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MARKOVIAN HETEROMORPHIC LIMITED SPACE QUEUE WITH A RANDOM COUNT INPUT SOURCES OPERATING AT A TIME

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ABSTRACT

In the under mentioned model, the stream of Poisson-type units arrive at a singular service station with two distinct arrival rates λ_1 and λ_2 from two separate input sources. The two separate input sources make an input source for a singular server. The input source switches between three presumable states, (i) operative, (ii) semi operative, and (iii) inoperative. When the units arrive at a service facility with mean rate $(\lambda_1+\lambda_2)$, the input is said to be in operative state (o,o). When units arrive with arrival rate λ_1 or λ_2, the input is said to be in semi operative state (o,i) or (i,o), and when units do not arrive, i.e., mean arrival rate is 0, the input is assumed to be in inoperative state (i,i). The service time is considered to be exponentially distributed with parameter μ for all states of the input.

Laplace transforms of the various probability generating functions are acquired and the steady state outcomes are clearly derived.

KEY WORDS : Queuing theory, Markovian process, Exponential distribution, Poisson distribution, Probability generating function

1. INTRODUCTION

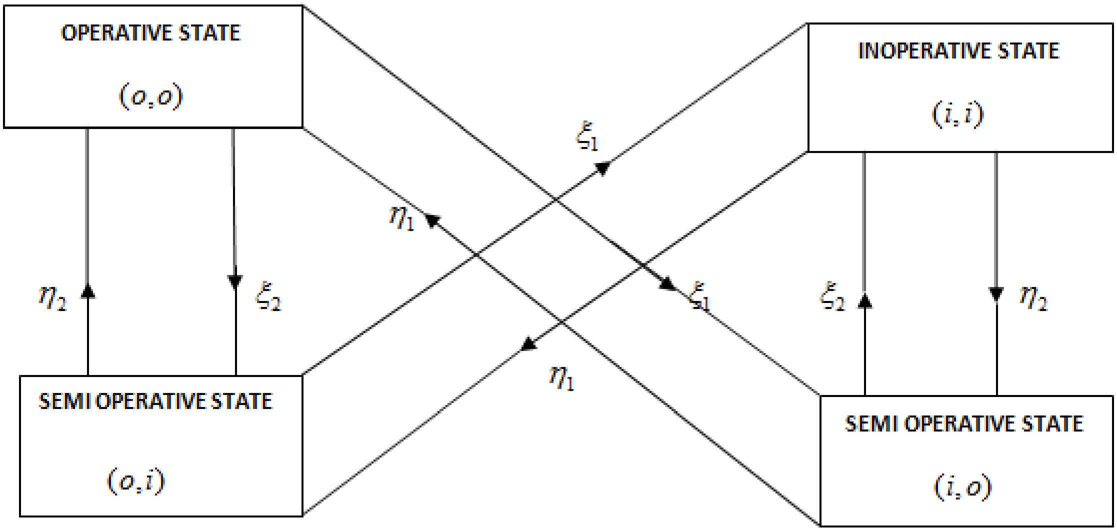
In the present day society, human beings are very busy in day by day recurring works and no person loves to wait. Every second waiting at a bus stop, gasoline centre, a traffic signal, post office, banks, grocery store or an elevator, all have waiting issues. Waiting lines are playing very vital role in our day by day life. Waiting of time is believed to be a nugatory of time. So study to decrease the waiting time is the call for our society. Scarcity of waiting time calls for greater investments. Before decisive whether to invest or not, it is play key position to understand the impact of the investment on the waiting time..

A.K. Erlang firstly unearths out queue in 1913 in the reference of telephone facilities. Queuing theory is a mathematical access in Operations Research used to the explication of queue. It plays a crucial function in modeling real life issues combining congestions in massive areas of science, technology and management. The waiting frames with heterogeneous system are extremely compatible as comparison to their homogeneous counterparts, therefore in real-life phenomena's the arrival pattern work at different rates. Morse [1] brings the concept of heterogeneous service. The heterogeneous service mechanisms are programming structures that permit customers to get several quality of service. Most of the manipulations in manufacturing systems have heterogeneous service mechanism. That is why; the queuing systems with heterogeneous servers have reached meaningful contemplation in the literature. Saaty [2] forward argues Morse's saying and draw conclusion the steady-state probabilities and the mean number in the system. Sharma and Dass [3] unearth the initial busy time of multichannel Markovian queueing framework and find the mainifestation of its density function in closed form. Dharmaraja [4] makes the transient solution of a two-processor heterogeneous system with Poisson arrival of jobs having exponentially distributed processing times. Krishanamoorthy [6] make consideration for a Poisson queue with two-heterogeneous servers with modified queue disciplines. Singh [7] fit up the work of Krishanamoorthy [6] on heterogeneous servers by incorporating balking to compare inferences with homogeneous servers queue to show that the conditions under which the heterogeneous system is preferable than relevant homogeneous system. Kumar and Sharma [8] get the transient solution of a two-heterogeneous servers Markovian queuing model with retention of reneging customers. Numerous other researches [13], [14], [15], [16] etc. have processed queuing models in the domain of heterotypical in arrival and services.

2. STATEMENT OF QUEUING MODEL

We consider a queuing frame where into, the count input sources operating at a time is random count which may take value 0, 1 or 2. For instance it, we imagine two officers in an institution, are given a common steno typist. Let the officers wear the label 1 and 2. If the officer of label 1 emits a steady Poisson stream of letters with mean arrival rate λ_1 and the officer of label 2 emits a steady Poisson stream of letters with mean arrival rate λ_2, then the arrival rate of letters to steno typist is $(\lambda_1+\lambda_2)$. Here we consider that two officers are the two input sources for steno-typist. When both the officers are in functioning (functioning in the sense that both are emitting a steady Poisson stream of letters with rate λ_1 and λ_2 respectively, then the number of input sources acting at a time is 2 (and the system is assumed to be in operative state). If officer of label 1 is in working and the officer of label 2 is out for some reason, then a input source of label 1 is operating and of label 2 is inoperative and vice-versa. In this case, the mean rate of arrival to the service facility is either λ_1 or λ_2 and the number of input sources operating at a time is 1 (and the system is considered to be in semi operative state). Also, if both the officers are out, e.g., when they are attending a meeting called by higher officer of the institution, then not a single letter is emitted by the officers and the arrival rate of letters to steno-typist become zero. In this case, the number of input source acting at a time is 0 (and system is considered to be in operative state). Thus, the following model consists of two input sources out of which no input source or one input source or two input sources are functioning at a period. The system under view strikes the below mentioned characteristics:

A flow of Poisson-type units reach at a singular employ station with two different coming rates λ_1 and λ_2 from two distinct input sources. The two distinct input sources make an input source for a singular server. The input source switches between three presumable states, (i) operative, (ii) semi operative, and (iii) inoperative. When the units reach at a employ facility with mean rate $(\lambda_1+\lambda_2)$, the input is said to be in operative state. When units arrive with arrival rate λ_1 or λ_2, the input is said to be in semi operative state, and when units do not arrive, i.e., mean arrival rate is 0, the input is considered to be in inoperative state. For the sake of facility, we notify the operative state by (o,o), when both the input sources are operating. There are two stages of semi operative state, viz., (o,i) and (i,o) i.e., when the first or the second input source is operating. The state (i,i) is that when both the input sources are not operating. The time gap during which the input sources affair in any one case is an exponentially distributed random variable and the transition rate from one case to another case is display in the down diagram:



The operative state (o,o) tends to move to the semi operative state (o,i) with rate ξ_2 and to semi operative state (i,o) with rate ξ_1. Thus, the time during which both the input sources are operating exponentially distributed with average rate $\left(\frac{1}{\xi_1+\xi_2}\right)$.

The inoperative state (i,i) tends to move to the semi operative state (o,i) with rate η_1 or to the semi operative state (i,o) with rate η_2. Thus, the time during which none of the input sources are operating is exponentially distributed with average period $\left(\frac{1}{\eta_1+\eta_2}\right)$. Similarly the times during which only the first input source is operating or only the second input source is operating are exponentially distributed with average periods $\left(\frac{1}{\eta_2+\xi_1}\right)$ and $\left(\frac{1}{\eta_1+\xi_2}\right)$ respectively.

Further, the service period is assumed to be exponentially distributed with parameter μ for all cases of the input. It is also supposed that if at any instant the units in the system are N, then the coming unit will be accepted lost for the channel. The stochastic processes involved, viz., inter-arrival period of clientages are unmatched of each other.

3. SOLUTION OF QUEUING MODEL

In this section, the computed framework of the queuing model is presented. The time-dependent and steady state conclusion of the problem have been deduced.

3.1 TRANSIENT SOLUTION

Define,

$P_{o,o,n}(t) \equiv$ The probability that both the input sources are operating, system has n units at time t;

$P_{o,i,n}(t) \equiv$ The probability that the input source of label 1 is operating, and the input source of label 2 is not operating, system has n units at time t;

$P_{i,o,n}(t) \equiv$ The probability that the input source of label 1 is not operating, and the input source of label 2 is operating, system has n units at time t;

$P_{i,i,n}(t) \equiv$ The probability that both the input sources are not operating, system has n units at time t;

$R_n(t) \equiv$ The probability that system has n units at time t.

Obviously,

$$R_n(t) = P_{o,o,n}(t) + P_{o,i,n}(t) + P_{i,o,n}(t) + P_{i,i,n}(t)$$

Let the time be computed from the instantly when the queue distance is zero, and both the input sources are operating. The initial conditions become

$$P_{o,o,n}(0) = \begin{cases} 1 & , \quad n = 0 \\ 0 & , \quad othrewise \end{cases}$$

$$P_{o,i,n}(0) = 0, P_{i,o,n}(0) = 0\ ,\ P_{i,i,n}(0) = 0; \qquad \forall n \geq 0$$

Define the under described probability generating function of $P_{o,o,n}(t), P_{o,i,n}(t), P_{i,o,n}(t), P_{i,i,n}(t)$ *and* $R_n\ (t)$

$$P_{o,o}(z,t) = \sum_{n=0}^{N} z^n P_{o,o,n}(t)$$

$$P_{o,i}(z,t) = \sum_{n=0}^{N} z^n P_{o,i,n}(t)$$

$$P_{i,o}(z,t) = \sum_{n=0}^{N} z^n P_{i,o,n}(t)$$

$$P_{i,i}(z,t) = \sum_{n=0}^{N} z^n P_{i,i,n}(t)$$

$$R(z,t) = \sum_{n=0}^{N} z^n R_n(t)$$

KOLMOGOROV'S forward equations representing the model lead to the under-described differential equations:

$$\frac{d}{dt} P_{o,o,0}(t) = -(\xi_1 + \xi_2 + \lambda_1 + \lambda_2) P_{o,o,0}(t) + \mu P_{o,o,1}(t) + \eta_1 P_{i,o,0}(t) + \eta_2 P_{o,i,0}(t) \tag{1.1}$$

$$\frac{d}{dt} P_{o,o,n}(t) = -(\xi_1 + \xi_2 + \lambda_1 + \lambda_2 + \mu) P_{o,o,n}(t) + (\lambda_1 + \lambda_2) P_{o,o,n-1}(t) + \mu P_{o,o,n+1}(t) + \eta_1 P_{i,o,n}(t) + \eta_2 P_{o,i,n}(t)$$

$$1 \leq n < N \tag{1.2}$$

$$\frac{d}{dt} P_{o,o,N}(t) = -(\xi_1 + \xi_2 + \mu) P_{o,o,N}(t) + (\lambda_1 + \lambda_2) P_{o,o,N-1}(t) + \eta_1 P_{i,o,N}(t) + \eta_2 P_{o,i,N}(t) \tag{1.3}$$

$$\frac{d}{dt}P_{o,i,0}(t)=-(\xi_1+\eta_2+\lambda_1)P_{o,i,0}(t)+\mu P_{o,i,1}(t)+\eta_1 P_{i,i,0}(t)+\xi_2 P_{o,o,0}(t), \quad (1.4)$$

$$\frac{d}{dt}P_{o,i,n}(t)=-(\xi_1+\eta_2+\lambda_1+\mu)P_{o,i,n}(t)+\lambda_1 P_{o,i,n-1}(t)+\mu P_{o,i,n+1}(t)+\eta_1 P_{i,i,n}(t)+\xi_2 P_{o,o,n}(t) \quad 1\le n<N \quad \ldots\ldots(1.5)$$

$$\frac{d}{dt}P_{o,i,N}(t)=-(\xi_1+\eta_2+\mu)P_{o,i,N}(t)+\lambda_1 P_{o,i,N-1}(t)+\eta_1 P_{i,i,N}(t)+\xi_2 P_{o,o,N}(t) \quad (1.6)$$

$$\frac{d}{dt}P_{i,o,0}(t)=-(\xi_2+\eta_1+\lambda_2)P_{i,o,0}(t)+\mu P_{i,o,1}(t)+\eta_2 P_{i,i,0}(t)+\xi_1 P_{o,o,0}(t) \quad (1.7)$$

$$\frac{d}{dt}P_{i,o,n}(t)=-(\xi_2+\eta_1+\lambda_2+\mu)P_{i,o,n}(t)+\lambda_2 P_{i,o,n-1}(t)+\mu P_{i,o,n+1}(t)+\eta_2 P_{i,i,n}(t)+\xi_1 P_{o,o,n}(t)$$

$$1\le n<N \quad (1.8)$$

$$\frac{d}{dt}P_{i,o,N}(t)=-(\xi_2+\eta_1+\mu)P_{i,o,N}(t)+\lambda_2 P_{i,o,N-1}(t)+\eta_2 P_{i,i,N}(t)+\xi_1 P_{o,o,N}(t) \quad (1.9)$$

$$\frac{d}{dt}P_{i,i,0}(t)=-(\eta_1+\eta_2)P_{i,i,0}(t)+\mu P_{i,i,1}(t)+\xi_1 P_{o,i,0}(t)+\xi_2 P_{i,o,0}(t) \quad (1.10)$$

$$\frac{d}{dt}P_{i,i,n}(t)=-(\eta_1+\eta_2+\mu)P_{i,i,n}(t)+\mu P_{i,i,n+1}(t)+\xi_1 P_{o,i,n}(t)+\xi_2 P_{i,o,n}(t) \quad 1\le n<N \quad (1.11)$$

$$\frac{d}{dt}P_{i,i,N}(t)=-(\eta_1+\eta_2+\mu)P_{i,i,N}(t)+\xi_1 P_{o,i,N-1}(t)+\xi_2 P_{i,o,N}(t) \quad (1.12)$$

Multiplying (1.1) to (1.12) by apposite powers of z, applying their related probability generating function (p.g.f.'s), taking $L.T.'^{S}$ and applying initial conditions, we get

$$K_{o,o}(z,s)\bar{P}_{o,o}(z,s)=z+\mu(z-1)\bar{P}_{o,o,0}(s)+z\eta_1\bar{P}_{i,o}(z,s)+z\eta_2\bar{P}_{o,i}(z,s)-z^{N+1}(z-1)(\lambda_1+\lambda_2)\bar{P}_{o,o,N}(s) \quad (1.13)$$

$$K_{o,i}(z,s)\bar{P}_{o,i}(z,s)=\mu(z-1)\bar{P}_{o,i,0}(s)+z\eta_1\bar{P}_{i,i}(z,s)+z\xi_2\bar{P}_{o,o}(z,s)-z^{N+1}(z-1)\lambda_1\bar{P}_{o,i,N}(s) \quad (1.14)$$

$$K_{i,o}(z,s)\bar{P}_{i,o}(z,s)=\mu(z-1)\bar{P}_{i,o,0}(s)+z\eta_2\bar{P}_{i,i}(z,s)+z\xi_1\bar{P}_{o,o}(z,s)-z^{N+1}(z-1)\lambda_2\bar{P}_{i,o,N}(s) \quad (1.15)$$

$$K_{i,i}(z,s)\bar{P}_{i,i}(z,s)=\mu(z-1)\bar{P}_{i,i,0}(s)+z\xi_1\bar{P}_{o,i}(z,s)+z\xi_2\bar{P}_{i,o}(z,s) \quad (1.16)$$

Where,

$$K_{o,o}(z,s)=\left[z\left\{s+(\lambda_1+\lambda_2)(1-z)+\xi_1+\xi_2+\mu\right\}-\mu\right]$$

$$K_{o,i}(z,s)=\left[z\left\{s+\lambda_1(1-z)+\xi_1+\eta_2+\mu\right\}-\mu\right]$$

$$K_{i,o}(z,s)=\left[z\left\{s+\lambda_2(1-z)+\xi_2+\eta_1+\mu\right\}-\mu\right]$$

$$K_{i,i}(z,s)=\left[z\left(s+\eta_1+\eta_2+\mu\right)-\mu\right]$$

Solving equations (1.13) – (1.16)

$$\bar{P}_{o,o}(z,s)=\frac{\begin{array}{l}zK_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^3\eta_2\xi_2K_{o,i}(z,s)\\-z^3\eta_1\xi_1K_{i,o}(z,s)+\mu(z-1)[\bar{P}_{o,o,0}(s)\{K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)\\-z^2\eta_2\xi_2K_{o,i}(z,s)-z^2\eta_1\xi_1K_{i,o}(z,s)\}-\bar{P}_{o,i,0}(s)\{z^3\eta_2^2\xi_2\\-z\eta_2K_{i,o}(z,s)K_{i,i}(z,s)-z^3\eta_1\eta_2\xi_1\}+\bar{P}_{i,o,0}(s)\{z\eta_1K_{o,i}(z,s)K_{i,i}(z,s)\\-z^3\eta_1^2\xi_1+z^3\eta_1\eta_2\xi_2\}+\bar{P}_{i,i,0}(s)\{z^2\eta_1\eta_2K_{i,o}(z,s)+z^2\eta_1\eta_2K_{o,i}(z,s)\}]\\-z^{N+1}(z-1)[(\lambda_1+\lambda_2)\bar{P}_{i,i,N}(s)\{K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2K_{o,i}(z,s)\\-z^2\eta_1\xi_1K_{i,o}(z,s)\}-\lambda_1\bar{P}_{o,i,N}(s)\{z^3\eta_2^2\xi_2-z\eta_2K_{i,o}(z,s)K_{i,i}(z,s)-z^3\eta_1\eta_2\xi_1\}\\+\lambda_2\bar{P}_{i,o,N}(s)\{z^3\eta_1\eta_2\xi_2-z^3\eta_1^2\xi_1+z\eta_1K_{o,i}(z,s)K_{i,i}(z,s)\}]\end{array}}{\begin{array}{l}K_{o,o}(z,s)K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2\{K_{o,o}(z,s)K_{o,i}(z,s)+K_{i,o}(z,s)K_{i,i}(z,s)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z,s)K_{i,o}(z,s)+K_{o,i}(z,s)K_{i,i}(z,s)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \tag{1.17}$$

$$\bar{P}_{o,i}(z,s)=\frac{\begin{array}{l}z^2\xi_2K_{i,o}(z,s)K_{i,i}(z,s)-z^4\eta_2\xi_2^2+z^4\eta_1\xi_1\xi_2+\mu(z-1)[\bar{P}_{o,o,0}(s)\{z\xi_2K_{i,o}(z,s)K_{i,i}(z,s)\\-z^3\eta_2\xi_2^2+z^3\eta_1\xi_1\xi_2\}+\bar{P}_{o,i,0}(s)\{K_{o,o}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2K_{o,o}(z,s)\\-z^2\eta_1\xi_1K_{i,i}(z,s)\}+\bar{P}_{i,o,0}(s)\{z^2\eta_1\xi_2K_{o,o}(z,s)+z^2\eta_1\xi_2K_{i,i}(z,s)\}+\bar{P}_{i,i,0}(s)\\\{z\eta_1K_{o,o}(z,s)K_{i,o}(z,s)+z^3\eta_1\eta_2\xi_2-z^3\eta_1^2\xi_1\}]-z^{N+1}(z-1)[(\lambda_1+\lambda_2)\bar{P}_{o,o,N}(s)\\\{z\xi_2K_{i,o}(z,s)K_{i,i}(z,s)-z^3\eta_2\xi_2^2+z^3\eta_1\xi_1\xi_2\}+\lambda_1\bar{P}_{o,i,N}(s)\{K_{o,o}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)\\-z^2\eta_2\xi_2K_{o,o}(z,s)-z^2\eta_1\xi_1K_{i,i}(z,s)\}+\lambda_2\bar{P}_{i,o,N}(s)\{z^2\eta_1\xi_2K_{o,o}(z,s)+z^2\eta_1\xi_2K_{i,i}(z,s)\}]\end{array}}{\begin{array}{l}K_{o,o}(z,s)K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2\{K_{o,o}(z,s)K_{o,i}(z,s)+K_{i,o}(z,s)K_{i,i}(z,s)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z,s)K_{i,o}(z,s)+K_{o,i}(z,s)K_{i,i}(z,s)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \tag{1.18}$$

$$\bar{P}_{i,o}(z,s)=\frac{\begin{array}{l}z^2\xi_1\{z^2\eta_2\xi_2+K_{o,i}(z,s)K_{i,i}(z,s)-z^2\eta_1\xi_1\}+\mu(z-1)[\bar{P}_{o,o,0}(s)\{z\xi_1K_{o,i}(z,s)K_{i,i}(z,s)\\-z^3\eta_1\xi_1^2+z^3\eta_2\xi_1\xi_2\}+\bar{P}_{o,i,0}(s)\{z^2\eta_2\xi_1K_{o,o}(z,s)+z^2\eta_2\xi_1K_{i,i}(z,s)\}+\bar{P}_{i,o,0}(s)\\\{K_{o,o}(z,s)K_{o,i}(z,s)K_{i,i}(z,s)-z^2\eta_1\xi_1K_{o,o}(z,s)-z^2\eta_2\xi_2K_{i,i}(z,s)\}+\bar{P}_{i,i,0}(s)\\\{z\eta_2K_{o,o}(z,s)K_{o,i}(z,s)+z^3\eta_1\eta_2\xi_1-z^3\eta_2^2\xi_2\}]-z^{N+1}(z-1)[(\lambda_1+\lambda_2)\bar{P}_{o,o,N}(s)\\\{z\xi_1K_{o,i}(z,s)K_{i,i}(z,s)-z^3\eta_1\xi_1^2+z^3\eta_2\xi_1\xi_2\}+\lambda_1\bar{P}_{o,i,N}(s)\{z^2\eta_2\xi_1K_{o,o}(z,s)\\+z^2\eta_2\xi_1K_{i,i}(z,s)\}+\lambda_2\bar{P}_{i,o,N}(s)\{K_{o,o}(z,s)K_{o,i}(z,s)K_{i,i}(z,s)-z^2\eta_1\xi_1K_{o,o}(z,s)\\-z^2\eta_2\xi_2K_{i,i}(z,s)\}]\end{array}}{\begin{array}{l}K_{o,o}(z,s)K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2\{K_{o,o}(z,s)K_{o,i}(z,s)+K_{i,o}(z,s)K_{i,i}(z,s)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z,s)K_{i,o}(z,s)+K_{o,i}(z,s)K_{i,i}(z,s)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \tag{1.19}$$

$$\bar{P}_{i,i}(z,s)=\frac{\begin{array}{l} z^3\xi_1\xi_2\{K_{i,o}(z,s)+K_{o,i}(z,s)\}+\mu(z-1)[\bar{P}_{o,o,0}(s)\{z^2\,\xi_1\xi_2K_{i,o}(z,s)+z^2\,\xi_1\xi_2K_{o,i}(z,s)\} \\ +\bar{P}_{o,i,0}(s)\{z\xi_1K_{o,o}(z,s)K_{i,o}(z,s)+z^3\,\eta_2\xi_1\xi_2-z^3\,\eta_1\xi_1^2\xi_2\}+\bar{P}_{i,o,0}(s)\{z\xi_2K_{o,o}(z,s)K_{o,i}(z,s) \\ +z^3\eta_1\xi_1\xi_2-z^3\,\eta_2\xi_2^2\}+\bar{P}_{i,i,0}(s)\{K_{o,o}(z,s)K_{o,i}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2K_{i,o}(z,s) \\ -z^2\eta_1\xi_1K_{o,i}(z,s)\}]-z^{N+1}(z-1)[(\lambda_1+\lambda_2)\bar{P}_{o,o,N}(s)\{z^2\,\xi_1\xi_2K_{i,o}(z,s) \\ +z^2\xi_1\xi_2K_{i,o}(z,s)\}+\lambda_1\bar{P}_{o,i,N}(s)\{z\xi_1K_{o,o}(z,s)K_{i,o}(z,s)+z^3\,\eta_2\xi_1\xi_2 \\ -z^3\,\eta_1\xi_1^2\}+\lambda_2\bar{P}_{i,o,N}(s)\{z\xi_2K_{o,o}(z,s)K_{o,i}(z,s)+z^3\,\eta_1\xi_1\xi_2-z^3\,\eta_2\xi_2^2\}] \end{array}}{\begin{array}{l} K_{o,o}(z,s)K_{o,i}(z,s)K_{i,o}(z,s)K_{i,i}(z,s)-z^2\eta_2\xi_2\{K_{o,o}(z,s)K_{o,i}(z,s)+K_{i,o}(z,s)K_{i,i}(z,s)\} \\ -z^2\eta_1\xi_1\{K_{o,o}(z,s)K_{i,o}(z,s)+K_{o,i}(z,s)K_{i,i}(z,s)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2) \end{array}} \quad (1.20)$$

3.2 STEADY STATE SOLUTION

The steady state solution can be obtained by the well – known property of the L, T., viz.,

$$\lim_{s\to 0} s\bar{F}(s)=\lim_{t\to\infty} F(t) \quad (1.21)$$

If the limit on the right exists.

Thus, if

$$\lim_{t\to\infty} P_n(t)=P_n$$

We have,

$$\lim_{s\to 0} s\bar{P}_n(s)=P_n \quad \text{etc.}$$

Using property (1.21) to equations (1.13) - (1.16), we obtain

$$K_{o,o}(z)P_{o,o}(z)=\mu(z-1)P_{o,o,0}(z)+z\eta_1P_{i,o}(z)+z\eta_2P_{o,i}(z)-z^{N+1}(z-1)(\lambda_1+\lambda_2)P_{o,o,N} \quad (1.22)$$

$$K_{o,i}(z)P_{o,i}(z)=\mu(z-1)P_{o,i,0}+z\eta_1P_{i,i}(z)+z\xi_2P_{o,o}(z)-z^{N+1}(z-1)\lambda_1P_{o,i,N} \quad (1.23)$$

$$K_{i,o}(z)P_{i,o}(z)=\mu(z-1)P_{i,o,0}+z\eta_2P_{i,i}(z)+z\xi_1P_{o,o}(z)-z^{N+1}(z-1)\lambda_2P_{i,o,N} \quad (1.24)$$

$$K_{i,i}(z)P_{i,i}(z)=\mu(z-1)P_{i,i,0}+z\xi_1P_{o,i}(z)+z\xi_2P_{i,o}(z) \quad (1.25)$$

Where,

$$K_{o,o}(z)=\left[z\left\{(\lambda_1+\lambda_2)(1-z)+\xi_1+\xi_2+\mu\right\}-\mu\right]$$

$$K_{o,i}(z)=\left[z\left\{\lambda_1(1-z)+\xi_1+\eta_2+\mu\right\}-\mu\right]$$

$$K_{i,o}(z)=\left[z\left\{\lambda_2(1-z)+\xi_2+\eta_1+\mu\right\}-\mu\right]$$

$$K_{i,i}(z)=\left[z\left(\eta_1+\eta_2+\mu\right)-\mu\right]$$

Fixing $z=1$ in equations (1.22) - (1.25) give,

$$(\xi_1+\xi_2)P_{o,o}(1)=\eta_1P_{i,o}(1)+\eta_2P_{o,i}(1) \quad (1.26)$$

$$(\xi_1+\eta_2)P_{o,i}(1)=\eta_1P_{i,i}(1)+\xi_2P_{o,o}(1) \quad (1.27)$$

$$(\xi_2+\eta_1)P_{i,o}(1)=\eta_2P_{i,i}(1)+\xi_1P_{o,o}(1) \quad (1.28)$$

$$(\eta_1+\eta_2)P_{i,i}(1)=\xi_1 P_{i,o}(1)+\xi_2 P_{i,o}(1) \quad (1.29)$$

Equations (1.26) - (1.29) give,

$P_{o,o}(1)\equiv$ The steady state probability that the system will be in the state where both the inputs are operating,

$$=\frac{\eta_1\eta_2}{(\eta_1+\xi_1)(\eta_2+\xi_2)} \quad (1.30)$$

$P_{o,i}(1)\equiv$ The steady state probability that the system will be in the state in which input label 1 is operating and input of label 2 is not operating,

$$=\frac{\eta_1\xi_2}{(\eta_1+\xi_1)(\eta_2+\xi_2)} \quad (1.31)$$

$P_{i,o}(1)\equiv$ The steady state probability that the system will be in the state in which input label 1 is not operating and input of label 2 is operating,

$$=\frac{\eta_2\xi_1}{(\eta_1+\xi_1)(\eta_2+\xi_2)} \quad (1.32)$$

$P_{i,i}(1)\equiv$ The steady state probability that the system will be in the state in which both the inputs are not operating,

$$=\frac{\xi_1\xi_2}{(\eta_1+\xi_1)(\eta_2+\xi_2)} \quad (1.33)$$

Computing equations (1.22) - (1.25), we obtain,

$$P_{o,o}(z)=\frac{\begin{aligned}&\mu(z-1)[P_{o,o,0}\{K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2K_{o,i}(z)\\&-z^2\eta_1\xi_1K_{i,o}(z)\}-P_{o,i,0}\{z^3\eta_2^2\xi_2-z\eta_2K_{i,o}(z)K_{i,i}(z)\\&-z^3\eta_1\eta_2\xi_1\}+P_{i,o,0}\{z\eta_1K_{o,i}(z)K_{i,i}(z)-z^3\eta_1^2\xi_1+z^3\eta_1\eta_2\xi_2\}\\&+P_{i,i,0}\{z^2\eta_1\eta_2K_{i,o}(z)+z^2\eta_1\eta_2K_{o,i}(z)\}]-z^{N+1}(z-1)\\&[(\lambda_1+\lambda_2)P_{i,i,N}\{K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2K_{o,i}(z)\\&-z^2\eta_1\xi_1K_{i,o}(z)\}-\lambda_1P_{o,i,N}\{z^3\eta_2^2\xi_2-z\eta_2K_{i,o}(z)K_{i,i}(z)\\&-z^3\eta_1\eta_2\xi_1\}+\lambda_2P_{i,o,N}\{z^3\eta_1\eta_2\xi_2-z^3\eta_1^2\xi_1+z\eta_1K_{o,i}(z)K_{i,i}(z)\}]\end{aligned}}{\begin{aligned}&K_{o,o}(z)K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2\{K_{o,o}(z)K_{o,i}(z)+K_{i,o}(z)K_{i,i}(z)\}\\&-z^2\eta_1\xi_1\{K_{o,o}(z)K_{i,o}(z)+K_{o,i}(z)K_{i,i}(z)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{aligned}} \quad (1.34)$$

$$P_{o,i}(z)=\frac{\begin{array}{l}\mu(z-1)[P_{o,o,0}\{z\xi_2K_{i,o}(z)K_{i,i}(z)-z^3\eta_2\xi_2^2+z^3\eta_1\xi_1\xi_2\}\\+P_{o,i,0}\{K_{o,o}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2K_{o,o}(z)-z^2\eta_1\xi_1K_{i,i}(z)\}\\+P_{i,o,0}\{z^2\eta_1\xi_2K_{o,o}(z)+z^2\eta_1\xi_2K_{i,i}(z)\}+P_{i,i,0}\{z\eta_1K_{o,o}(z)K_{i,o}(z)\\+z^3\eta_1\eta_2\xi_2-z^3\eta_1^2\xi_1\}]-z^{N+1}(z-1)[(\lambda_1+\lambda_2)P_{o,o,N}\{z\xi_2K_{i,o}(z)K_{i,i}(z)\\-z^3\eta_2\xi_2^2+z^3\eta_1\xi_1\xi_2\}+\lambda_1P_{o,i,N}\{K_{o,o}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2K_{o,o}(z)\\-z^2\eta_1\xi_1K_{i,i}(z)\}+\lambda_2P_{i,o,N}\{z^2\eta_1\xi_2K_{o,o}(z)+z^2\eta_1\xi_2K_{i,i}(z)\}]\end{array}}{\begin{array}{l}K_{o,o}(z)K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2\{K_{o,o}(z)K_{o,i}(z)+K_{i,o}(z)K_{i,i}(z)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z)K_{i,o}(z)+K_{o,i}(z)K_{i,i}(z)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \quad (1.35)$$

$$P_{i,o}(z)=\frac{\begin{array}{l}\mu(z-1)[P_{o,o,0}\{z\xi_1K_{o,i}(z)K_{i,i}(z)-z^3\eta_1\xi_1^2+z^3\eta_2\xi_1\xi_2\}+P_{o,i,0}\{z^2\eta_2\xi_1K_{o,o}(z)\\+z^2\eta_2\xi_1K_{i,i}(z)\}+P_{i,o,0}\{K_{o,o}(z)K_{o,i}(z)K_{i,i}(z)-z^2\eta_1\xi_1K_{o,o}(z)\\-z^2\eta_2\xi_2K_{i,i}(z)\}+P_{i,i,0}\{z\eta_2K_{o,o}(z)K_{o,i}(z)+z^3\eta_1\eta_2\xi_1-z^3\eta_2^2\xi_2\}]\\-z^{N+1}(z-1)[(\lambda_1+\lambda_2)P_{o,o,N}\{z\xi_1K_{o,i}(z)K_{i,i}(z)-z^3\eta_1\xi_1^2+z^3\eta_2\xi_1\xi_2\}\\+\lambda_1P_{o,i,N}\{z^2\eta_2\xi_1K_{o,o}(z)+z^2\eta_2\xi_1K_{i,i}(z)\}+\lambda_2P_{i,o,N}\{K_{o,o}(z)K_{o,i}(z)K_{i,i}(z)\\-z^2\eta_1\xi_1K_{o,o}(z)-z^2\eta_2\xi_2K_{i,i}(z)\}]\end{array}}{\begin{array}{l}K_{o,o}(z)K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2\{K_{o,o}(z)K_{o,i}(z)+K_{i,o}(z)K_{i,i}(z)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z)K_{i,o}(z)+K_{o,i}(z)K_{i,i}(z)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \quad (1.36)$$

$$P_{i,i}(z)=\frac{\begin{array}{l}\mu(z-1)[P_{o,o,0}\{z^2\xi_1\xi_2K_{i,o}(z)+z^2\xi_1\xi_2K_{o,i}(z)\}+P_{o,i,0}z\xi_1K_{o,o}(z)K_{i,o}(z)\\+z^3\eta_2\xi_1\xi_2-z^3\eta_1\xi_1^2\xi_2\}+P_{i,o,0}\{z\xi_2K_{o,o}(z)K_{o,i}(z)+z^3\eta_1\xi_1\xi_2-z^3\eta_2\xi_2^2\}\\+P_{i,i,0}\{K_{o,o}(z)K_{o,i}(z)K_{i,i}(z)-z^2\eta_2\xi_2K_{i,o}(z)-z^2\eta_1\xi_1K_{o,i}(z)\}]\\-z^{N+1}(z-1)[(\lambda_1+\lambda_2)P_{o,o,N}\{z^2\xi_1\xi_2K_{i,o}(z)+z^2\xi_1\xi_2K_{i,o}(z)\}\\+\lambda_1P_{o,i,N}\{z\xi_1K_{o,o}(z)K_{i,o}(z)+z^3\eta_2\xi_1\xi_2-z^3\eta_1\xi_1^2\}+\lambda_2P_{i,o,N}\\\{z\xi_2K_{o,o}(z)K_{o,i}(z)+z^3\eta_1\xi_1\xi_2-z^3\eta_2\xi_2^2\}]\end{array}}{\begin{array}{l}K_{o,o}(z)K_{o,i}(z)K_{i,o}(z)K_{i,i}(z)-z^2\eta_2\xi_2\{K_{o,o}(z)K_{o,i}(z)+K_{i,o}(z)K_{i,i}(z)\}\\-z^2\eta_1\xi_1\{K_{o,o}(z)K_{i,o}(z)+K_{o,i}(z)K_{i,i}(z)\}+z^4(\eta_1^2\xi_1^2+\eta_2^2\xi_2^2-2\eta_1\eta_2\xi_1\xi_2)\end{array}} \quad (1.37)$$

CONCLUSION

In the present research paper, the both transient and steady state analysis of a Markovian queuing system with heterogeneity in arrival process are clearly examined. In transient solution, equations (1.17) – (1.20) obtain the values of L.T.'s of p.g.f. of the distribution of the count of units for several cases of the input source. These are clearly known if the seven unknowns, viz., $\bar{P}_{o,o,0}(s), \bar{P}_{o,i,0}(s), \bar{P}_{i,o,0}(s), \bar{P}_{i,i,0}(s), \bar{P}_{o,o,N}(s), \bar{P}_{o,i,N}(s)$ and $\bar{P}_{i,o,N}(s)$ mixed in these equations, are determined. We observe that higher degree of z in the numerator of each of the above cited equations is $(N+7)$, and higher degree of z in their denominator is seven. Additionally, $P_{o,o}(z,t)$ etc. are polynomial of N degree. Therefore, seven zeros in denominators of (1.17) – (1.20) must vanish their numerators giving rise to seven equations in seven above written unknowns. Solving this set of seven equations, we obtain their values. Therefore, $\bar{P}_{o,o}(z,s)$, $\bar{P}_{o,i}(z,s)$, $\bar{P}_{i,o}(z,s)$ and $\bar{P}_{i,i}(z,s)$ are known and $\bar{R}(z,s)=\bar{P}_{o,o}(z,s)+\bar{P}_{o,i}(z,s)+\bar{P}_{i,o}(z,s)+\bar{P}_{i,i}(z,s)$ can be completely evaluated. In steady state solution, equations (1.35) - (1.38), give the p.g.f. of the distribution of the number of units in the system for various cases of the input. Moreover, each is a polynomial of degree N. Therefore, all zeros in the denominator must satisfy its numerator. Denominator of each equation has seven zeros of z and numerator of each equation has $(N+7)$ zeros of z. Thus seven zeros in the denominator must vanish, the numerator of each $P_{o,o}(z), P_{o,i}(z), P_{i,o}(z)$ and $P_{i,i}(z)$ giving rise to six equations in seven unknowns viz., $P_{o,o,0}, P_{o,i,0}, P_{i,o,0}, P_{i,i,0}, P_{o,o,N}, P_{o,i,N}$ and $P_{i,o,N}$; because equation corresponding to $z=1$ becomes an identity. On solving the set of six equations along with the normalizing condition, one easily obtains the clear values of these unknowns and $R(z) = P_{o,o}(z) + P_{o,i}(z) + P_{i,o}(z) + P_{i,i}(z)$ is teetotally computed.

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