

FUZZY GAME PROBLEM USING OCTAGONAL FUZZY NUMBERS

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Abstract:

In this paper, we consider a two player zero sum game with infinite value in pay-off matrix. Now we determine on solving a fuzzy game problem using octagonal fuzzy number. To solve a problem using Pascal's triangular graded mean integration for finding a solution. We can convert a fuzzy valued game problem into a crisp valued game problem by changing the payoffs.

Keywords: Fuzzy set, Octagonal Fuzzy Number, Crisp Set, Pascal's Triangle Graded Mean Approach.

1. Introduction

Game theory has played an important role in the fields of decision making theory such as economics, management etc. This paper uses the concept of Pascal's triangle graded mean integration. It mainly uses octagonal fuzzy number to solve a fuzzy game problem. The defuzzified value can then be converted into a crisp value query, which can then be answered using the conventional approach.

In this paper, First section includes some basic definition and second section includes mathematical formulation of fuzzy game problem and the third section includes algorithm for the problem and some numerical examples and in the final section we conclude the conclusion.

2. Preliminaries

In this section, we represent basic idea about the fuzzy numbers and definitions which involved in the research work.

Definition 1:[3] A fuzzy set \tilde{A} in X (set of real numbers) is a set of ordered pairs $\tilde{A} = \{(x, \mu(\tilde{A})) / x \in X\}$ is called membership function of x in \tilde{A} which maps X to $[0, 1]$.

Definition 2:[6, 7] A fuzzy collection A based on real numbers. If the membership function $\mu_A: R \rightarrow [0, 1]$ of R has the following properties, it is said to be a fuzzy number.

(1) A is normal. It implies that $X \in R$ exists such that $\mu_A(x) = 1$

(2) A is convex. It implies that for every $x_1, x_2 \in R$

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}, \lambda \in [0, 1]$$

(3) μ_A is upper semi-continuous

(4) $\text{Supp}(A)$ is bounded in R .

Definition 3:[4] A crisp set is a special case of a fuzzy set, in which the membership function only takes two values, commonly defined as 0 and 1.

Definition 4:[4] A fuzzy number \tilde{A} is a normal octagonal fuzzy number denoted by

$(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8)$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7 \leq a_8$ are real numbers and its membership function $\mu_A(x)$ is given below

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a_1, \\ k \left(\frac{x-a_1}{a_2-a_1} \right) & \text{for } a_1 \leq x \leq a_2, \\ k & \text{for } a_2 \leq x \leq a_3, \\ k + (1-k) \left(\frac{x-a_3}{a_4-a_3} \right) & \text{for } a_3 \leq x \leq a_4, \\ 1 & \text{for } a_4 \leq x \leq a_5, \\ k + (1-k) \left(\frac{a_6-x}{a_6-a_5} \right) & \text{for } a_5 \leq x \leq a_6, \\ k & \text{for } a_6 \leq x \leq a_7, \\ k \left(\frac{a_8-x}{a_8-a_7} \right) & \text{for } a_7 \leq x \leq a_8, \\ 0 & \text{for } x < a_8 \end{cases}$$

where $0 < k < 1$

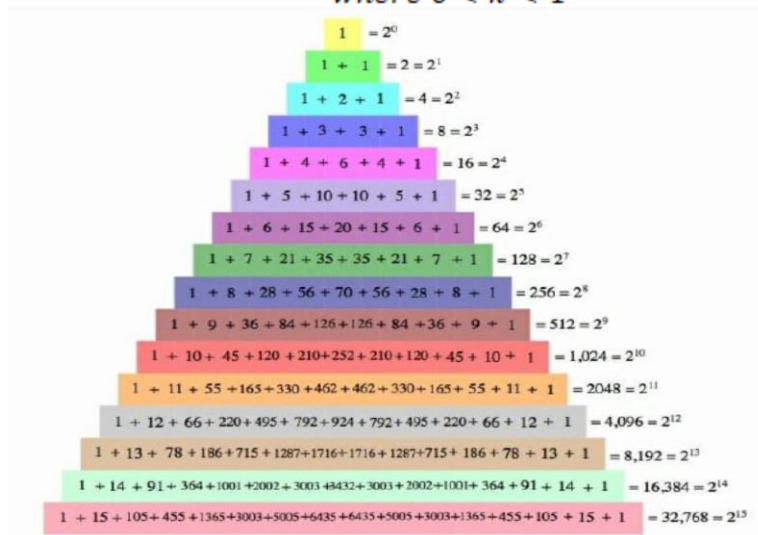


Figure 1: Pascal's Triangle Graded Mean Approach

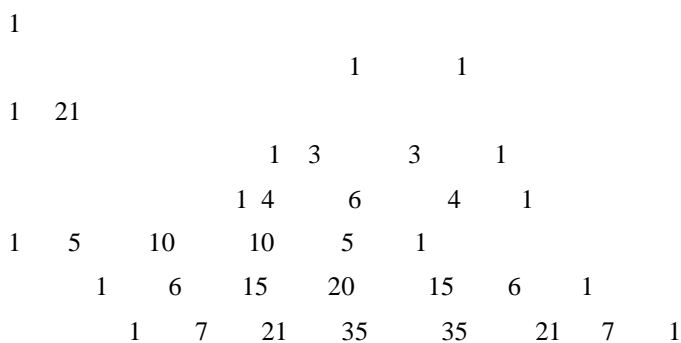


Figure: Pascal's Octagonal Approach

The following is the octagonal solution of Pascal:

Let $A=(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ be two octagonal fuzzy numbers then we can take the coefficient of fuzzy numbers from the triangle of Pascal and apply the approach.

$$p(A) = \frac{x_1 + 7x_2 + 21x_3 + 35x_4 + 35x_5 + 21x_6 + 7x_7 + x_8}{128}$$

The coefficients $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)$ are 1,7,21,35,35,21,7,1. This method can also be applied to the octagonal fuzzy order of n-dimensional Pascal.

0s3.Mathematical formulation of fuzzy game problem

Consider two competitors (called Players) in the game. Let Player A has m strategies A_1, A_2, \dots, A_m and Player B have n strategies B_1, B_2, \dots, B_n . Here, it is assumed that each Player A has his choices from amongst the pure strategies. Assume that Player A is therefore considered to always be the winner and then the loser in any game is Player B. That is, in terms of Player A, all payoffs are presumed. If Player A selects approach A_i and Player B will choose strategy B_j . The payout matrix for player A follows.

		Player B			
		B_1	\dots	B_n	
Player A	A_1	a_{11}	\dots	a_{1n}	
	\dots	\dots	\dots	\dots	
	A_m	a_{m1}	\dots	a_{mn}	

Definition 5:[9] Particular course of action that are selected by players, is called pure strategies.

Definition 6:[10] Course of action that are to be selected on a particular occasion with some fixed probability are called mixed strategies.

Definition 7:[3] The maximin.=minimax, the corresponding strategies which give the saddle point are called optimal strategies.

Definition 8:[11] Rectangular games with two teams are called zero-sum games. In this case, one player's loss (gain) is exactly equal to the other player's gain (loss).

Definition 9:[3] If the maximin value equals the minimax value, then the game is said to have a saddle point.

Definition 10:[13] A zero-sum game is one in which the algebraic sum of all player's profits and losses equals zero. The game is otherwise called a non-zero-sum-game.

Definition 11:[4] The solution of 2*2 games without saddle point is

Player B

$$\text{Player A} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\text{Optimum Mixed strategy } S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix}, S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$$

$$p_1 = \frac{a_{22} - a_{21}}{\lambda}$$

$$p_2 = 1 - p_1$$

$$q_1 = \frac{a_{22} - a_{12}}{\lambda}$$

$$q_2 = 1 - q_1$$

$$\lambda = (a_{11} + a_{22}) - (a_{12} + a_{21})$$

$$\text{Value of the game } \gamma = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

4. Algorithm for solving a problem

Step 1: By using Pascal's Triangle Graded Mean Approach .We can convert given octagonal fuzzy numbers to a crisp values.

Step 2: If the saddle point exist we obtain a value of the game. If saddle point does not exist go to next step.

Step 3: Optimum Mixed strategy $S_A = \begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix}$, $S_B = \begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$

$$p_1 = \frac{a_{22} - a_{21}}{\lambda}$$

$$p_2 = 1 - p_1$$

$$q_1 = \frac{a_{22} - a_{12}}{\lambda}$$

$$q_2 = 1 - q_1$$

$$\lambda = (a_{11} + a_{22}) - (a_{12} + a_{21})$$

$$\text{Value of the game } \gamma = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

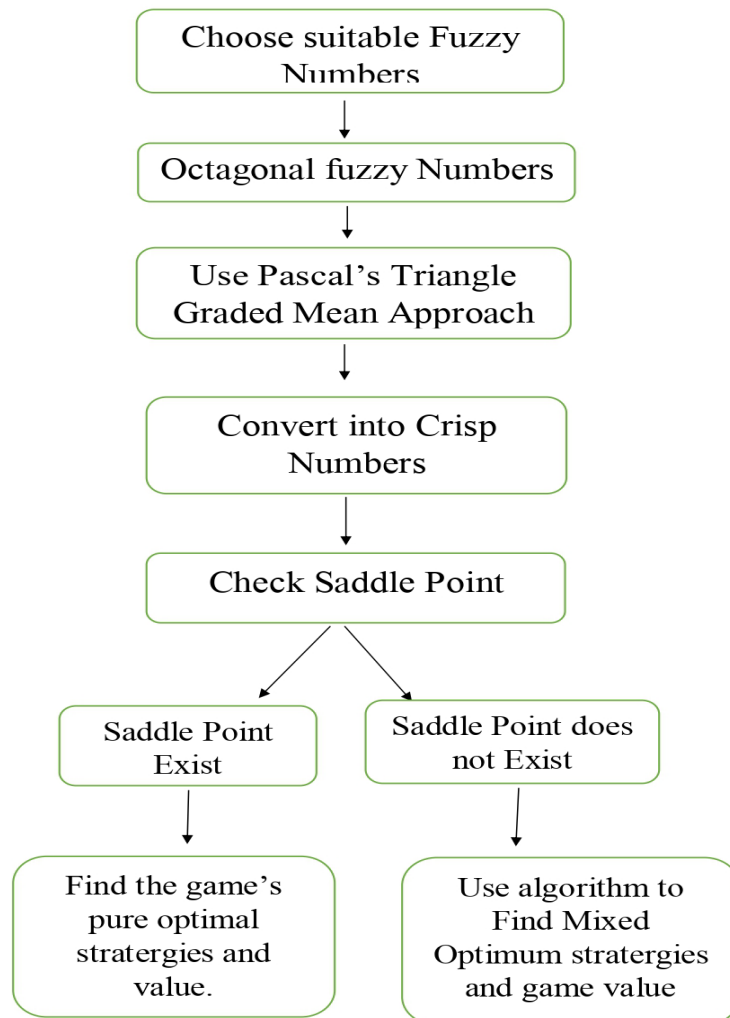


Figure 2:Flow chart for solving fuzzy game problem

4.1 Numerical example

Consider the following fuzzy game problem

$$\text{Player A} \begin{matrix} & \text{Player B} \\ \begin{pmatrix} (5, 7, 0, 1, 2, 8, 9, -6) \\ (6, 6, 3, 4, 6, 2, 0, 9) \end{pmatrix} & \begin{pmatrix} (-1, -5, -3, 0, 7, 1, 3, 4) \\ (5, 10, 6, 7, 6, 5, 1, 0) \end{pmatrix} \end{matrix}$$

Step 1: Use Pascal's Triangle Graded Mean Approach to convert a given fuzzy problem into a crisp value (Table 1).

$$p(A) = \frac{x_1 + 7x_2 + 21x_3 + 35x_4 + 35x_5 + 21x_6 + 7x_7 + x_8}{128}$$

Step 2: The payoff matrix

$$\text{Player A} = \begin{pmatrix} 3 & 1.5 \\ 4 & 6 \end{pmatrix}$$

minimum value of 1st row = 1.5

maximum value of 1st column = 4

minimum value of 2nd row = 4

maximum value of 2nd column = 6

maximin value = maximum(minimum value) = maximum(1.5, 4) = 4

minimax value = minimum(maximum value) = minimum(4, 6) = 4

saddle point is (4, 4)

Therefore value of the game $\gamma = 4$.

Therefore the game is said to be strictly determinable.

Table 1

Crisp value for the fuzzy problem

$a_{11} = (5, 7, 0, 1, 2, 8, 9, -6)$	$p(a_{11}) = \frac{5+7*7+21*0+35*1+35*2+21*8+7*9-6}{128} = 3$
$a_{12} = (-1, -5, -3, 0, 7, 1, 3, 4)$	$p(a_{12}) = \frac{-1+7*-5+21*-3+35*0+35*7+21*1+7*3+4}{128} = 1.5$
$a_{21} = (6, 6, 3, 4, 6, 2, 0, 9)$	$p(a_{21}) = \frac{6+7*6+21*3+35*4+35*6+21*2+7*0+9}{128} = 4$
$a_{22} = (5, 10, 6, 7, 6, 5, 1, 0)$	$p(a_{22}) = \frac{5+7*7+21*6+35*7+35*6+21*5+7*1+0}{128} = 6$

4.2 Numerical example

Consider the following fuzzy game problem

$$\text{Player A} \begin{pmatrix} \text{Player B} \\ \begin{pmatrix} (6,5,5,6,8,3,8,13) & (10,9,2,0,7,11,14,15) \\ (5,7,10,3,4,5,13,15) & (6,7,9,11,5,2,6,8) \end{pmatrix} \end{pmatrix}$$

Step 1: Use Pascal's Triangle Graded Mean Approach to convert a given fuzzy problem into a crisp value (Table 2).

$$p(A) = \frac{x_1 + 7x_2 + 21x_3 + 35x_4 + 35x_5 + 21x_6 + 7x_7 + x_8}{128}$$

Step 2: The payoff matrix

$$\text{Player A} = \begin{pmatrix} \text{Player B} \\ \begin{pmatrix} 6 & 5.5 \\ 5.625 & 7 \end{pmatrix} \end{pmatrix}$$

minimum value of 1st row = 5.5

maximum value of 1st column = 6

minimum value of 2nd row = 5.625

maximum value of 2nd column = 7

maximin value = maximum(minimum value) = maximum(5.5, 5.625) = 5.625

minimax value = minimum(maximum value) = minimum(6, 7) = 6

It does not exist saddle point .

Step 3: To find the optimum mixed strategy and value of the game. Here $a_{11} = 6$,

$a_{12} = 5.5$, $a_{21} = 5.625$, $a_{22} = 7$.

$$\lambda = (a_{11} + a_{22}) - (a_{12} + a_{21}) = (6+7) - (5.5+5.625) = 1.875$$

$$p_1 = \frac{a_{22} - a_{21}}{\lambda} = \frac{7 - 5.625}{1.875} = 0.73$$

$$p_2 = 1 - p_1 = 1 - 0.73 = 0.27$$

$$q_1 = \frac{a_{22} - a_{12}}{\lambda} = \frac{7 - 5.5}{1.875} = 0.8$$

$$q_2 = 1 - q_1 = 1 - 0.8 = 0.2$$

$$\text{Value of the game } \gamma = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \frac{42 - 30.9375}{13 - 11.125} = \frac{9.0625}{1.875} = 4.83$$

Value of the game $\gamma = 4.83$

Table 2

Crisp value for the fuzzy problem

$a_{11}=(6,5,5,6,8,3,8,13)$	$p(a_{11}) = \frac{6+7*5+21*5+35*6+35*8+21*3+7*8+13}{128} = 6$
$a_{12}=(10,9,2,0,7,11,14,15)$	$p(a_{12}) = \frac{10+7*9+21*2+35*0+35*7+21*11+7*14+15}{128} = 5.5$
$a_{21}=(5,7,10,3,4,5,13,15)$	$p(a_{21}) = \frac{5+7*7+21*10+35*3+35*4+21*5+7*13+15}{128} = 5.625$
$a_{22}=(6,7,9,11,5,2,6,8)$	$p(a_{22}) = \frac{6+7*7+21*9+35*11+35*5+21*2+7*6+8}{128} = 7$

5. Conclusion

In this paper, solving a fuzzy game problem using Pascal’s Triangle Graded Mean Approach. To solve any 2*2 Matrix with its values as octagonal fuzzy number. These values can be converted to crisp values using Pascal’s Triangle Graded Mean Approach. It will be useful in the future problems involving octagonal fuzzy numbers.

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