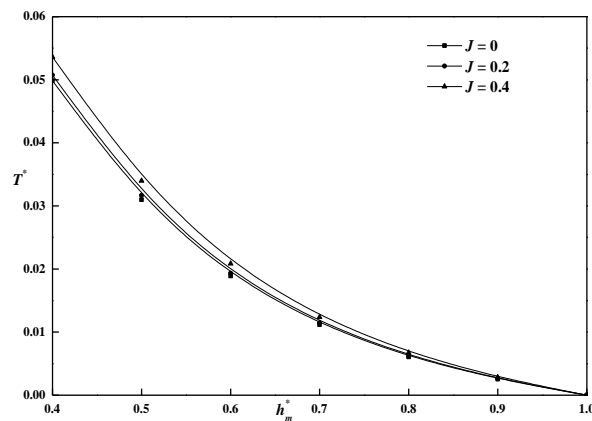


Figure 11: Variation of non-dimensional Pressure  $T^*$  with  $h_m^*$  for different values of  $J$  with  $\beta = 0.004$  and  $K = 0.6$

### 4. Conclusion:

From the above work Squeeze film lubrication between secant plates with Rabinowitsch fluid model has been studied and by graphical representation we conclude as follows:

- By increasing the upper & lower curvature parameter the pressure has been falling off and upgrades respectively.
- In regard to the upper curved circular plates the load support increases by increasing  $J$  and with respect to the lower curved circular plate the load support decreases by increasing  $K$ .
- By increasing the value of  $K$  the squeeze film time decreases and it increases by increasing the value of  $J$ .

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