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STUDY OF TWO SERVER'S WHEN ONE SERVER IS RESERVED IN VARIOUS QUEUEING MODELS.

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Abstract

Queueing theory is a well-developed branch of applied probability theory. In general, we do not like to wait. But reduction of the waiting time usually requires extra investments to decide whether or not to invest. In this chapter we use the concept of standby server. If the main server fails then we switch over to standby so that the task is not affected. Here we study two server queue with standby. There are two servers, one server is busy with the main line of service and the other is kept reserved as standby.

Keywords: Main server, standby server.

INTRODUCTION

Many queueing systems with re-orientation time were investigated in the previous chapter with only one server. However, we all face the same problem of server failure at some point in our lives. A server failure occurs, requiring the use of a standby server. We quickly switch over to standby if the main server fails so that the task is not affected, i.e. When the main power supply fails, the generator turns in. for the sake of our rescue In order to earn the customers' trust, When the queue extends from a a predetermined number, the standby can begin working.

The provision of standby assistance is recommended in order to increase the system's level of service. Every time it is not possible to have the necessary number of spare servers due to volume/cost constraints. In such instances, having extra repairmen on hand is recommended to achieve the appropriate reliability/availability. The availability and reliability of redundant reparable systems are largely dependent on the usual cause failure of operational and backup units in queueing systems. Many environmental variables, like humidity, temperature, vibration, or voltage fluctuation, which are common in much applications, have been seen to induce immediate failure of some or all units of the queueing system. Jain et al. (2003) and Vaurio (2003) are two researchers who have used this notion in their research (2003). Using a two-unit queueing system with a shared failure cause was suggested by Dhillon and Yang in 1992. It can be seen, system dependability is a crucial performance analysis aspect for every manufacturing/production system when it comes to service. Gupta and Tyagi (1986) investigated the dependability of a variety of complicated queueing systems under various failure situations.

Many writers have used the thought of standbys in queueing models to increase the system's operational availability. Goyal and Sharma (1980) Shawky (2000), and Jain et al. (2003) presented stochastic analyses of two unit standby queueing systems with failure of two modes. Sharma and Sharma (1997) investigated the repair of a Markovian queueing system with finite server and standbys with three modes of failure. With a limited number of servers and two potential sources of failure, Jain et al. (2000) and Wang and Wu(1995) provided a cost analysis for the queueing modelling of the Markovian machine repair problem . Based on a queueing model with standbys, Mishra and Shukla (2010) created an optimal analysis for the machine interference problem. The system's total cost function was optimised using the N-R method. Ke et al.(2011) examined a standby and homogenous machine maintenance issue under the presumption that different persons are in charge of overseeing these machines. MRP queueing analysis was performed by Jain and Preeti (2014) using standby machines, server working vacations, and breakdowns. Kumar and Jain (2014) looked into a bi-level policy for dealing with many standbys in a machine repair scenario. They calculated the transient state probabilities using Runge-approach Kutta's and produced several system performance indices. Furthermore, some researchers have included specific types of standbys in their queueing models in order to analyse the realistic behaviour of queueing models.

In this chapter we study two server queue with standby. There are two servers, one server is busy with the main line of service and the other is kept reserved as standby.

The description of the queueing model is as follows:

(i) Arrival pattern is single with Poisson rate λ.

(ii) There are two servers, one is main and the other is standby. The service rate of main server is μ_1, although it is susceptible to failure with a probability of θ (independent from the time it is working). When the main server fails, the standby server's service rate goes to μ_2. As far as the queue length reaches a certain number K, standby additionally begins operating. The main unit's repair time distribution is exponential with parameter ϕ.

(iii)

SOUTION OF THE MODEL

Define :

$Q_0(t)$ = Probability of zero unit in the system at a time 't' when Main server gets failed and standby is working.

$P_0(t)$ = Probability of zero unit in the system at a time 't' when the Main server is operating .

$P_{m,n}(t)$ = Probability of one unit in the system at a time 't' and n unit are waiting when Main server is in operating state.

$P_{B,n}(t)$ = Probability of having n units in the system at a time 't' and both servers are working .

$P_{S,n}(t)$ = Probability that at a time 't' there are n units in the system and standby server is operating.

Difference differential equations representing the system are as follows:

$$Q'_0(t) = -(\lambda+\phi)\, Q_0(t) + \theta\, P_0(t) + \mu_2 P_{S,0}(t) \qquad (6.1)$$

$$P'_0(t) = -(\lambda+\theta)\, P_0(t) + \mu_1 P_0(t) + \phi Q_0(t) \qquad (6.2)$$

$$P'_{M,0}(t) = -(\lambda+\theta+\mu_1)\, P_{M,0}(t) + \mu_1 P_{M,1}(t) + \lambda P_0(t) + \mu_2 P_{S,0}(t) + \phi P_{S,0}(t) \qquad (6.3)$$

$$P'_{M,n}(t) = -(\lambda+\theta+\mu_1)\, P_{M,n}(t) + \mu_1 P_{M,n-1}(t) + \lambda P_{M,n-1}(t) + \mu_2 P_{B,n}(t) + \phi P_{S,n}(t) \qquad (6.4)$$

$$P'_{M,K-1}(t) = -(\lambda+\theta+\mu_1)\, P_{M,K-1}(t) + \mu_2 P_{B,K-1}(t) + \lambda P_{M,K-2}(t) + \phi P_{S,K-1}(t) \qquad (6.5)$$

$$P'_{B,0}(t) = -(\lambda+\theta+\mu_1+\mu_2)\, P_{B,0}(t) + \mu_1 P_{B,1}(t) \qquad (6.6)$$

$$P'_{B,n}(t) = -(\lambda+\theta+\mu_1+\mu_2)\, P_{B,n}(t) + \mu_1 P_{B,n+1}(t) + \lambda P_{B,n-1}(t) \qquad (6.7)$$

$$P'_{B,K-1}(t) = -(\lambda+\theta+\mu_1+\mu_2)\, P_{B,K-1}(t) + (\mu_1+\mu_2) P_{B,K}(t) + \lambda P_{M,K-1}(t) \qquad (6.8)$$

$$P'_{B,n}(t) = -(\lambda+\theta+\mu_1+\mu_2)\, P_{B,n}(t) + (\mu_1+\mu_2) P_{B,n+1} \quad n \geq 1 \qquad (6.11)$$

Define the generating function

$$P_m(z,t) = \sum_{n=0}^{K-1} P_{M,n}(t) z^n \qquad (6.12)$$

$$P_B(z,t) = \sum_{n=0}^{\infty} P_{B,n}(t) z^n \qquad (6.13)$$

$$P_S(z,t) = \sum_{n=0}^{\infty} P_{S,n}(t) z^n \qquad (6.14)$$

Multiply eq. (6.1) to eq. (6.5) by proper power of z and taking Laplace transform and using eq. (6.12), we get

$$\bar{P}_M(z,t) = \frac{\lambda z \bar{P}_0(S) - \mu_1 \bar{P}_{M,0}(S) - \lambda \bar{P}_{M,K-1}(S) z^{K+1} + \phi \sum_{n=0}^{K-1} \bar{P}_{S,n}(S) z^{n+1} + \mu_2 \sum_{n=0}^{K-1} \bar{P}_{B,n}(S) z^{n+1}}{z(\lambda+\theta+\mu_1) - \mu_1 - \lambda z^2} \qquad (6.15)$$

Similarly multiplying eq. (6.6) to eq. (6.11) by suitable power of z and taking Laplace transform and using eq. (6.13), (6.14), we get

$$\bar{P}_B(z,t) = \frac{\lambda \bar{P}_{M,K-1}(S) z^K - \mu_1 \bar{P}_{B,0}(S)}{z(\lambda+\theta+\mu_1+\mu_2) - \mu_1 - \lambda z^2} \qquad (6.16)$$

$$\bar{P}_S(z,t) = \frac{z\mu_2 P_{s,1}(t) + \theta z^2 P_B(z,t) - \mu_2 P_0(z,t) + \lambda Q_0(t)}{z(\lambda+\phi+\mu_2) - \mu_2} \qquad (6.17)$$

STEADY STATE SOLUTION OF THE SYSTEM

We can find the probability generating function (p.g.f) in steady state from (6.15) to (6.17) by using the property of Laplace Transform $\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t)$ we get

$$P_M(z) = \frac{\lambda z P_0 - \mu_1 P_{M,0} - \lambda P_{M,K-1} z^{K+1} + \phi \sum_{n=0}^{K-1} P_{S,n} z^{1+n} + \mu_2 \sum_{n=0}^{K-1} P_{B,n} z^{n+1}}{z(\lambda+\theta+\mu_1) - \mu_1 - \lambda z^2} \qquad (6.18)$$

$$P_B(z) = \frac{\lambda P_{M,K-1} z^K - \mu_1 P_{B,0}}{z(\lambda+\theta+\mu_1+\mu_2) - \mu_1 - \lambda z^2} \qquad (6.19)$$

$$P_S(z) = \frac{z\mu_2 P_{s,1} + \theta z^2 P_B(z) - \mu_2 P_0 + \lambda Q_0}{z(\lambda+\phi+\mu_2) - \mu_2} \qquad (6.20)$$

Eq. (6.1) ,(6.2), (6.6) and (6.11) gives

$$P_{S,0} = \frac{\lambda+\phi}{\mu_2} Q_0 - \frac{\theta}{\mu_2} \qquad (6.21)$$

$$P_{M,0} = \frac{\lambda+\phi}{\mu_1} P_0 - \frac{\phi}{\mu_2} Q_0 \qquad (6.22)$$

$$P_{B,1} = \frac{\lambda+\theta+\mu_1+\mu_2}{\mu_1} P_{B,0} \qquad (6.23)$$

$$P_{B,1} = r^2 P_{B,0}$$

$$P_{S,n} = \left(\frac{\lambda+\mu_2+\phi}{\mu_2}\right)^n P_{S,0} - \frac{\phi}{\mu_1} \sum_{i=0}^{n-1} \left(\frac{\lambda+\mu_2+\phi}{\mu_2}\right)^{n-i-2} P_{B,i} \qquad (6.24)$$

Where

$$r = \frac{(\lambda+\theta+\mu_1+\mu_2) \pm ((\lambda+\theta+\mu_1+\mu_2)^2 - 4\lambda\mu_1)^{1/2}}{2\mu_1} \qquad (6.25)$$

If we put the values of $P_{S,0}$, $P_{M,0}$, $P_{B,n}$ *and* $P_{S,n}$ (n=0,1,2.......K-1) in eq. (6.19) to (6.21) only four unknowns $P_0, Q_0, P_{B,0}$ *and* $P_{M,K-1}$ are left to be determined.

As $P_M(z)$ is a polynomial function of variable z so the numerator must vanish for two zeroes r_1 and r_2 of the denominator. This gives

$$\lambda r_j P_0 - \mu_1 P_{M,0} - \lambda P_{M,K-1} r_j^{K+1} + \phi \sum_{n=0}^{K-1} P_{S,n} r_j^{n+1} + \mu_2 \sum_{n=0}^{K-1} P_{B,n} r_j^{n+1} = 0 \qquad (6.26)$$

(j = 1, 2)

Also, $P_B(z)$ is convergent series in the plane $|z| < 1$ so it must vanish for the roots of denominator those lies inside the unit circle (say r_3) . It gives

$$\lambda P_{M,K-1} r_3^K - \mu_1 P_{B,0} = 0 \qquad (6.27)$$

Since

$P_B(1) + P_S(1) + P_M(1) = 1$, this gives

$$\lambda P_0 - \mu_1 P_{M,0} - \lambda P_{M,K-1} + \phi \textstyle\sum_{n=0}^{K-1} P_{S,n} + \mu_2 \sum_{n=0}^{K-1} P_{B,n} \quad (\mu_2 + \theta)(\lambda + \phi) + (\lambda P_{M,K-1} - \mu_1 P_{B,0})(\lambda + \phi)\phi + (\mu_2 P_{s,1} + \theta P_B(z) - \mu_2 P_0 + \lambda Q_0)(\mu_2 + \theta)\phi = \phi(\mu_2 + \theta)(\lambda + \phi) \qquad (6.28)$$

On Solving eq.(6.26), (6.27) and (6.28), we can find $P_0, Q_0, P_{B,0}$ and $P_{M,K-1}$

OPERATIONAL CHARACTERISTICS

1 Average number of units in the system

Let L_1, L_2, L_3 be the average number of units when both systems are in operative state or when only standby server is working or when the main server is working in the system respectively

$$L_1 = {P'_B(1)}/{P_B(1)}$$

$$P'_B(1) = \frac{(\mu_2+\theta)\lambda P_{M,K-1} - (\lambda P_{M,K-1} - \mu_1 P_{B,0})(\mu_1+\mu_1+\theta-\lambda)}{(\mu_2+\theta)^2} \qquad (6.29)$$

$$L_2 = {P'_S(1)}/{P_S(1)}$$

$$P'_S(1) = \frac{(\lambda+\phi)\,(\mu_2 P_{S,1} + 2\theta P'_B(1)) - (\mu_2 P_{S,1} + \theta P_B(1) - \mu_1 P_0 + \lambda Q_0)(\mu_2+\phi+\lambda)}{(\lambda+\phi)^2} \qquad (6.30)$$

$$\boldsymbol{L_3 = {P'_M(1)}/{P_M(1)}}$$

$$P'_M(1) = \frac{\phi\begin{pmatrix}\lambda P_0 - \lambda(K+1)P_{M,K-1} + \phi\sum_{n=0}^{k-1}(n+1)P_{S,n} + \mu_2\sum_{n=0}^{K-1}(n+1)P_{B,n}) - (\lambda P_0 - \mu_1 P_{M,0} - \lambda P_{M,K-1} + \\ \phi\sum_{n=0}^{k-1}P_{S,n} + \mu_2\sum_{n=0}^{K-1}P_{B,n}\end{pmatrix}(\mu_1+\phi-\lambda)}{(\phi)^2} \qquad (6.31)$$

2. Proportion of time when the main server , both server and standby server remains in operative state

Let E_1 , E_2 , E_3 be the proportion of time when the main server is in working state , both server are operative and standby server is operative respectively.

$$E_1 = P_M(1) = \frac{\lambda P_0 - \mu_1 P_{M,0} - \lambda P_{M,K-1} + \phi \sum_{n=0}^{K-1} P_{S,n} + \mu_2 \sum_{n=0}^{K-1} P_{B,n}}{\phi} \quad (6.32)$$

$$E_2 = P_B(1) = \frac{\lambda P_{M,K-1} - \mu_1 P_{B,0}}{(\mu_2 - \theta)} \quad (6.33)$$

$$E_3 = P_S(1) = \frac{\mu_2 P_{s,1} + \theta P_B(1) - \mu_2 P_0}{(\lambda + \phi)} \quad (6.34)$$

PARTICULAR CASES

When both of the servers are alike i.e. $\mu_1 = \mu_2 = \mu$

Then the equations (6.29) to eq.(6.34) gives

$$P'_B(1) = \frac{(\mu+\theta)\lambda P_{M,K-1} - (\lambda P_{M,K-1} - \mu P_{B,0})(2\mu+\theta-\lambda)}{(\mu+\theta)^2}$$

$$P'_S(1) = \frac{(\lambda+\phi)(\mu P_{S,1} + 2\theta P'_B(1)) - (\mu P_{S,1} + \theta P_B(1) - \mu P_0 + \lambda Q_0)(\mu+\phi+\lambda)}{(\lambda+\phi)^2}$$

$$P'_M(1) = \frac{\phi\left(\begin{array}{c}\lambda P_0 - \lambda(K+1)P_{M,K-1} + \phi \sum_{n=0}^{k-1}(n+1)P_{S,n} + \mu \sum_{n=0}^{K-1}(n+1)P_{B,n}) - (\lambda P_0 - \mu P_{M,0} - \lambda P_{M,K-1} \\ + \phi \sum_{n=0}^{k-1} P_{S,n} + \mu \sum_{n=0}^{K-1} P_{B,n}\end{array}\right) (\mu + \phi - \lambda)}{(\phi)^2}$$

$$E_1 = P_M(1) = \frac{\lambda P_0 - \mu P_{M,0} - \lambda P_{M,K-1} + \phi \sum_{n=0}^{K-1} P_{S,n} + \mu \sum_{n=0}^{K-1} P_{B,n}}{\phi}$$

$$E_2 = P_B(1) = \frac{\lambda P_{M,K-1} - \mu P_{B,0}}{(\mu_2 - \theta)}$$

$$E_3 = P_S(1) = \frac{\mu (P_{s,1} - P_0) + \theta P_B(1)}{(\lambda + \phi)}$$

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