ISSN: 0974-5823 Vol. 7 No. 4 April, 2022

International Journal of Mechanical Engineering

SOLVING TRANSPORTATION MODEL BASED ON MEAN DEVIATION

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ABSTRACT:

In this paper a new method has been developed to find the optimal solution for transportation models using mean deviation. The transportation costs are found for different problems of varying dimensions and the results are compared with the Vogel's Approximation Method (VAM). The new method holds high efficiency where the transportation costs are equal or lesser when compared with VAM.

KEYWORDS: Transportation problem, Mean Deviation, Vogel's approximation method (VAM)

INTRODUCTION:

Operations research is a decision science which helps in the management of making decisions. The objectives of OR is improving decision making quality, identifying optimal solution, improving the objectivity of analysis, minimizing the cost and maximization of profit and improving the productivity.

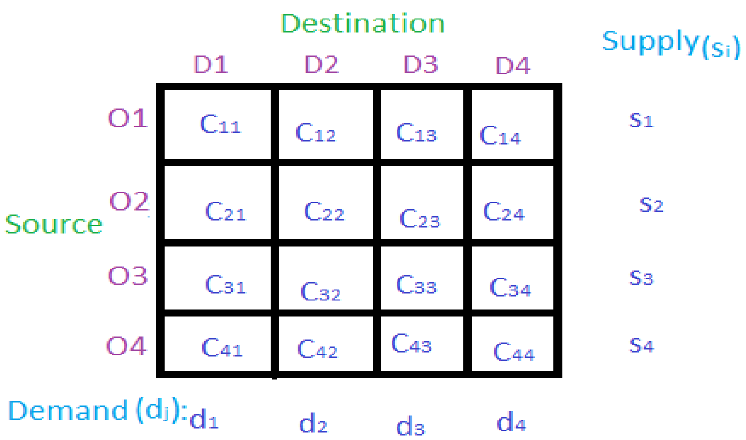
Some of the areas where OR techniques have been successfully applied are Allocation and Distribution, Production and Facility Planning, Procurement, Marketing, Finance, Personnel, Research and Development, etc.,

Linear programming problem helps in decision making and planning for the optimal allocation of resources. LPP deals with maximization or minimization of a function of variables (objective function) subject to a set of linear equations/inequalities(constraints).

Transportation problem is a special kind of Linear Programming Problem (LPP) in which products are transported from several sources of origins to different destinations at a minimum total cost. The classical transportation problem was first introduced by Hitchcock F.L. in 1941.

Transportation problems has versatile applications in real life such as scheduling, production planning, waste management, plant location problems, etc.,

BASIC STRUCTURE OF A TRANSPORTATION PROBLEM:



Here,

D1, D2, D3 and D4 are different destinations where the products are to be delivered from different sources S1, S2, S3 and S4.

S_i is the supply from the source O_i.

d_j is the demand of the destination D_j.

C_{ij} is the cost of the product which has to be delivered from source S_i to destination D_j.

DEFINITION OF THE TRANSPORTATION MODEL:

Suppose that there are 'm' sources and 'n' destinations. Let a_i be the number of supply units available at source i (i=1, 2, 3, …m) and let b_j be the number of demand units required at destination j (j=1, 2, 3, ….n). Let c_{ij} denotes the unit transportation cost for transporting the units from source i to destination j. The main objective is to determine the number of units to be transported from source i to destination j at a minimum transportation cost. Also that, the supply limits and the demand requirements must be equal.

If x_{ij} ($x_{ij} \geq 0$) is the number of units transported from source i to destination j, then the linear programming model will be

Find x_{ij} (i=1, 2, 3, …., m; j=1, 2, 3, …., n) in order to

Minimize $Z=\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = a_i$, i= 1, 2, ……, m,

and $\sum_{i=1}^{m} x_{ij} = b_j$, j= 1, 2, ……, n,

where $x_{ij} \geq 0$.

The system is balanced if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. A transportation problem will have a feasible solution only if the above condition is satisfied.

Problems which satisfies this condition are referred as balanced transportation problems. And any non-standard or unbalanced transportation problem in which the supplies and demand are not balanced, must be converted to a balanced transportation problem by using a dummy source/ destination.

FEASIBLE SOLUTION:

A feasible solution to a transportation problem is a set of non-negative allocations, x_{ij} that satisfies the rim (row and column) restrictions.

BASIC FEASIBLE SOLUTION:

A feasible solution to a transportation problem is called a basic feasible solution if it contains not more than m+n-1 non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.

OPTIMAL SOLUTION:

A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

NON-DEGENERATE BASIC FEASIBLE SOLUTION:

A basic feasible solution to a (m × n) transportation problem is said to be degenerate if,

a) The total number of non-negative allocations is exactly m+n-1 (i.e., number of independent constraint equations), and
b) These m+n-1 allocations are in independent positions.

DEGENERATE BASIC FEASIBLE SOLUTION:

A basic feasible solution in which the total number of non-negative allocations is less than m+n-1 is called degenerate basic feasible solution.

METHODS FOR SOLVING TRANSPORTATION PROBLEM

There are three methods for finding the solutions of balanced transportation problem:

1. Northwest Corner method
2. Least cost method
3. Vogel's approximation method

North-West Corner Method (NWC)

Step 1: First, select the North-west corner cell (upper left) of the table and allocate the maximum possible units between the demand requirements and the supply.

Step 2: Delete that row or column in which the supply/ demand values is fully exhausted.

Step 3: Again, select the North-west corner cell with the new reduced table and allocate the available values.

Step 4: Repeat the steps until all the supply and demand values exhaust.

Least Cost Method (LCM)

Step 1: Select the least value in the entire table and allocate maximum possible units of the supply and demand.

Step 2: Delete that row/column in which the demand/ supply values has exhausted.

Step 3: In the reduced table select the smallest cost cell and allocate. In case, if there are more than one minimum costs, select the cell where maximum allocation could be made.

Step 4: Redo the steps until all the allocations are over.

Vogel's Approximation Method (VAM)

Step 1: First, Calculate the penalties for each row and column by taking the difference between two lowest costs available in that row/column. If there are two lowest costs, then the penalty is 0.

Step 2: Then, Select the row/column, with the largest penalty and make allocation in the cell having the minimum cost in that selected row/column. If there exist two or more equal penalties then select the row/column which contains the least unit cost. If there is a tie again then select the one where maximum allocation could be made.

Step 3: Delete the row/column, in which the supply and demand is zero.

Step 4: Repeat the steps until the entire supply and demands exhaust.

Mean Deviation: Definition

The mean deviation is a statistical measure used to calculate the average deviation from the mean value of a given set of values.

Mean deviation = $\frac{1}{n}\sum_{i=1}^{n}\left|x_i - m(X)\right|$

m(X) = mean

n = total number of the given set of data x_i= data values in the given set

Mean Deviation Method:

Step (1): Calculate the Mean deviation for each row and column.

Step (2): Select the row or column with the maximum mean deviation and then select the cell having least cost in that row or column. If two or more equal mean deviation exist, select the row/column contains minimum unit cost. If there is a tie again then, select the cell where maximum allocation could be made.

Step (3): Allocate the maximum possible units between the supply and demand. Delete that row or column, which has satisfied the supply or demand values.

Step (4): Repeat the steps until all demand and supply values exhaust.

EXAMPLES:

1) Find the initial basic feasible solution for the following transportation problem.

	D1	D2	D3	Supply
S1	160	100	150	8
S2	100	120	100	6
Demand	5	5	4	

Solution:

Here Total Demand = Total Supply

Vogel's Approximation method:

Iteration 1:

	D1	D2	D3	Supply	Row penalty
S1	160	100	150	8	50
S2	100 (5)	120	100	6	0
Demand	5	5	4		
Column penalty	60	20	50		

The largest penalty occurs in column D1.

The minimum cost in this column is 100. So, allocate maximum possible units, i.e., 5 and eliminate the column D1.

Iteration 2:

	D2	D3	Supply	Row penalty
S1	100 (5)	150	8	50
S2	120	100	1	20
Demand	5	4		
Column penalty	20	50		

The maximum penalty occurs in the row S1 and the column D3. The smallest cost is also same in both the row and column. Hence, we are choosing the cell where the maximum allocation is possible i.e., at (S1, D2) and we eliminate the column D2.

Iteration 3:

	D3	Supply
S1	150 (3)	3
S2	100 (1)	1
Demand	4	

Hence, we have the following allocations,

	D1	D2	D3	Supply
S1	160	100 (5)	150 (3)	8
S2	100 (5)	120	100 (1)	6
Demand	5	5	4	

Transportation cost = (100×5) + (100×5) + (150×3) + (100×1) = 1550

Mean deviation method:

Iteration 1:

	D1	D2	D3	Supply	Mean deviation of row
S1	160	100	150	8	24.4
S2	100 (5)	120	100	6	8.9
Demand	5	5	4		
Mean deviation of column	30	10	25		

The largest mean deviation occurs in column D1.

The minimum cost in this column is 100. So, allocate maximum possible units, i.e., 5 and eliminate the column D1.

Iteration 2:

	D2	D3	Supply	M.D of row
S1	100 (5)	150	8	25
S2	120	100	1	10
Demand	5	4		
M.D of column	10	25		

The maximum mean deviation occurs in the row S1 and the column D3. The smallest cost is also same in both the row and column. Hence, we are choosing the cell where the maximum allocation is possible i.e., at (S1, D2) and we eliminate the column D2.

Iteration 3:

	D3	Supply
S1	150 (3)	3
S2	100 (1)	1
Demand	4	

Hence, we have the following allocations,

	D1	D2	D3	Supply
S1	160	100 (5)	150 (3)	8
S2	100 (5)	120	100 (1)	6
Demand	5	5	4	

Transportation cost = $(100\times5) + (100\times5) + (150\times3) + (100\times1) = 1550$

2) Find the transportation cost for the given problem

	P	Q	R	Availability
A	5	7	8	70
B	4	4	6	30
C	6	7	7	50
Requirement	65	42	43	

VAM:

The overall allocation is given by

	P	Q	R	Availability
A	5 (65)	7 (5)	8	70
B	4	4 (30)	6	30
C	6	7 (7)	7 (43)	50
Requirement	65	42	43	

Transportation cost = (5×65) + (7×5) + (4×30) + (7×7) + (7×43) = 830

Mean deviation method:

The overall allocation is given by

	P	Q	R	Availability
A	5 (65)	7 (5)	8	70
B	4	4 (30)	6	30
C	6	7 (7)	7 (43)	50
Requirement	65	42	43	

Transportation cost = (5×65) + (7×5) + (4×30) + (7×7) + (7×43)

=830

3) Solve the following transportation model

	P	Q	R	S	Supply
X	5	4	2	6	20
Y	8	3	5	7	30
Z	5	9	4	6	50
Demand	10	40	20	30	

VAM:

The overall allocation is given by

	P	Q	R	S	Supply
X	5	4	2 (20)	6	20
Y	8	3 (30)	5	7	30
Z	5 (10)	9 (10)	4	6(30)	50
Demand	10	40	20	30	

Transportation cost = (2×20) + (3×30) + (5×10) + (9×10) + (6×30)

=450

Mean Deviation method:

The overall allocation is given by

	P	Q	R	S	Supply
X	5	4 (10)	2 (10)	6	20
Y	8	3 (30)	5	7	30
Z	5 (10)	9	4 (10)	6 (30)	50
Demand	10	40	20	30	

Transportation cost = (4×10) + (2×10) + (3×30) + (5×10) + (4×10) + (6×30)

= 420

COMPARISON OF RESULTS WITH VOGEL'S APPROXIMATION METHOD AND MEAN DEVIATION METHOD:

Problem	Dimension of problem	Methods	Minimum Z
1	2×3	VAM Mean deviation	1550 1550
2	3×3	VAM Mean deviation	830 830
3	3×4	VAM Mean deviation	2850 2850
4	4×4	VAM Mean deviation	415 410
5	4×3	VAM Mean deviation	120 120
6	3×3	VAM Mean deviation	820 820
7	3×4	VAM Mean deviation	28 28
8	4×3	VAM Mean deviation	80 80
9	3×4	VAM Mean deviation	1020 1020
10	3×4	VAM Mean deviation	450 420

CONCLUSION:

A new method for finding the optimal solution for transportation problems using mean deviation has been proposed and the results are analyzed and compared with Vogel's approximation method as VAM obtains the best or near optimal solution.

From the results obtained we conclude that the suggested method gives more better solution for instance the problem 4 and 10.

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