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A case study on the optimization of multi-objective functions transportation model for public transport authority (Egypt).

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Abstract

The government Transport Authority in Cairo, Egypt (CTA) is one of the biggest public transport networks in the capital city and most of the people depend on it for their daily life. So to help the government to continue providing good service to the citizens, we investigated a transportation network for trying redistribution of the buses from their garages to beginning station aims to minimize total travel time and the total transportation cost.

This paper presents a newly proposed method called interactive fuzzy goal programming (IFGB). This leads to a strong method of solving and avoiding all the shortcomings of the interactive fuzzy Multi-objective Transportation Problem (IFMOTP). Thus, an interactive fuzzy goal programming was proposed aiming to find the preferred compromise solution to minimize the total travel time of the buses from their garages to the beginning station and therefore the total transportation cost of the transportation network. The research work shows that the proposed model can be useful for transportation network design through a case study based on the data was collected from the company. The research shows that applying the proposed distribution to a part of the Cairo Transport Authority system can save the time of buses transporting by about 12124 hr per year. This means the proposed distribution could save about 551880 LE per year.

Key Words: Initial feasible solution, interactive Fuzzy Multi-objective Transportation Problem technique, optimal solution.

1. Introduction

Transportation economics is a broad term that includes many applications, such as transportation problems, vehicle replacement strategies, bus scheduling policies, shortest route problems, traffic lights design, and more other applications.

The transportation problem (TP) is classified as a special case of linear programming (LP) and deals with the problems of passenger and goods transportation and distribution. It is important to apply methods of solving the transportation problem, which provides the main three elements included the source of goods, destination, and the transport unit cost of goods from each source to each destination [1].

The classical single objective transportation problem model with equality constraints uses to minimize the total shipping cost which satisfies total supply and demands. This model discusses the fixed amount of supply and demand. But in real transportation problems, most of the problems have inequality and equality type constraints with multiple objectives for example in production inventory, job scheduling, allocation problems, and investment analysis [2,3]. Such problems which often arise as a result of mathematical modeling of many real-life situations are called optimization problems. Hence it is not possible to formulate and solve the transportation problem using traditional methods.

Multi-objective programming (MOTP) or multi-objective optimization is the process of simultaneously optimizing many objectives subject to certain constraints. This means that no one solution can be said to be the optimal solution but a set of efficient solutions presented to the decision-maker (DM) to make the most appropriate decision for his organization.

There are many applications of Multi-objective optimization problems in the real world such as aircraft design, the oil and gas industry, automobile design. In addition, maximizing profit and minimizing the cost of a product; maximizing performance and minimizing vehicle fuel consumption, and minimizing weight while maximizing the strength of a particular component [4].

At the partially fuzzy transportation problem, the fuzziness could exist is in the R.H.S (i.e. in the availabilities of sources and the requirements of destinations) or in the objective function. While in the fully fuzzy transportation problem there is fuzziness in both the R.H.S and the objective function parameters. The process of systematic improvement or optimization of a set of objective functions for the transportation problem is called the multi-objective transportation problem. In this case, it is not necessary to obtain the optimal solution for a single objective and to unify the optimal solution. But it is important to take into consideration a set of objectives (2 or more), to find efficient solutions to these multiple objectives.

Fuzzy sets introduced in 1965 by Zadeh is applied to tackle such uncertain environments [5]. Later on, large different algorithms by other researchers started to solve the fuzzy transportation problem, whether partially or fully.

Lushu and Lai [6] use ordinary optimization techniques to solve fuzzy compromise programming models to obtain a non-dominated compromise solution. Paratane and Bit [7] applied the fuzzy programming techniques with a new exponential membership function to solve multi-objective transportation problems with mixed constraints in-equations instead of usual equations. The proposed method gives efficient solutions and the best compromise solution for multi-objective functions. M. Bagheri et al [8] proposed a fuzzy data envelopment analysis (DEA) approach to solve the fuzzy multi-objective transportation problem by converting it into a standard fuzzy transportation problem. The use of the fuzzy DEA approach provides the simultaneous use of maximization functions in modeling fuzzy multi-objective transportation problems. M. Afwat et al [9] illustrated a new approach called the product approach to solving the multi-objective transportation problems. The proposed method depends on using fuzzy programming to convert different penalty units to membership value, and then aggregate them by product. Jayesh M. [10] presented a genetic algorithm (GA) based solution of a fuzzy transportation problem with multi-objective functions. In addition, the exponential membership function uses to give more feasibility to a decision-maker for better decisions [11, 12].

In a multi-objective transportation problem, all objectives are taken into account for a set of efficient solutions that take into account all constraints and objectives. While there are many methods in which one of the objectives can be weighted at the expense of the other, while the results may not satisfy the decision-maker. Therefore, the decision-maker needs to participate interactively with the programmer and the researcher. This is a so-called interactive fuzzy Multi-objective Transportation Problem (IFMOTP). Abd El-Wahed et al [13] presented an advanced approach to solving interactive fuzzy multi-objective transportation problems. This approach depends on combining the basis of Interactive approaches, goal programming approaches, and fuzzy programming approaches. Tien- Fu Liang [14] developed a novel interactive fuzzy multi-objective linear programming (IFMOLP) model for solving transportation decision (TPD) problems with multiple fuzzy objectives. The proposed model outputs more wide-ranging decision information than other models. Therefore is provides greater computational efficiency.

This paper clarified the limitations and shortcomings of each of the previously mentioned approaches. It also illustrated the advantages of combining these approaches in the proposed method called interactive fuzzy goal programming (IFGB). This leads to a strong method of solving and avoiding all the shortcomings of each approach. The purpose of this method is to find the "preferred compromise solution". This is done by treating the fuzziness in the input data, finding the preferred solution, and presenting the solutions to the decision-maker to reach his satisfactory level. The author supported this paper with numerical examples and solved them using each approach independently. Then these examples are solved by the new method "IFGB" and compared with the optimal solution. The author found that the new method gives better results and is very close to the optimal solution.

2. Transportation Problem

Traditional transportation problems are interested in the distribution of any products from supply points to demand points to minimize the cost of transporting. Assuming (ai) is source parameter may be production facilities, warehouse, etc., whereas (bj) is destination parameter may be a warehouse, sales outlet, etc. The penalty (Cij) that is, the co-efficient of the objective functions, could represent the cost of transportation, consuming time, amount of goods transposed, unfulfilled demand, and many others.

2.1 Multi-Objective Transportation Problem (MOTSP)

In the real application of transportation problems, in reality, the transportation problem involves multiple objective functions. This type of problem is called a multi-objective transportation problem. The mathematical model of multi-objective transportation problems can be modeled are follows.

$$\min f^k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^k X_{ij} \qquad ; \; k = 1,2,\dots,p \tag{5}$$

Subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i, \qquad i = 1,2,\dots,m \tag{6}$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1,2,\dots,n \tag{7}$$

$$x_{ij} \geq 0 \quad \forall \quad i = 1,2,\dots,m, \quad j = 1,2,\dots,n, \quad k = 1,2,\dots,p. \tag{8}$$

Where:

- Cij is the cost of transporting one unit of the commodity from origin i to destination j.
- Xij is the amount to be shipped from origin i to destination j.
- ai is the supply availabilities at origin I.
- bj is the demand requirements at destination j.

The superscript on $f^k(x)$ and C_{ij}^k are used to identify the different objective functions and their coefficients, $a_i > 0$ for all i, $b_j > 0$ for all j, $C_{ij}^k > 0$ for all (i, j and k). The index k denotes the number of objectives.

The balanced condition is both necessary and sufficient for solving the transportation problem in both cases with single and multiple objectives.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{9}$$

In a fuzzy transportation problem, if a multi-objective problem of k objectives is considered, this is called a fuzzy multi-objective transportation problem. So, to obtain the solution, each objective is considered at a time to get the lower and upper bounds for that objective [15]. Let for k^{th} objective, and $l_k \; and \; u_k$ are the lower (min) and upper (max) bounds. The linear membership functions for each fuzzy objective function is defined by equation (10) [5]. The graph of the linear membership functions for equation (10) is shown in figure (1):

$$\mu_k\big(F_k(x)\big) = \begin{cases} 1 & \text{if} \quad F_k(x) \leq L_k \\ \dfrac{U_k - F_k(x)}{U_k - L_k} & \text{if} \quad L_k < F_k(x) < U_k \\ 0 & \text{if} \quad F_k(x) \geq U_k \end{cases} \tag{10}$$

Using the membership function defined above and following the fuzzy decision of Bellman and Zadeh (1970), the MOLPP can be interpreted as follows:

Maximize $\min \mu_k\big(F_k(x)\big)$

Subject to $g_i(x)\{\leq, =, \geq\}b_i$, $i = 1,2,3,\dots,m$ and $x \geq 0$

Introducing an auxiliary variableλ, it can be reduced to the following conventional Linear Programming Problem and can be solved.

Maximize λ

Subject to $\lambda \leq \mu_k\big(F_k(x)\big), j = 1,2,3,\dots k$

$g_i(x)\{\leq, =, \geq\}b_i$, $i = 1,2,3,\dots,m$ and $x \geq 0$

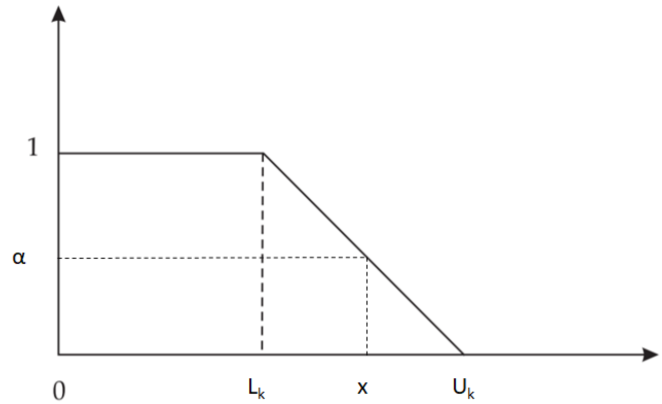

Figure (1) linear membership function

3. Case Study

Cairo Transportation Authority (CTA) considers one of the largest and oldest transportation companies in the Middle East. CTA was originated in 1966. According to the company data, the company fleet of busses includes 3,000 busses travels close to 365,000 kilometers every day. These busses are working on three governorates to serve about 1.5 Million persons every single day. The fleet consumes about 6.5 Million liters of fuel monthly which costs about 40 million Egyptian pounds. On the other hand, the maintenance cost of this fleet reaches about 200 Million Pounds and the tire costs reach about 70 Million Pounds every year.

The company is working on two shifts: Morning and afternoon. The first shift starts it work early morning when the buses are moving from different garages that are owned by the company, where the busses stay to be serviced and maintained, to the start stations where busses begin their daily journey. Each of these garages serves a certain number of busses. The distances that the vehicles travel from garages to the beginning stations are non-productive kilometers. The busses consume a lot of fuel through these distances without any income. In this thesis, we will study this problem to minimize these distances to reduce the fleet fuel consumption and the exhaust gases emission by redistributing these buses on their garages using the transportation technique based on two objectives, the traveling time and cost of fuel consumption are the two objectives that are studied. We need to generate the most ideal distribution based on the two objectives together and then try to improve one of the objectives on the other according to the opinion of the decision-maker which is known as Interactive.

The collected data from the company are not accurate and the numbers of busses which travel from a garage to a starting station are not certain but lie in a range. Therefore, it will use the Interactive Fuzzy Multi-objective Transportation Problem technique (IFMOTP) to solve the problem of the CTA fleet of busses.

4. Methodology

This thesis is studied just 4 sectors out of 7 sectors of the CTA that exist all over the three governorates. The studied 4 sectors are north and south of Cairo, and north and south of Giza. These sectors include 7 CTA garages and 22 starting bus stations.

The interactive fuzzy multi-objective transportation problem technique is used to solve the CTA problem to find the best distribution between CTA garages and starting stations. Both transportation problems, fuzzy, multi-objective, and interactive models. The following is the solution algorithm in step by step to illustrate how the interactive fuzzy multi-objective transportation problem technique is applied. Models and results of each solution step are illustrated in the next sub-tiles below.

1. Collect and prepare all the needed data.
2. Use an Excel solver or any operations research package to find the optimal solution of the time transportation problem which is prepared in Table (7). The right-hand side of availabilities (busses at each garage) and requirements (busses at each starting station) are considered fuzzy numbers as illustrated in Tables (1, 2). From the solution of this problem summarize its basic variables (x_1) and the minimum transportation time in minutes $Z_1(x_1)$.
3. Similarly, find the optimal solution of the transportation problem for the cost of fuel consumption which is prepared in Table (8). Again, the right-hand side of availabilities (busses at each garage) and requirements (busses at each starting station) are considered fuzzy numbers as illustrated in Tables (1, 2). From the solution of this problem summarize its basic variables (x_2) and the minimum transportation cost in LE $Z_2(x_2)$.
4. From steps 2 and 3, calculate the upper and lower bound of each objective by calculating: $Z_1(x_1)$, $Z_1(x_2)$, $Z_2(x_1)$ and $Z_2(x_2)$, where:

$$Z_1(x_1) = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_1 \tag{11}$$

$$Z_1(x_2) = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_2 \tag{12}$$

$$Z_2(x_1) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_1 \tag{13}$$

$$Z_2(x_2) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_2 \tag{14}$$

Where:

- t_{ij}: is the transportation time that was prepared in the time matrix, Table (7).
- c_{ij}: is the cost of fuel consumption which was prepared in the cost matrix, Table (8).
- x_1: is the solution of the time transportation problem as illustrated previously in step 2.
- x_2: is the solution of the cost transportation problem as illustrated previously in step 3.
- m and n represents the number of garages and the starting stations that is (m=7 and n=22) in our case study.

5. Use the values of $Z_1(x_1)$, $Z_1(x_2)$, $Z_2(x_1)$ and $Z_2(x_2)$ to find the lower and upper bounds of each objective L_k and U_k to generate new membership functions (μ_1) and (μ_2) to be used for the fuzzy multi-objective transportation problem as shown in equation (10):
6. Prepare the fuzzy multi-objective transportation problem model based on the values of (μ_1) and (μ_2) and solve it. From the generated basic variables of this solution (x_{ij}), calculate Z_1 for total time and Z_2 for the total cost of fuel consumption.
7. Show the solution and the calculated values of Z_1 and Z_2 to the decision-maker. If the decision-maker agrees to these results, stop. Otherwise, go to step 8 below.
8. Prepare the interactive fuzzy multi-objective transportation problem model based on the objective which the decision-maker needs to improve, and solve it. Calculate the new values of Z_1 and Z_2 and show these values to the decision-maker. If the decision-maker agrees for these results, stop. Otherwise, repeat this step (step 8) till there is a solution that meets the decision-maker agreement.

The following sections illustrate the models and the solutions of each step of the solution algorithm.

- Data Collection

The collected data can be summarized to

1. Numbers of garages in chosen sectors
2. Numbers of the starting stations.
3. The capacity of buses in each garage.
4. The capacity of buses in starting bus stations
5. The distance between each garage and starting station.
6. The current plan for the distribution of buses in their garages.
7. Average fuel consumption for buses according to their life.

Some of the required data were taken from the CTA and other data can't be taken from CTA, so it was obtained from another way. This data is the distance from the garage to each starting bus station.

- Bus Capacity in Garages and Starting Bus Stations
- According to the collected data about the bus capacity between the garages and their starting bus stations, the number of busses at leaves garages and reaches busses destinations changes from a day today, it is illustrated in the following two Tables (1,2). Table (1) shows the maximum and the minimum number of busses that leave each garage every day to the starting bus stations. Table (2) illustrates the maximum and the minimum number of busses that reach the starting bus stations every day.

Table (1) Upper and lower bus capacity leave the garages.

	G1	G2	G3	G4	G5	G6	G7
Upper	58	170	272	84	170	288	114
lower	48	126	188	74	124	160	90

Table (2) Upper and lower bus capacity for the destinations

Destination	Upper	lower	Destination	Upper	lower
S1	90	64	S12	30	20
S2	58	40	S13	18	12
S3	12	10	S14	30	20
S4	34	26	S15	64	44
S5	50	36	S16	76	52
S6	40	28	S17	114	82
S7	160	114	S18	20	14
S8	50	34	S19	46	32
S9	30	20	S20	12	10
S10	34	24	S21	64	44
S11	24	16	S22	100	68

4.1 The distances matrix

The distances matrix between each garage and each starting bus station was created, from which the transportation network will be configured for traveling time and cost of fuel consumption. Due to the inability to obtain distances between garages and starting stations from the company's data, it has been calculated manually using the Cairo Map and Google Maps. Table (6) illustrates the measured distances between each garage and each starting station in kilometers (Km).

4.2 Time Matrix between Garages and Starting Stations

According to CTA data, buses in the morning do not increase the speed of the journey from the garage to the starting bus stations more than 60 km/h and not less than 40 km/h. So, due to lower and higher speeds and stop times at traffic lights and for other reasons, the average speed of the busses was considered to be 50 km/has a fixed speed for all buses.

Based on the distance matrix in Table (6) and the average speed of 50 km/h, the time matrix was prepared (time = distance/speed) and constructed in the following table. Table (7) gives the time required for a bus to reach from each garage to each starting bus station in minutes (min).

5. Actual Distribution

The current (actual) distribution of buses from the company's garages to the starting stations varies almost daily. The current distribution of busses in one day on the studied sectors could be collected. Then both the related total time matrix and the related total cost of fuel consumption matrix were calculated. Both the total time and total cost of fuel consumption (Z_1 and Z_2) were obtained by multiplying c_{ij} of each object (time and cost) by x_{ij} (current distribution of buses). The values of Z_1 and Z_2 were found to be: (Z_1 = 7685 minutes and Z_2 = 6003 LE). Table (9) summarizes the current distribution of buses from CTA garages to the starting bus stations.

6. Solution of the Time Transportation Problem

Since the right-hand side of the problem availabilities and requirements are in fuzzy nature, the problem is formulated and solved as a fuzzy T.P. The fuzzy transportation problem model with fuzzy numbers on the right-hand side is as follows:

Model (1):

$$\text{Min. } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad (15)$$

Subject To:

$$\sum_{i=1}^{m} x_{ij} = \left(\underline{b_j}, \overline{b_j}\right), j = 1,2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = \left(\underline{a_j}, \overline{a_j}\right), j = 1,2, \ldots, m \quad (16)$$

$x_{ij} \geq 0$, i=1,2,…,m and j=1,2,…,n

Where c_{ij} is the transportation unit costs. It is the transportation time (t_{ij}) for this section. These data are prepared in Table (7). x_{ij} is are the number of locations between sources and destinations. This represents the number of busses that are allocated from CTA garages to the starting stations. These amounts need to be calculated by the model.

$\underline{b_j}$ and $\overline{b_j}$ are the lower and upper limits (respectively) of the right-hand side of the destinations. These are the lower and upper number of busses that reach CTA starting stations. These data were prepared in Table (1). $\underline{a_j}$ and $\overline{a_j}$ are the lower and upper limits (respectively) of the right-hand side of the sources. These are the lower and upper number of busses that leave CTA garages. These data were prepared in Table (2). Where m and n donate for the number of sources and destinations. These are 7 and 22 respectively.

Excel Solver was used to solving model (1) to find the optimal distribution of busses that minimizes the total transportation time. Table (9) illustrates the solution of model (1).

7. Transportation problem solution with fuel consumption costs

The right-hand side of the problem availabilities and requirements are in fuzzy nature, the problem is formulated and solved as a fuzzy T.P. As mentioned above, the fuzzy transportation problem model with fuzzy numbers on the right-hand side. Model (1) is applied to solve this problem as well but with the following definitions:

c_{ij} is the transportation unit costs. This is the cost of fuel consumption (c_{ij}) in this section. These data are prepared in Table (8). While x_{ij} is the number of locations between sources and destinations. This represents the number of busses that are allocated from CTA garages to the starting stations. These amounts need to be calculated by the model.

$\underline{b_j}$ and $\overline{b_j}$ are the lower and upper limits (respectively) of the right-hand side of the destinations. These are the lower and upper number of busses that reach CTA starting stations. These data were prepared in Table (2). $\underline{a_j}$ and $\overline{a_j}$: are the lower and upper limits (respectively) of the right-hand side of the sources. These are the lower and upper number of busses that leave CTA garages. These data were prepared in Table (1). While m and n: the number of sources and destinations. These are 7 and 22 respectively.

Excel Solver was used to solving model (1) to find the optimal distribution of busses that minimizes the total cost of fuel consumption. Table (11) illustrates the solution of model (1).

8. Membership Functions for the Fuzzy Multi-Objective Model

After solving the fuzzy single objective problems of both time and fuel consumption cost, it is time to prepare the model of the fuzzy multi-objective transportation problem. Before we start we need to calculate the upper and lower bounds for each objective.

Assuming that time is the first objective and cost of fuel consumption is the second objective, the upper and lower limits are calculated as follows:

- The lower bound for the first objective "Time" is generated from the optimal solution for its single objective model in section (6). Let us call it $Z_1(x_1)$ and it equals 5677 minutes.
- The lower bound for the second objective "Cost" is generated from the optimal solution for its single objective model in section (6) Let us call it $Z_2(x_2)$ and it equals 4469 LE.
- The upper bound for the first objective is obtained by multiplying t_{ij} for the first objective (time required for the bus to reach from each garage to each starting station) by x_{ij} for the second objective (the optimal distribution of buses depending on the cost of fuel consumption). Let us call it $Z_1(x_2)$ and it equals 5778 minutes.
- The upper bound for the second objective is obtained by multiplying c_{ij} for the second objective (cost of fuel consumption per bus from each garage to each service start station) by x_{ij} for the first objective (optimal distribution of buses depending on time). Let us call it $Z_2(x_1)$ and it equals 4609 L.E.

$Z_1(X_1)$	5677
$Z_1(X_2)$	5778
$Z_2(X_1)$	4609
$Z_2(X_2)$	4469

Therefore, the aspiration levels for each objective are defined from the above values by evaluating the maximum and minimum values of each objective. These aspiration levels are:

$$5677 <= F_1 <= 5778$$

$$4469 <= F_2 <= 4609$$

Now, the membership functions of the first and second objective functions can be generated based on equation (10), the two membership functions are prepared as follows:

$$\mu_1(F_1(x)) = \begin{cases} 1 & \text{if} \quad F_1(x) \leq 5677 \\ \dfrac{5778 - F_1(x)}{101} & \text{if} \quad 5677 < F_1(x) < 5778 \\ 0 & \text{if} \quad F_1(x) \geq 5778 \end{cases} \tag{17}$$

$$\mu_2(F_2(x)) = \begin{cases} 1 & \text{if} \quad F_2(x) \leq 4469 \\ \dfrac{4609 - F_2(x)}{140} & \text{if} \quad 4469 < F_2(x) < 4609 \\ 0 & \text{if} \quad F_2(x) \geq 4609 \end{cases} \tag{18}$$

9. Fuzzy Multi-objective Transportation Problem Model

In this step, we will solve the fuzzy multi-objective transportation problem (FMOTP) based on the previous step. Before introducing the FMOTP model, let’s introduce the multi-objective transportation problem MOTP, without fuzzy numbers, first. MOTP formulated as follows [12]:

Model (2):

$$\text{Min. } Z^k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k \, x_{ij} \tag{19}$$

Subject To:

$$\sum_{j=1}^{n} x_{ij} = a_i \,, i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \,, j = 1, 2, \ldots, n \tag{20}$$

$$x_{ij} \geq 0, i = 1,2, \ldots, m; j = 1,2, \ldots, n; \text{and } k = 1, 2, \ldots, K$$

Model (2) is modified by introducing an auxiliary variable (λ) where ($0 \leq \lambda \leq 1$), to generate the following fuzzy multi-objective transportation problem (FMOTP) model.

Model (3):

$$\text{Max. } \lambda$$
$$\lambda \le \mu_k(F_k(x))$$
$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\dots,m.$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\dots,n. \qquad (21)$$
$$x_{ij} \ge 0 \ \forall\, j = 1,2,\dots,n, \ \ k = 1,2,\dots,K.$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$
$$a_i > 0 \ \forall\, i, \quad b_j > 0\ \forall\, j, \quad c_{ij}^k > 0 \ \forall\, i,j.$$

After solving the above-extended form, we obtain the values of (F_1, F_2) for both objectives by multiplying the unit cost of each object (Time and Cost) by x_{ij} (the new generated efficient distribution from the FMOTP model). These values of F_1 and F_2 were found to be F_1 = 5692 minutes, F_2 = 4491 L.E.

10. Interactive Fuzzy Multi-objective Transportation Problem

The idea of the interactive approach is to generate an efficient solution for the multi-objective problem. Then show this solution to the company's decision-maker to confirm if the generated solution meets his/her satisfaction or not based on the values of F1 and F2. If the decision-maker is satisfied with the solution, that is fine. Otherwise, another new solution should be generated to minimize the objective value that does not meet his/her satisfaction. Continuous improvements with new solutions should be carried out till the decision-maker is completely satisfied.

In general, with the generated new solutions with the interactive model, one of the two objectives is improved at the expense of the other. This means that one of the obtained values (F1, F2) from MOTP improves (decreases) while the other value of objective increases.

To illustrate how the interactive approach works, a solution algorithm for IFMOTP is prepared then two scenarios are below.

11. IFMOTP Solution Algorithm

Here are the steps of the solution algorithm:

Step 1: Solve FMOTP as a single fuzzy objective transportation problem taking each time only one objective function.

Step 2: Find the value of each objective function at each solution computed in step

Step 3: Decide the aspiration levels for each objective function by evaluating the maximum and minimum values of each objective function. So, determine its best lower bounds (L_k) and worst upper bounds (U_k) according to the set of optimal solutions.

Step 4: Construct the membership function for each objective function using the formulas in section (7).

Step 5: Construct model (3) to solve the fuzzy multi-objective transportation problem and solve it for integer solution.

Step 6: Give the solution to the decision-maker. If the decision-maker accepts it, go to step 8. Otherwise, go to step 7.

Step 7: Find the value of each objective function at the solution obtained in step 5. Compare the upper bound of each objective function with the new value of the objective function. If the new value is lower than the upper bound and the decision-maker is not fully satisfied with the solution, then there are two possibilities for the decision-maker to accept the solution:

i. If the decision-maker accepts the solution, then the algorithm is finished then go to step 8.

ii. If the decision-maker is partially satisfied with the solution and needs improvement in the value of some objective functions only. Consider new upper bound only for the function that requires the improvement. Otherwise, keep the old one as it is.

Step 8: STOP.

Figure (2) below illustrates a flow chart indicating how to apply the IFMOTP solution algorithm for real problems.

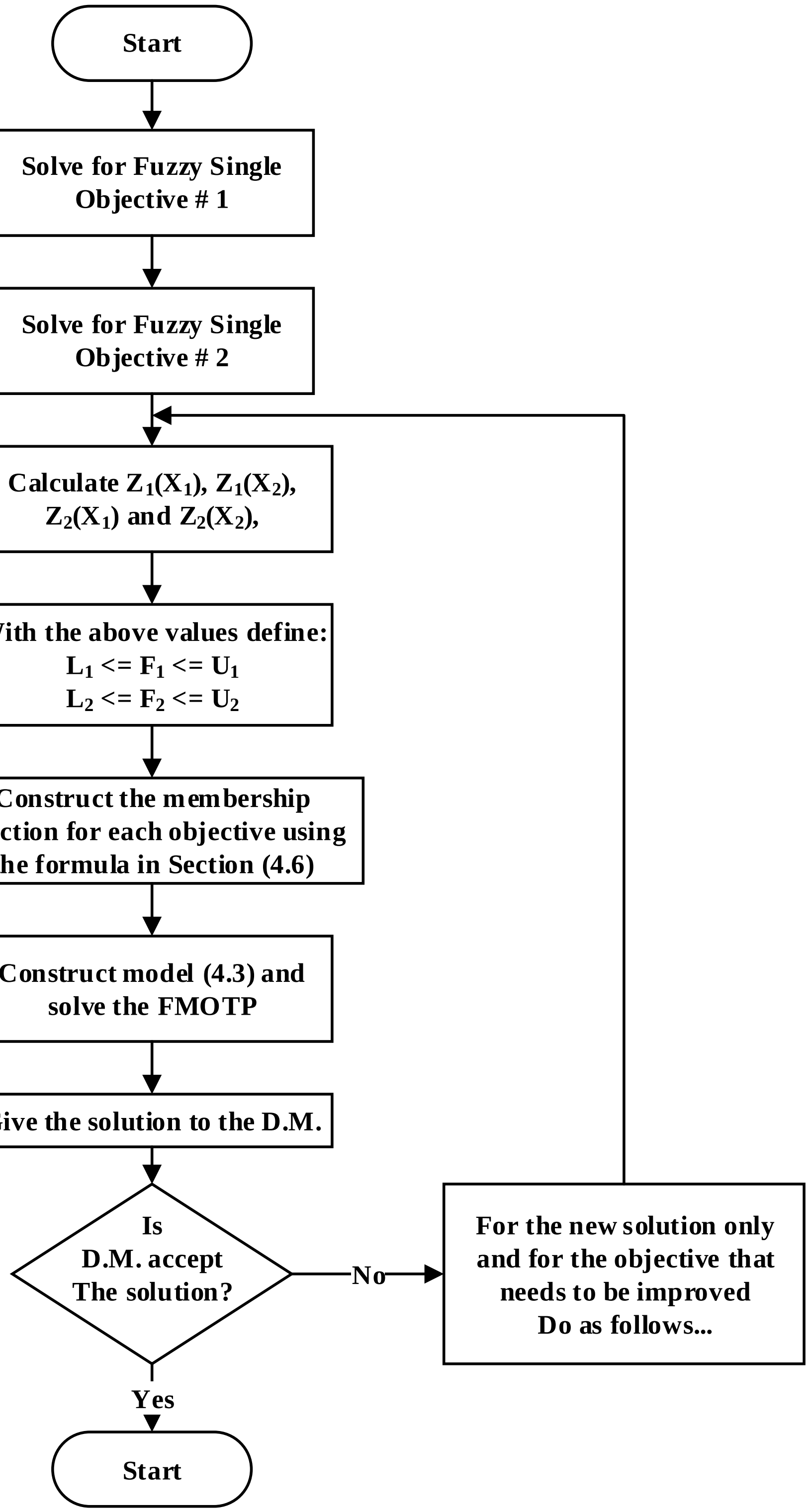

Figure (2) IFMOTP flowchart.

12. Interactive FMOTP "Scenario A"

In scenario "A" the decision-maker needs more improvement for the second objective value. The decision-maker here needs to reduce the costs of fuel consumption.

So, the upper and lower bounds of each objective function can be re-written as follows:

$$5677 <= F_1 <= 5778$$

$$4469 <= F_2 <= 4491$$

The membership function for first objective will not change but the second one will be:

$$\mu_2(F_2(x)) = \begin{cases} 1 & \text{if} \quad F_2(x) \leq 4469 \\ \dfrac{4491 - F_2(x)}{22} & \text{if} \quad 4469 < F_2(x) < 4491 \\ 0 & \text{if} \quad F_2(x) \geq 4491 \end{cases} \quad (22)$$

Table (12) illustrates the ideal distribution of IFMOTP "Scenario A" results. Then we obtain the ideal solution (F_1, F_2) by multiplying c_{ij} for each object (time and cost of fuel consumption) by x_{ij} (ideal distribution for IFMOTP Scenario A). These values were found to be:

$F_1 = 5701$ minutes and $F_2 = 4474$ L.E.

13. Interactive FMOTP "Scenario B"

In this scenario, the decision-maker needs improvements for the first objective only, after solving the FMOTP model.

So, the upper and lower bounds of each objective can be re-written as follows:

$$5677 <= F_1 <= 5692$$

$$4469 <= F_2 <= 4609$$

The membership function for the second objective will not change but the first one will be:

$$\mu_1(F_1(x)) = \begin{cases} 1 & \text{if} \quad F_1(x) \leq 5677 \\ \dfrac{5692 - F_1(x)}{15} & \text{if} \quad 5677 < F_1(x) < 5692 \\ 0 & \text{if} \quad F_1(x) \geq 5692 \end{cases} \quad (23)$$

Table (13) illustrates the ideal distribution of IFMOTP "Scenario B" results. Then we obtain the ideal solution (F_1, F_2) by multiplying c_{ij} for each object (time and cost of fuel consumption) by x_{ij} (ideal distribution for IFMOTP Scenario B). These values were found to be:

$F_1 = 5681$ minutes and $F_2 = 4515$ L.E.

14. Results and discussion

Fuzzy objective 2, multi-objective, inter scenario A and scenario B for the value of the first objective in minutes and the related improvements at each step in percentage. Tables (3:5) illustrate a comparison between current “actual” distribution, fuzzy objective 1, fuzzy objective 2, multi-objective, inter scenario A, and scenario B for the value of the second objective in Egyptian Pounds (L.E.) and the related improvements at each step in percentage. Figure (2) illustrates these comparisons as a diagram.

Table (3) Comparison of different steps results for the first objective.

	Z_1	Improvement Percentage
Current “Actual” Distribution	7685	0.00%
Fuzzy Objective 1	5677	26.13%
Fuzzy Objective 2	5778	24.81%
Multi-objective	5692	25.93%
Inter Scenario A	5701	25.82%
Inter Scenario B	5681	26.08%

Table (4) Comparison of different steps results for the second objective.

	Z2	Improvement Percentage
Current “Actual” Distribution	6003	0.00%
Fuzzy Objective 1	4609	23.22%
Fuzzy Objective 2	4469	25.55%
Multi-objective	4491	25.19%

Inter Scenario A	4474	25.47%
Inter Scenario B	4515	24.79%

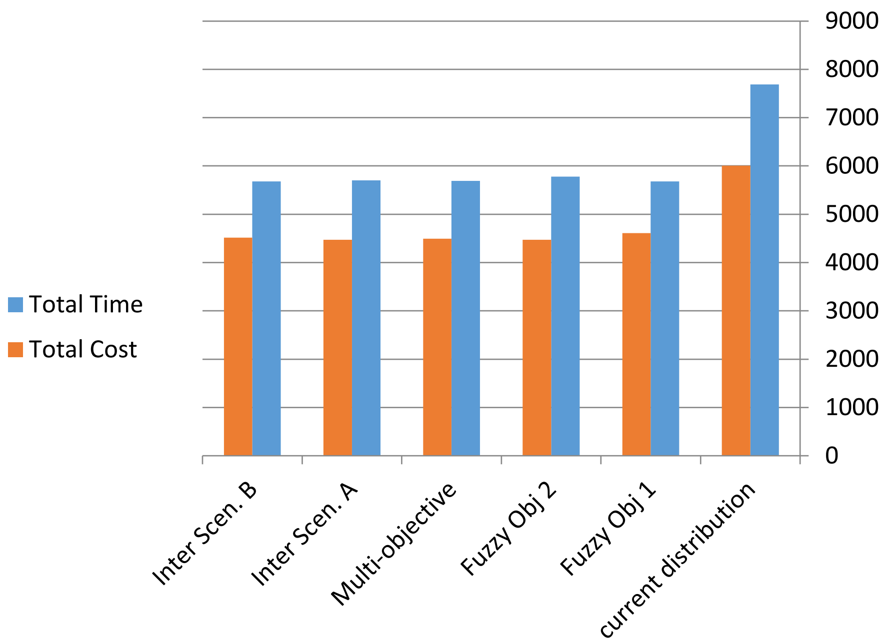

Figure (2) Comparison of different steps results for both objectives.

Table (5) Improvement over the fuel consumption cost of the network per year in L.E. and the total times dissipated.

	Time (min)	Time (hour)	Cost (LE)
Current "Actual" Distribution	-	-	-
Fuzzy Objective 1	732920	12215	508810
Fuzzy Objective 2	696055	11601	559910
Multi-objective	727445	12124	551880
Inter Scenario A	724160	12069	558085
Inter Scenario B	731460	12191	543120

15. Conclusions

In most real-world situations, the relevant parameters like supply, demand, transportation cost, and transportation time, are imprecise due to the impact of different reasons, such as incomplete information. To overcome such situations, TP with fuzzy multi-objective functions has been used in a way that functions would contradict each other. This paper proposed a new method called interactive fuzzy goal programming (IFGB). The purpose of this method is to find the "preferred compromise solution". This is done by treating the fuzziness in the input data, finding the preferred solution, and presenting the solutions to the decision-maker to reach his satisfactory level. By using the interactive fuzzy goal programming (IFGB) in the present case study, we could achieve an improvement ratio of 24% up to 26% of the current distribution for these objectives, this means, the proposed distribution could save about 12124 hrs per year or about 551880 LE per year.

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