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COEFFICIENT BOUNDS FOR CERTAIN SUBCLASSES OF ANALYTIC AND BI-UNIVALENT FUNCTIONS

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Abstract

In this paper we introduce two new subclass of analytic and bi-univalent function in the open unit disk U. Further,we find estimate on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$. for functions in this subclass.

1 Introduction

Let A denote the class of functions f(z) which are analytic in the open disk

$$U = z\epsilon C : |z| < 1$$

that have the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1.1)$$

Let S be the subclass of A consisting of univalent function in U. An important member of the class S is the Koebe function. The function

$$w = f(z) = f_\theta(z) = \frac{z}{(1-e^{i\theta}z)^2} = z + \sum_{n=2}^{\infty} ne^{i(n-1)\theta} z^n$$

Where $\{\theta \epsilon [0,2\pi)\}$.This function was studied by p koebe[1]. The koebe function map the disc $\{|z| < 1\}$ onto the w-plane with a slit along the ray starting at the point $\{-e^{-i\theta}/4\}$, its extension containing the point w=0. The koebe function in a number of problems in the theory of univalent fuctions.The function

$$\phi(z) = \frac{1-z}{\sqrt{1-2z\cos\alpha+z^2}}$$

is in P for every real α where P is the caratheodory class defined by P=$\{p(z): R(p(z)) > 0, z\epsilon u\}$

$$p(z) = 1 + c_1 z + c_2 z^2 + \ldots..$$

then,

$$\phi(z) = 1 + \sum_{n=1}^{\infty} [p_n(\cos\alpha) - p_n - 1(\cos\alpha)]z^n \qquad (1.2)$$

$$= 1 + \sum_{n=1}^{\infty} B_n z^n, z\epsilon u$$

If we consider,

$$\frac{1}{(\phi(z))^2} = \frac{1-2z\cos\alpha+z^2}{(1-z)^2}$$

$$= 1 + 2(1-\cos\alpha)\frac{z}{(1-z)^2}$$

By the geometric ptoperties of koebe function ϕ maps onto the right plane $R(w) > 0$ minus the slit along positive real axis from $\frac{1}{|\cos\frac{\alpha}{2}|}$ $\phi(u)$ is univalent,symmetric with respect to real axis and starlike with respect to $\phi(0) = 1$by using one- quarter theorem[4]. Thus every functions $f\epsilon A$ has an inverse f^{-1} ,which is defined by

$$f^{-1}(f(z)) = z(z\epsilon U)$$

and

$$f(f^{-1}(w)) = w(|w| < r_0(f); r_0(f) \geq \frac{1}{4})$$

In fact,the inverse function f^{-1} is given by

$$f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \ldots.$$

A function $f \epsilon A$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U. Let $\sum$ denote the class of bi-univalent functions in the U given by (1.1). For a brief history and interesting example of function in the class $\sum$, see[3]. Brannan and Taha [1] introduced the following two subclasses of the bi-univalent function class $\sum$ and obtained non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in each of these subclasses.

Definition 1 *Let function $f \epsilon \sigma$ belongs to the class $L_\sigma(\lambda, \phi)$ with $0 \leq \lambda \leq 1$ if the following subordination condition are satisfied.*

$$\lambda(1 + \frac{zf''(z)}{f'(z)} + (1-\lambda)(\frac{zf'(z)}{f(z)} < \phi(z)(z \epsilon U)$$

and

$$\lambda(1 + \frac{wg''(w)}{g'(w)} + (1-\lambda)(\frac{wf'(w)}{g(w)} < \phi(w)(w \epsilon U)$$

2 Results

$$u(z) = \sum_{n=1}^{\infty} b_n z^n$$

and

$$v(w) = \sum_{n=1}^{\infty} c_n w^n$$

Then

$$\phi(u(z)) = 1 + B_1b_1z + (B_1b_2 + B_2b_1^2)z^2 + (B_1b_3 + 2b_1b_2B_2 + B_3b_1^3)z^3 + \ldots.$$

and

$$\phi(v(w)) = 1 + B_1c_1w + (B_1c_2 + B_2c_1^2)w^2 + (B_1c_3 + 2c_1c_2B_2 + B_3c_1^3)w^3 + \ldots.$$

where

$$B_1 = \cos\alpha - 1$$

$$B_2 = \frac{1}{2}(\cos\alpha - 1)(1 + 3\cos\alpha)$$

$$B_3 = \frac{1}{2}(5\cos^3\alpha - 3\cos^2\alpha + 1)$$

Theorem 1 *Let the function $f \epsilon L_\sigma(\lambda, \phi)$ .Then*

$$|a_2| \leq \frac{\sqrt{2}(1-\cos\alpha)}{\sqrt{(1+\lambda)[3\lambda(\cos\alpha+1)+5+\cos\alpha]}} \qquad (2.1)$$

and

$$|a_3| \leq [1 - \frac{(1+\lambda)^2}{2(1+2\lambda)|B_1|}]|a_2|^2 + \frac{|B_1|}{2(1+2\lambda)} \qquad (2.2)$$

and for $z \epsilon u$

$$|a_3 - \xi a_2^2| \leq \begin{cases} \frac{|\cos\alpha - 1|}{2} & if |\xi - 1| \leq \frac{(\cos\alpha - 1)}{(1+\lambda)[(\cos\alpha-1)-2(1+\lambda)(1+3\cos\alpha)]} \\ \frac{|\cos\alpha - 1|}{2(1+2\lambda)} & if |\xi - 1| \geq \frac{(\cos\alpha - 1)}{(1+\lambda)[(\cos\alpha-1)-2(1+\lambda)(1+3\cos\alpha)]} \end{cases}$$

Proof. Let $f \epsilon L_\sigma(\lambda, \phi)$ then,

$$|a_3 - \xi a_2^2| \leq \left|\frac{B_1}{4(1+2\lambda)}(b_2 - c_2) - \xi \frac{B_1^3}{2(1+\lambda)[B_1^2 - (1+\lambda)B_2]}(b_2 + c_2)\right| \quad (2.3)$$

By expanding and re-arranging we get,

$$|a_3 - \xi a_2^2| \leq \frac{B_1}{2}\left[\left(\Psi - \xi \frac{B_1{}^2}{(1+\lambda)[B_1^2 - (1+\lambda)B_2]}\right) b_2\right] - \frac{B_1}{2}\left[\left(\Psi + \xi \frac{B_1^2}{(1+\lambda)[B_1^2 - (1+\lambda)B_2]}\right) c_2\right]$$

Applying result to the above equation and after simple calculations,We get

$$|a_3 - \xi a_2^2| \leq \begin{cases} \frac{|\cos\alpha - 1|}{2} & if |\xi - 1| \leq \frac{(\cos\alpha - 1)}{(1+\lambda)[(\cos\alpha - 1) - 2(1+\lambda)(1+3\cos\alpha)]} \\ \frac{|\cos\alpha - 1|}{2(1+2\lambda)} & if |\xi - 1| \geq \frac{(\cos\alpha - 1)}{(1+\lambda)[(\cos\alpha - 1) - 2(1+\lambda)(1+3\cos\alpha)]} \end{cases} \quad (2.4)$$

3 Conclusion

In this paper, we introduce a new subclass of bi-univalent function. Moreover we find estimate on the frist two Taylor-Maclaurin coefficient $|a_2|$ and $|a_3|$ for the functions belonging to these new subclasses.

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