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Performance Analysis of an M/M/1 Queue with Server in Differentiated Phase Subject to Customer Impatience

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Abstract: A Markovian single server queue in which server is in distinct phases with impatience customers is studied. The server stays in active mode and in sleep mode alternatively for random amount of time. In active mode, either server is fully active or partially active (slow and active mode). Arrivivg customers get service with FCFS queue discipline while server is in Full active mode. Server can offer service only in lesser rate whenever it switches from full active to slow active mode. During slow active mode, no customer is allowed to join the system and each customer independently activates impatient timer and leaves the system if he does not get service before the expiry of the timer. Server goes to the sleep mode immediately after serving the last available customer either in active or slow mode. All customers who are all arriving during the sleep mode are lost. At the end of the sleep mode, the server enters into full active mode. We obtain expressions for the time-independent state probabilities.We also obtain some performance measures of the system.

Keywords: Single server queueing system, Server in different phases, Matrix geometric method, Customer removal.

1 Introduction

Significant delay for the customers in the queue before getting the service may lead to the to develop impatience despite of the server being available in the system. Consideration of Impatient customers in the Queueing models have been studied by several authors [1-5] . These models have been applicable in many real time situations such as in treating critical patients, transmitting data packets, production inventory systems ,Call centers etc.Impatience is studied by allotting a random time for each customer who enters the system.If the customer does not get service within his allotted time,his time expires then customer leaves the queue and is lost forever. Literatures relating to this topic can be found in several surveys by [6-9].

Queueing systems with impatient customers have been concentrated by number of authors.There is an board measures of literature based on this kind of model and we refer the reader to [10-13]. In this papers, the wellspring of fretfulness has continuously been taken to be either a significant delay as of now experienced upon landing in a line, or a huge delay expected by a customer upon arrival.

In[14-16] analysed Queues with system disasters and impatient customers.Udayabaskaran and Dora Pravina[17] studied the Transient analysis of single server queue with server operating in three modes,namely active,maintanence and sleep modes.Perel and Yechiali[18] studied Queues with slow servers with impatient customers.That is,server is in two phases(fast and slow). It is not give due importance for the sleep mode in their model,since it is essential to consider the queueing models concentrated on in power saving mechanism.To fill this gap,Here it is proposed the realistic single server queueing model with server operating in three modes subject to customers impatience.

The sections of this article is arranged as follows: In Section 2 the model is described. Section 3 is devoted for writting the balance equations to arrive time-independent probabilities of the system. Explicit expressions for the steady state probabilities are arrived in Section 4. In section 5, we obtain the system performance measures. Section 6 devoted for the numerical illustrations to validate the model.

2 Assumptions and Description of the Model

Considered a single server which is operating in three phases namely active phase, slow phase(only service no arrival) and sleep phase. In active phase, we assume that the customers arrival pattern follows Poisson distribution with rate λ. In active and slow phases, service rates are μ_1 and μ_2 respectively such that $\mu_2 < \mu_1$. The server moves to sleep phase with rate α after it served all the available customers in FCFS basis when it is in active phase. During the sleep phase no customer is allowed to join the system. The server spends for a random amount of time in sleep mode which is exponentially distributed with mean $1/\beta$. Immediately after the sleep period, the server ready to serve the customers with active phase. If the server is in active phase and the customers are there for service, then the server can switch over from active phase to slow phase with rate γ.No customers are allowed to join the system while the server is in slow phase. The server enter in to the sleep phase after completed all the customers who are all waiting for service when it is in slow phase. When the server is in slow phase, due to prolonged service time the customers may fall in impatience and leaves the system with the rate η.

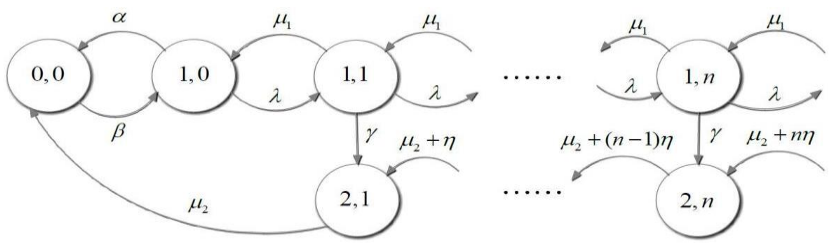
Let U be the phase of the server and let X be the number of customers in the system. Then the two dimensional stochastic process (U, X) defines a continuous time Markov process. We define the probability

$$\pi_{m,l} = \Pi[U = m, X = l | U = 0, X = 0], m = 0,1,2.; l = 0,1,2, \cdots.$$

denote the steady-state probabilities of the random process(U, X) In the next section, we derive the balance equations.

The state transition diagram is given below:

Figure 1: **State transition diagram**



3 Balance Equations

$$\beta\pi_{0,0} \quad = \alpha\pi_{1,0} + \mu_2\pi_{2,1} \tag{1}$$

$$\lambda + \alpha)\pi_{1,0} = \beta\pi_{0,0} + \mu_1\pi_{1,1} \tag{2}$$

$$\lambda + \mu_1 + \gamma)\pi_{1,l} = \lambda\pi_{1,l-1} + \mu_1\pi_{1,l+1}, l = 1,2, \cdots. \tag{3}$$

$$\mu_2 + (l-1)\eta)\pi_{2,l} = \gamma\pi_{1,l} + (\mu_2 + l\eta)\pi_{2,l+1}, l = 1,2, \cdots. \tag{4}$$

4 Steady State Solutions

The corresponding transition rate matrix

$$\mathcal{Q} = \begin{pmatrix} \mathcal{D}_0 & \mathcal{A}_0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathcal{A}_{2,1} & \mathcal{A}_{1,1} & \mathcal{A}_0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \mathcal{A}_{2,2} & \mathcal{A}_{1,2} & \mathcal{A}_0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \mathcal{A}_{1,l-1} & \mathcal{A}_0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & \mathcal{A}_{2,l} & \mathcal{A}_{1,l} & \mathcal{A}_0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{5}$$

where

$$\mathcal{D}_0 = \begin{pmatrix} -(\lambda + \alpha) & \alpha \\ \beta & -\beta \end{pmatrix} \tag{6}$$

$$\mathcal{A}_0 = \begin{pmatrix} \lambda & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

$$\mathcal{A}_{1,l} = \begin{pmatrix} -(\lambda + \mu_1 + \gamma) & \gamma \\ 0 & -(\mu_2 + (l-1)\eta) \end{pmatrix} \tag{8}$$

$$\mathcal{A}_{2,l} = \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 + (l-1)\eta \end{pmatrix} \quad (9)$$

Let $\pi_l = (\pi_{0,0}, \pi_{1,l}, \pi_{2,l})$ be the existing partition of the stationary probability vector associated with the Markov process such that $\pi Q = 0$ and $\pi e = 1$ where e is the two dimensional column vector whose elements are all unity. From the matrix analytic method (Neuts [19]), it is easy to see that

$$\pi_0\big(\mathcal{D}_0 + \mathcal{R}\mathcal{A}_{2,l}\big) = 0 \quad \text{and} \quad \pi_0(I - \mathcal{R})^{-1}e = 1 \quad (10)$$

$$\pi_l = \pi_0 \mathcal{R}^l \qquad \text{for} \quad l = 0,1,2,\ldots \quad (11)$$

Where matrix $\mathcal{R}$ is the solution to the matrix -quadratic equations

$$\mathcal{R}^2\mathcal{A}_{2,l} + \mathcal{R}\mathcal{A}_{1,l} + \mathcal{A}_0 = 0 \quad (12)$$

$$\mathcal{R} = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix}$$

Substitute and simplify (12),we get

$$\mu_1(r_1^2 + r_2 r_3) - r_1(\lambda + \mu_1 + \gamma) + \lambda = 0 \quad (13)$$

$$\mu_2 + (l-1)\eta](r_1 r_2 + r_2 r_4) + \gamma r_1 - r_2(\mu_2 + (l-1)\eta) = 0 \quad (14)$$

$$\mu_1(r_3 r_1 + r_4 r_3) - r_3(\lambda + \mu_1 + \gamma) = 0 \quad (15)$$

$$\mu_2 + (l-1)\eta](r_2 r_3 + r_4^2) + \gamma r_3 - r_4(\mu_2 + (l-1)\eta) = 0 \quad (16)$$

Solving (13) to (16) and simplify we get

$r_3 = 0$ and $r_4 = 0$

$$r_1 = \frac{(\lambda+\mu_1+\gamma) - \sqrt{(\lambda+\mu_1+\gamma)^2 - 4\lambda\mu_1}}{2\mu_1}$$

$$r_2 = \frac{\gamma r_1}{(1-r_1)[\mu_2 + (l-1)\eta]}$$

Therefore

$$\mathcal{R} = \begin{pmatrix} r_1 & r_2 \\ 0 & 0 \end{pmatrix}$$

From (10) we get

$$\pi_0 = \begin{pmatrix} \pi_{1,0} & \pi_{0,0} \end{pmatrix}$$

$$\pi_0(\mathcal{D}_0 + \mathcal{R}\mathcal{A}_{2,l}) = 0$$

$$\begin{pmatrix} \pi_{1,0} & \pi_{0,0} \end{pmatrix} \begin{pmatrix} -(\lambda+\alpha) + \mu_1 r_1 & \alpha + r_2(\mu_2 + (l-1)\eta) \\ \beta & -\beta \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$(-(\lambda+\alpha) + \mu_1 r_1)\pi_{1,0} + \beta\pi_{0,0} = 0 \quad (17)$$

$$(\alpha + r_2(\mu_2 + (l-1)\eta))\pi_{1,0} - \beta\pi_{0,0} = 0 \quad (18)$$

From (18)

$$\pi_{1,0} = T\beta \quad (19)$$

$$\pi_{0,0} = T[\alpha + r2(\mu_2 + (l-1)\eta)] \tag{20}$$

To find T using (10)

$$\pi_0(I-\mathcal{R})^{-1}e = 1$$

$$\begin{pmatrix}\pi_{1,0} & \pi_{0,0}\end{pmatrix}\begin{pmatrix}\frac{1}{1-r_1} & \frac{r_2}{1-r_1}\\ 0 & 1\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix} = 1$$

$$\frac{1}{1-r_1}\pi_{1,0} + \frac{r_2}{1-r_1}\pi_{1,0} + \pi_{0,0} = 1$$

substitute $\pi_{1,0}$ and $\pi_{0,0}$ and simplify we get T

$$T\left[\frac{\beta(1+r_2)}{1-r_1} + \alpha + r_2(\mu_2 + (l-1)\eta)\right] = 1$$

$$T = \frac{(1-r_1)}{\beta(1+r_2)+(\alpha+r_2(\mu_2+(l-1)\eta))(1-r_1)} \tag{21}$$

From (11)

$$\pi_l = \pi_0\mathcal{R}^l$$

$$\pi_l = \begin{pmatrix}\pi_{1,0} & \pi_{0,0}\end{pmatrix}\begin{pmatrix}r_1 & r_2\\ 0 & 0\end{pmatrix}^l$$

$$\begin{pmatrix}\pi_{1,l} & \pi_{2,l}\end{pmatrix} = \begin{pmatrix}\pi_{1,0} & \pi_{0,0}\end{pmatrix}\begin{pmatrix}r_1^l & r_2^{l-1}r_2\\ 0 & 0\end{pmatrix}$$

$$\begin{pmatrix}\pi_{1,l} & \pi_{2,l}\end{pmatrix} = \begin{pmatrix}\pi_{1,0}r_1^l & \pi_{1,0}r_1^{l-1}r_2\end{pmatrix}$$

Equating we get

$$\pi_{1,l} = T\beta r_1^l, \qquad l = 1,2,\ldots \tag{22}$$

$$\pi_{2,l} = T\beta r_1^{l-1}r_2,\ l = 1,2,\ldots \tag{23}$$

Steady state probabilities of all the states are in equations (19),(20),(22)and (23) .

5 Effective Measures of System Performance

5.1 Expected customers in the active phase

Let the average customers in the active phase be denoted by E[R].

$$E[R] = \sum_{l=0}^{\infty} l\pi_{1,l} = \sum_{l=0}^{\infty} lT\beta r_1^l$$

$$E[R] = T\beta r_1[1 + 2r_1 + 3r_1^2 + \ldots]$$

$$E[R] = \frac{T\beta r_1}{(1-r_1)^2}$$

5.2 Expected customers in the slow phase

Let E[N] signifies the average number of customers in slow phase.

$$E[N] = \sum_{l=1}^{\infty} l\pi_{2,l} = \sum_{l=1}^{\infty} lT\beta r_1^{l-1}r_2 = T\beta r_2[1 + 2r_1 + 3r_1^2 + \ldots]$$

$$E[N] = \frac{T\beta r_2}{(1-r_1)^2}$$

5.3 Expected number of customers in the system

Let E[S] denotes the average number of customers in the system.

$$E[S] = E[R] + E[N] = \sum_{l=0}^{\infty} l\pi_{1,l} + \sum_{l=1}^{\infty} l\pi_{2,l} = \frac{T\beta r_1}{(1-r_1)^2} + \frac{T\beta r_2}{(1-r_1)^2}$$

$$E[S] = \frac{T\beta(r_1+r_2)}{(1-r_1)^2}$$

5.4 Average number of times server switches from active phase to sleep phase

Let $E[RS]$ signifies the expected number of times the server switching from active phase to sleep phase per unit time.

$$E[RS] = \alpha\pi_{1,0} = \alpha T\beta$$

5.5 Average number of times server switches from active phase to slow phase

Let E[RN] signifies the expected number of times the server switching from active phase to slow phase per unit time.

$$E[RN] = \gamma \sum_{l=1}^{\infty} \pi_{1,l} = \gamma \sum_{l=1}^{\infty} T\beta r_1^l = \gamma T\beta r_1[1 + r_1 + r_1^2 + \ldots]$$

$$E[RN] = \frac{\gamma T\beta r_1}{1-r_1}$$

5.6 Average number of times server switches from sleep phase to active phase

Let $E[SR]$ signifies the expected number of times the server switching from sleep phase to active phase per unit time.

$$E[SR] = \beta\pi_{0,0} = \beta T[\alpha + r_2(\mu_2 + (l-1)\eta)]$$

5.7 Average customers leaves the system by impatience

Let E[I] denotes mean customers left the system due to impatience per unit time.

$$E[I] = \eta \sum_{l=2}^{\infty} l\pi_{2,l} = \eta T\beta r_2 \sum_{l=2}^{\infty} l r_1^{l-1}$$

$$E[I] = \eta T\beta r_2[(1-r_1)^{-2} - 1]$$

$$E[I] = \eta T\beta r_2 \left[\frac{1-(1-r_1)^2}{(1-r_1)^2}\right]$$

$$E[I] = \eta T\beta r_2 \left[\frac{2r_1 - r_1^2}{(1-r_1)^2}\right]$$

5.8 Effective arrival rate

Let A be The effective arrival rate. It is defined as the total arrival when the server is available.Even if the server is available in either active phase or slow phase, the customers are allowed to join the system only when the server is in active phase.

Therefore,

$$\pi_{0,0} + \sum_{l=0}^{\infty} \pi_{1,l} + \sum_{l=1}^{\infty} \pi_{2,l} = 1$$

$$\sum_{l=0}^{\infty} \pi_{1,l} = 1 - \pi_{0,0} - \sum_{l=1}^{\infty} \pi_{2,l}$$

$$A = [1 - \pi_{0,0} - \sum_{l=1}^{\infty} \pi_{2,l}]\lambda$$

$$A = [1 - T[\alpha + r_2(\mu_2 + (l-1)\eta)] - T\beta r_1^{l-1} r_2]\lambda$$

5.9 Average time a customer spemds in the system

Let E[W] signifies the customers expect to spend in the system By Little's formula

$$E[W] = \frac{E[S]}{\lambda}$$

$$E[W] = \frac{T\beta(r_1+r_2)}{\lambda(1-r_1)^2}$$

6 Numerical Illustration

6.1 Steady state probabilities

By fixing $\lambda = 0.25; \mu_1 = 0.5; \mu_2 = 0.03; \eta = 0.8; \gamma = 1.2; \beta = 1.3; \alpha = 1.1.$ Arrived the steady-state probabilities by using (3.19),(3.20),(3.22)and (3.23), we get probability distribution which is given in Table 1:

Table 1: **Steady-state probability distribution**

(i,j)	π(i,j)	(i,j)	π(i,j)
(0,0)	0.4596	(2,1)	0.0031
(1,0)	0.4655	(2,2)	0.0036
(1,1)	0.0618	(2,3)	0.0036
(1,2)	0.0700	(2,4)	0.0036
(1,3)	0.0711	(2,5)	0.0036
(1,4)	0.0712	(2,6)	0.0036
(1,5)	0.0712	(2,7)	0.0036
(1,n),n=6,7,8,...	0.0712	(2,n),n=8,9,10,...	0.0036

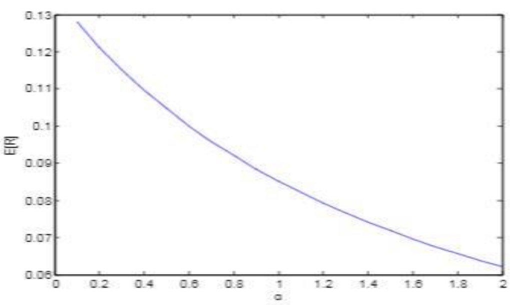
6.2 Mean number of customers in active phase against α

By fixing $\lambda = 0.25; \mu_1 = 0.5; \mu_2 = 0.03; \eta = 0.8; \gamma = 1.2; \beta = 1.3.$ and vary α from 0.1 to 3.0. Computing the steady-state probabilities of the average customers in the active phase. It is listed out in table 2 and depicted in figure 2:

Table 2: **Assortment of Mean number of customers in active phase against α**

α	$E[R]$	α	$E[R]$	α	$E[R]$
0.1	0.12796	1.1	0.082138	2.1	0.060481
0.2	0.1212	1.2	0.079299	2.2	0.058928
0.3	0.11511	1.3	0.076649	2.3	0.057452
0.4	0.10961	1.4	0.074171	2.4	0.056048
0.5	0.10461	1.5	0.071848	2.5	0.054711
0.6	0.10005	1.6	0.069666	2.6	0.053437
0.7	0.09587	1.7	0.067612	2.7	0.05222
0.8	0.092024	1.8	0.065676	2.8	0.051058
0.9	0.088475	1.9	0.063848	2.9	0.049946
1.0	0.085189	2.0	0.062119	3.0	0.048882

Figure 2: **Assortment of $E[R]$ versus α**



We observe from Table 2 and Figure 2, the customers in the active phase diminishes as the rate of the system to be in sleep phase increases.

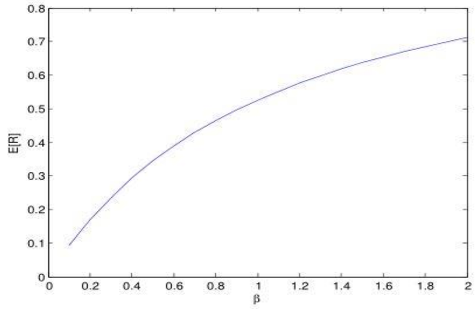
6.3 Average number of customers in active phase against β

We next fix $\lambda = 0.25; \mu_1 = 0.5; \mu_2 = 0.03; \eta = 0.8; \gamma = 1.2; \alpha = 1.1.$ and vary β from 0.1 to 3.0. The steady-state probabilities for the mean number of customers in the active phase. It is listed out in the following table 3 and depicted in figure 3:

Table 3: **variation of average number of customers in active phase against β**

β	$E[SR]$	β	$E[SR]$	β	$E[SR]$
0.1	0.091707	1.1	0.55145	2.1	0.72437
0.2	0.16937	1.2	0.57549	2.2	0.73591
0.3	0.23598	1.3	0.59754	2.3	0.74677
0.4	0.29375	1.4	0.61782	2.4	0.75702
0.5	0.34432	1.5	0.63655	2.5	0.76669
0.6	0.38896	1.6	0.65389	2.6	0.77584
0.7	0.42865	1.7	0.67000	2.7	0.78451
0.8	0.46419	1.8	0.68499	2.8	0.79274
0.9	0.49617	1.9	0.69899	2.9	0.80055
1.0	0.52512	2.0	0.71209	3.0	0.80799

Figure 3: **Variation of $E[SR]$ versus β**



We observe from Table 3 and Figure 3, the average customers in the active phase increases as the rate of residing sleep phase increases.

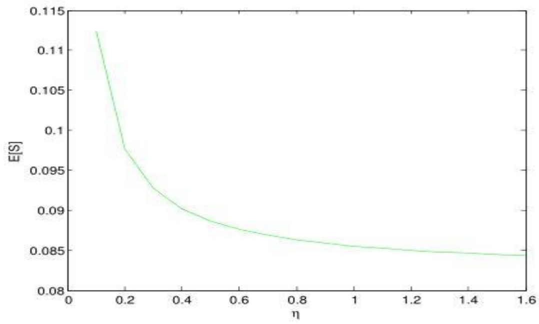
6.4 Mean number of customers in system against η

We next fix $\lambda = 0.25; \mu_1 = 0.5; \mu_2 = 0.03; \gamma = 1.2; \beta = 1.3, \alpha = 1.1$ and vary η from 0.1 to 3.0. To compute the steady-state probabilities of the average customers in the system, It is shown in the following table 4 and depicted in figure 4:

Table 4: **variation of average number of customers in system against η**

η	$E[S]$	η	$E[S]$	η	$E[S]$
0.1	0.11245	1.1	0.085259	2.1	0.083917
0.2	0.097722	1.2	0.085024	2.2	0.08385
0.3	0.09269	1.3	0.084826	2.3	0.083789
0.4	0.090151	1.4	0.084656	2.4	0.083733
0.5	0.08862	1.5	0.084508	2.5	0.083681
0.6	0.087596	1.6	0.084379	2.6	0.083633
0.7	0.086863	1.7	0.084265	2.7	0.083589
0.8	0.086312	1.8	0.084164	2.8	0.083548
0.9	0.085883	1.9	0.084073	2.9	0.083509
1.0	0.08554	2.0	0.083991	3.0	0.083474

Figure 4: **Variation of $E[S]$ versus η**



We observe from Table 4 and Figure 4, the average number of customers in the system decreases while the rate ofcustomer impatience increases.

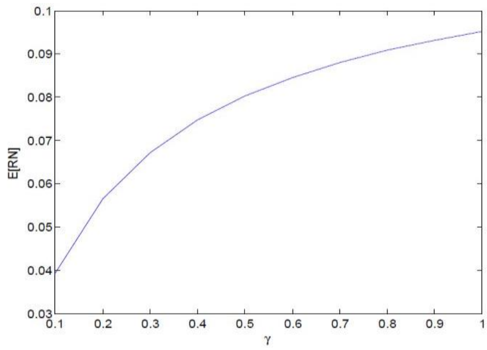
6.5 Variation in average number of customers in the system against γ

We next fix $\lambda = 0.25; \mu_1 = 0.5; \mu_2 = 0.03; \eta = 0.8; \beta = 1.3, \alpha = 1.1$ and vary γ from 0.1 to 3.0. Computing the average number of customers in the system, it is shown in the following table 5 and depicted in figure 5:

Table 5: **Variation in average number of customers in the system against γ**

γ	$E[RN]$	γ	$E[RN]$	γ	$E[RN]$
0.1	0.039072	1.1	0.097026	2.1	0.10688
0.2	0.056536	1.2	0.098566	2.2	0.10745
0.3	0.067251	1.3	0.099923	2.3	0.10798
0.4	0.074686	1.4	0.10113	2.4	0.10847
0.5	0.080216	1.5	0.10221	2.5	0.10893
0.6	0.084519	1.6	0.10318	2.6	0.10935
0.7	0.087979	1.7	0.10406	2.7	0.10976
0.8	0.090829	1.8	0.10486	2.8	0.11013
0.9	0.093222	1.9	0.10559	2.9	0.11049
1.0	0.095263	2.0	0.10626	3.0	0.11082

Figure 5: **Variation of $E[RN]$ versus γ**



We observe from Table 5 and Figure 5, there is an increase in the average number of customers while the rate of active phase increased.

7 Conclusions

Our study carried out on the steady state analysis for Single server queue with the consideration of three distinct phases of the server. In which, while the server serving the customers in slow phase, customers who are in queue become impatience and immediately lost from the system. System have been solved and gor expressions for steady state probabilities. Performance measures like Expected number of customers in the system which covers both the cases of the server is in active phase and slow phase. Expected number of times the server switches from one phase to another, Expected number of customers leaves the system due to the effect of implimentation of impatiency, Effective arrival rate of teh customers to the system and Expected waiting time of the arriving customer in the system. Effect of change in each parameter on the values of parameters are listed by appropriate figures and diagrams. This article laid the path to the researchers to take up the model with the consideration of the stay of the servers in each phase as a general distribution and may prioratice the customers.

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