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NEUTROSOPHIC BETA OMEGA MAPPING IN NEUTROSOPHIC TOPOLOGICAL SPACES

S. Pious Missier[#1], A. Anusuya[#2], A. Nagarajan[#3]

[#1] Head & Associate Professor, Department of Mathematics, Don Bosco College of Arts and Science,

(Affiliated to Manonmaniam Sundaranar University, Abishekabatti, Tirunelveli) Keela Eral, Thoothukudi,

Tamil Nadu-628 908, India.

[#2] Research Scholar(Reg.No-19222232092024), V.O.Chidambaram College,

(Affiliated to Manonmaniam Sundaranar University, Abishekabatti, Tirunelveli), Thoothukudi-628 008, India.

[#1] Head & Associate Professor, Department of Mathematics, V.O.Chidambaram College,

(Affiliated to Manonmaniam Sundaranar University, Abishekabatti, Tirunelveli), Thoothukudi-628 008

Abstract: In this paper, we introduce the concepts of neutrosophic beta omega open mapping,neutrosophic beta omega closed mapping. Also, we analyze the properties of these mappings. Furthermore, we study the relation between the neutrosophic beta omega open mapping with the already neutrosophic open mapping.

Keywords: neutrosophic beta omega open mapping, neutrosophic beta omega closed mapping.

I. Introduction

Fuzzy set theory introduced by Zadeh[15] has laid the foundation for the new mathematical theories in the research of mathematics. Later, The concept "neutrosophic set" was first given by Smarandache[5]. Open mapping and closed mapping play vital roles in neutrosophic topological space. In 2017, Arokiarani[2] established neutrosophic open map. Late, Atkinswestley[4] studied Neutrosophic, open map and closed map. In this paper, we studied about neutrosophic beta omega open mapping, neutrosophic beta omega closed mapping.

II. Preliminaries

Definition 2.1. [5] Let Δ_N be a non-empty fixed set. A neutrosophic set(NS) G_N is an object having the form $G_N = \{<\xi, \mu_{G_N}(\xi), \sigma_{G_N}(\xi), \upsilon_{G_N}(\xi)>: \xi \in \Delta_N\}$ where $\mu_{G_N}(\xi)$, $\sigma_{G_N}(\xi)$ and $\upsilon_{G_N}(\xi)$ represent the degree of membership, degree of indeterminacy and the degree of nonmembership respectively of each element $\xi \in \Delta_N$ to the set G_N. A neutrosophic set $G_N = \{<\xi, \mu_{G_N}(\xi), \sigma_{G_N}(\xi), \upsilon_{G_N}(\xi)>: \xi \in \Delta_N\}$ can be identified as an ordered triple $<\mu_{G_N}, \sigma_{G_N}, \upsilon_{H_N}>$ in $]^{-}0, 1^{+}[$ on Δ_N.

Definition 2.2. [2] For any two sets G_N and H_N,

1. $G_N \subseteq H_N \Leftrightarrow \mu_{G_N}(\xi) \le \mu_{H_N}(\xi), \sigma_{G_N}(\xi) \le \sigma_{H_N}(\xi)$ and $\upsilon_{G_N}(\xi) \ge \upsilon_{H_N}(\xi), \xi \in \Delta_N$
2. $G_N \cap H_N = <\xi, \mu_{G_N}(\xi) \wedge \mu_{H_N}(\xi), \sigma_{G_N}(\xi) \wedge \sigma_{H_N}(\xi), \upsilon_{G_N}(\xi) \vee \upsilon_{H_N}(\xi)>$
3. $G_N \cup H_N = <\xi, \mu_{G_N}(\xi) \vee \mu_{H_N}(\xi), \sigma_{G_N}(\xi) \vee \sigma_{H_N}(\xi), \upsilon_{G_N}(\xi) \wedge \upsilon_{H_N}(\xi)>$
4. $G_N{}^C = \{<\xi, \upsilon_{G_N}(\xi), 1 - \sigma_{G_N}(\xi), \mu_{G_N}(\xi)>: \xi \in \Delta_N\}$
5. $0_N = \{<\xi, 0, 0, 1>: \xi \in \Delta_N\}$
6. $1_N = \{<\xi, 1, 1, 0>: \xi \in \Delta_N\}$.

Definition 2.3. [14] A neutrosophic topology (NT) on a non-empty set Δ_N is a family τ_N of neutrosophic subsets in Δ_N satisfies the following axioms:

1. $0_N, 1_N \subseteq \tau_N$
2. $G_{N_1} \cap G_{N_2} \subseteq \tau_N$ for any $G_{N_1}, G_{N_2} \subseteq \tau_N$
3. $\cup G_{N_i} \subseteq \tau_N$ where $\{G_{N_i} : i \subseteq J\} \subseteq \tau_N$

Here, the pair (Δ_N, τ_N) is a neutrosophic topological space (NTS) and any neutrosophic set in τ_N is known as a neutrosophic open set (N-open set) in Δ_N. A neutrosophic set G_N is a neutrosophic closed set (N-closed set) if and only if its complement G_N^C is a neutrosophic open set in Δ_N.

Definition 2.4. [2] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic open (N-open) if $f(G_N)$ is N-open set in (Γ_N,σ_N) for every N-open set G_N in (Δ_N,τ_N).

Definition 2.5. [12] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic beta open (Nβ-open) if $f(G_N)$ is Nβ-open set in (Γ_N, σ_N) for every N-open set G_N in (Δ_N,τ_N).

Definition 2.6. [4] A mapping $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic generalized star open (NG*-open) if $f(G_N)$ is NG*-open set in (Γ_N,σ_N) for every N-open set G_N in (Δ_N,τ_N).

Definition 2.7. [13] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic pre open (NP-open) if $f(G_N)$ is NP-open set in (Γ_N,σ_N) for every N-open set G_N in (Δ_N,τ_N).

Definition 2.8. [8] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic generalized open (NG-open) if $f(G_N)$ is NG-open set in (Γ_N, σ_N) for every N-open set G_N in (Δ_N,τ_N).

Definition 2.9. [10] A mapping $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is called neutrosophic closed (N-Closed) if $f(G_N)$ is N-closed set in (Γ_N,σ_N) for every N-closed set G_N in (Δ_N,τ_N).

Definition 2.10. [4] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic generalized star closed(NG*-closed) if $f(G_N)$ is NG*-closed set in (Γ_N,σ_N) for every N-closed set G_N in (Δ_N,τ_N).

Definition 2.11. [13] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic pre closed (NP-closed) if $f(G_N)$ is NP-closed set in (Γ_N,σ_N) for every N-closed set G_N in (Δ_N,τ_N).

Definition 2.12. [8] A mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is called neutrosophic generalized closed (NG-closed) if $f(G_N)$ is NG-closed set in (Γ_N, σ_N) for every N-closed set G_N in (Δ_N,τ_N).

Definition 2.13. [11] A neutrosophic set G_N of a neutrosophic topological space (Δ_N,τ_N) is called neutrosophic beta omega closed (Nβω-Closed) if $\beta cl_N(G_N) \subseteq U_N$ whenever).$G_N \subseteq U_N$ and U_N is Nω-Open in (Δ_N,τ_N).

III. Neutrosophic Beta Omega Open Mapping

Definition 3.1. A mapping $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is Nβω-open if image of every N-open set of (Δ_N,τ_N) is Nβω-open set in (Γ_N,σ_N).

Example 3.1. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.7},\frac{\lambda_3}{0.6}\right),\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.6},\frac{\lambda_3}{0.7}\right),\left(\frac{\lambda_1}{0.3},\frac{\lambda_2}{0.2},\frac{\lambda_3}{0.3}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.7},\frac{\delta_3}{0.8}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.5},\frac{\delta_3}{0.6}\right),\left(\frac{\delta_1}{0.4},\frac{\delta_2}{0.3},\frac{\delta_3}{0.4}\right)>$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping.

Theorem 3.1. Every N-open mapping is Nβω-open but the converse may not be true.

Proof. Let $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \phi_N)$ be any N-open mapping. Let U_N be a N–open set in (Δ_N,τ_N). Then $f(U_N)$ is N-open, since f is N-open. This implies $f(U_N)$ isNβω-open. Hence f is a Nβω-open mapping.

Example 3.2. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.6},\frac{\lambda_2}{0.7},\frac{\lambda_3}{0.6}\right),\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.6},\frac{\lambda_3}{0.7}\right),\left(\frac{\lambda_1}{0.3},\frac{\lambda_2}{0.2},\frac{\lambda_3}{0.3}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.8},\frac{\delta_2}{0.7},\frac{\delta_3}{0.8}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.5},\frac{\delta_3}{0.6}\right),\left(\frac{\delta_1}{0.4},\frac{\delta_2}{0.3},\frac{\delta_3}{0.4}\right)>$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping. But f is not a N-open mapping, since G_N is N-open in (Δ_N,τ_N) but $f(G_N)$ is not N-open in (Γ_N,σ_N).

Theorem 3.2. Every NG*-open mapping is a Nβω-open mapping but not conversely.

Proof. Let U_N be a N-open set in (Δ_N, τ_N) and f be a NG*-open mapping. Then $f(U_N)$ is NG*-open. This implies $f(U_N)$ is Nβω-open. Hence f is a Nβω-open mapping.

Example 3.3. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.4},\frac{\lambda_2}{0.4},\frac{\lambda_3}{0.3}\right),\left(\frac{\lambda_1}{0.4},\frac{\lambda_2}{0.4},\frac{\lambda_3}{0.4}\right),\left(\frac{\lambda_1}{0.8},\frac{\lambda_2}{0.7},\frac{\lambda_3}{0.8}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.3},\frac{\delta_2}{0.2},\frac{\delta_3}{0.3}\right),\left(\frac{\delta_1}{0.3},\frac{\delta_2}{0.4},\frac{\delta_3}{0.3}\right),\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.7},\frac{\delta_3}{0.6}\right)>$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N,\tau_N)\rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping. But f is not a NG*-open mapping, since G_N is N-open in (Δ_N,τ_N) but $f(G_N)$ is not NG*-open in (Γ_N,σ_N).

Theorem 3.3. Every NP-open mapping is a Nβω-open mapping but not conversely.

Proof. Let U_N be a N-open set in (Δ_N, τ_N) and f be a NP-open mapping. Then $f(U_N)$is Nβω-open. This implies $f(U_N)$ is Nβω-open. Hence f is a Nβω-open mapping.

Example 3.4. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.2},\frac{\lambda_2}{0.3},\frac{\lambda_3}{0.2}\right),\left(\frac{\lambda_1}{0.2},\frac{\lambda_2}{0.3},\frac{\lambda_3}{0.2}\right),\left(\frac{\lambda_1}{0.6},\frac{\lambda_2}{0.7},\frac{\lambda_3}{0.6}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.2},\frac{\delta_3}{0.2}\right),\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.2},\frac{\delta_3}{0.2}\right),\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.8},\frac{\delta_3}{0.7}\right)>$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping. But f is not a NP-open mapping, since G_N is N-open in (Δ_N,τ_N)but $f(G_N)$ is not NP-open in (Γ_N,σ_N).

Theorem 3.4. Every Nβ-open mapping is a Nβω-open mapping but not conversely.

Proof. Let U_N be a N-open set in (Δ_N, τ_N) and f be a Nβ-open mapping. Then $f(U_N)$ is Nβ-open. This implies $f(U_N)$ is Nβω-open. Hence f is a Nβω-open mapping.

Example 3.5. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = < \xi, \left(\frac{\lambda_1}{0.1}, \frac{\lambda_2}{0.1}, \frac{\lambda_3}{0.1}\right), \left(\frac{\lambda_1}{0.2}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.2}\right), \left(\frac{\lambda_1}{0.8}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.8}\right) >$ and $H_N = < \xi, \left(\frac{\delta_1}{0.2}, \frac{\delta_2}{0.2}, \frac{\delta_3}{0.1}\right), \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}\right), \left(\frac{\delta_1}{0.7}, \frac{\delta_2}{0.6}, \frac{\delta_3}{0.7}\right) >$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping. But f is not a Nβ-open mapping, since G_N is N-open in (Δ_N, τ_N) but $f(G_N)$ is not Nβ-open in (Γ_N, σ_N).

Remark 3.1. NG-open mapping and Nβω-open mapping are independent.

Example 3.6. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = < \xi, \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.2}, \frac{\lambda_3}{0.3}\right), \left(\frac{\lambda_1}{0.3}, \frac{\lambda_2}{0.3}, \frac{\lambda_3}{0.3}\right), \left(\frac{\lambda_1}{0.9}, \frac{\lambda_2}{0.8}, \frac{\lambda_3}{0.9}\right) >$ and $H_N = < \xi, \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.4}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.4}\right) >$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a NG-open mapping. But f is not a Nβω-open mapping, since G_N is N-open in (Δ_N, τ_N) but $f(G_N)$ is not Nβω-open in (Γ_N, σ_N).

Example 3.7. Let $\Delta_N = \{\lambda_1, \lambda_2, \lambda_3\}$, $\Gamma_N = \{\delta_1, \delta_2, \delta_3\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = < \xi, \left(\frac{\lambda_1}{0.5}, \frac{\lambda_2}{0.4}, \frac{\lambda_3}{0.5}\right), \left(\frac{\lambda_1}{0.5}, \frac{\lambda_2}{0.5}, \frac{\lambda_3}{0.5}\right), \left(\frac{\lambda_1}{0.6}, \frac{\lambda_2}{0.6}, \frac{\lambda_3}{0.6}\right) >$ and $H_N = < \xi, \left(\frac{\delta_1}{0.8}, \frac{\delta_2}{0.7}, \frac{\delta_3}{0.8}\right), \left(\frac{\delta_1}{0.6}, \frac{\delta_2}{0.5}, \frac{\delta_3}{0.6}\right), \left(\frac{\delta_1}{0.4}, \frac{\delta_2}{0.3}, \frac{\delta_3}{0.4}\right) >$. Then τ_N and σ_N are NTs. Define a mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ by $f(\lambda_1) = \delta_1$, $f(\lambda_2) = \delta_2$ and $f(\lambda_3) = \delta_3$. Then f is a Nβω-open mapping. But f is not a NG-open mapping, since G_N is N-open in (Δ_N, τ_N) but $f(G_N)$ is not NG-open in (Γ_N, σ_N).

Theorem 3.5. A mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ is Nβω-open if and only if for every N-open set G_N of (Δ_N, τ_N), $f(int_N(G_N)) \subseteq N\beta\omega int_N(f(G_N))$.

Proof. Necessity: Let f be a Nβω-open mapping and G_N is a N-open set in (Δ_N, τ_N). Now $int_N(G_N) = G_N$ which implies that $f(int_N(G_N) \subseteq f(G_N)$. Since f is a Nβω-open mapping, $f(int_N(G_N)$ is Nβω-open set in (Γ_N, σ_N) such that $f(int_N(G_N) \subseteq f(G_N)$. Therefore $f(int_N(G_N)) \subseteq N\beta\omega int_N(f(G_N))$.

Sufficiency: For the converse, suppose that G_N is a N-open set of (Δ_N, τ_N). Then $f(G_N) = f(int_N(G_N)) \subseteq N\beta\omega int_N(f(G_N))$. But $N\beta\omega int_N(f(G_N)) \subseteq f(G_N)$. Consequently, $f(G_N) = N\beta\omega int_N(f(G_N))$ which implies that $f(G_N)$ is a Nβω-open set of (Γ_N, σ_N) and hence f is a Nβω-open.

Theorem 3.6. If $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ is a Nβω-open mapping then $int_N(f^{-1}(G_N)) \subseteq f^{-1}(N\beta\omega int_N(G_N))$ for every neutrosophic set G_N of (Γ_N, σ_N).

Proof. Let G_N is a neutrosophic set of (Γ_N, σ_N). Then $int_N(f^{-1}(G_N))$ is a N-open set in (Δ_N, τ_N). Since f is Nβω-open, $f(int_N(f^{-1}(G_N))$ is Nβω-open in (Γ_N, σ_N). Hence by theorem 3.5., $f(int_N f^{-1}(G_N)) \subseteq N\beta\omega int_N(f(f^{-1}(G_N)) \subseteq N\beta\omega int_N(G_N)$. Thus $int_N(f^{-1}(G_N)) \subseteq f^{-1}(N\beta\omega int_N(G_N))$.

Theorem 3.7. A mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ is Nβω-open if and only if for each neutrosophic set G_N of (Γ_N, σ_N) and for each N-closed set U_N of (Δ_N, τ_N) containing $f^{-1}(G_N)$, there is a Nβω-closed V_N of (Γ_N, σ_N) such that $G_N \subseteq V_N$ and $f^{-1}(V_N) \subseteq U_N$.

Proof. Necessity: Suppose that f is a Nβω-open mapping. Let G_N be the neutrosophic set of (Γ_N, σ_N) and U_N is a N-closed set of (Δ_N, τ_N) such that $f^{-1}(G_N) \subseteq U_N$. Then $V_N = (f(U_N^C))^C$ is Nβω-closed set of (Γ_N, σ_N) such that $f^{-1}(V_N) \subseteq U_N$.

Sufficiency: For the converse, suppose that F_N is a N-open set of (Δ_N, τ_N). Then $(f^{-1}((f(F_N))^C \subseteq F_N^C$ and F_N^C is N-closed set in (Δ_N, τ_N). By hypothesis there is a Nβω-closed set V_N of (Γ_N, σ_N) such that $(f(F_N))^C \subseteq V_N$ and $f^{-1}(V_N) \subseteq F_N^C$. Therefore $F_N \subseteq f^{-1}(V_N))^C$. Hence $V_N^C \subseteq f(F_N) \subseteq f((f^{-1}(V_N))^C) \subseteq V_N^C$ which implies $f(F_N) = V_N^C$. Since V_N^C is Nβω-open set of (Γ_N, σ_N). Hence $f(F_N)$ is Nβω-open in (Γ_N, σ_N) and thus f is Nβω-open mapping.

Theorem 3.8. A mapping $f : (\Delta_N, \tau_N) \to (\Gamma_N, \phi_N)$ is Nβω-open if and only if $f^{-1}(N\beta\omega cl(G_N)) \subseteq cl(f^{-1}(G_N))$ for every neutrosophic set G_N of (Γ_N, σ_N).

Proof. Necessity: Suppose that f is a Nβω-open mapping. For any neutrosophic set G_N of (Γ_N, σ_N), $f^{-1}(G_N) \subseteq cl(f^{-1}(G_N))$. Therefore there exists a Nβω-closed set F_N in (Γ_N, σ_N) such that $G_N \subseteq F_N$ and $f^{-1}(F_N) \subseteq cl(f^{-1}(G_N))$. Therefore, we obtain that $f^{-1}(N\beta\omega cl(G_N)) \subseteq f^{-1}(F_N) \subseteq cl(f^{-1}(G_N))$.

Sufficiency: For the converse, suppose that G_N is a neutrosophic set of (Γ_N, σ_N) and F_N is a N-closed set of (Δ_N, τ_N) containing $f^{-1}(G_N)$. Put $H_N = cl_N(G_N)$. Then we have $G_N \subseteq H_N$ and H_N is Nβω-closed and $f^{-1}(H_N) \subseteq cl(f^{-1}(G_N)) \subseteq F_N$. Then by theorem 3.3, f is Nβω-open mapping.

Theorem 3.9. If $f : (\Delta_N, \tau_N) \to (\Gamma_N, \sigma_N)$ and $g : (\Gamma_N, \sigma_N) \to (\Omega_N, \phi_N)$ be two neutrosophic mappings and $gof : (\Delta_N, \tau_N) \to (\Omega_N, \phi_N)$ is Nβω-open. If $g : (\Gamma_N, \sigma_N) \to (\Omega_N, \phi_N)$ is Nβω-irresolute then f is Nβω-open mapping.

Proof. Let H_N be a N-open set of neutrosophic topological space (Δ_N, τ_N). Then $(gof)(H_N)$ is Nβω-open set of Ω_N because gof is Nβω-open mapping. Now since $g : (\Gamma_N, \sigma_N) \to (\Omega_N, \phi_N)$ is Nβω-irresolute and $(gof)(H_N)$ is Nβω-open set of Ω_N, $g^{-1}(gof(H_N)) = f(H_N)$ is Nβω-open set in neutrosophic topological space (Γ_N, σ_N). Hence f is Nβω-open mapping.

Theorem 3.10. If f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is a N-open mapping and g : $(\Gamma_N,\sigma_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open mappings then gof : $(\Delta_N,\tau_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open.

Proof. Let H_N be a N-open set of a neutrosophic topological space (Δ_N,τ_N). Then $f(H_N)$is N-open set of (Γ_N, σ_N) because f is neutrosophic open mapping. Now since g : $(\Gamma_N, \sigma_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω–open, $g(f(H_N)) = (gof)(H_N)$ is Nβω-open set. Hencegof is Nβω-open mapping.

Theorem 3.11. If f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ and g : $(\Gamma_N,\sigma_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open mappings such that (Γ_N,σ_N) is $T_{N\beta\omega}$-space then gof : $(\Delta_N,\tau_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open.

Proof. Let H_N be a N-open set of neutrosophic topological space (Δ_N,τ_N). Thenf(H_N) is Nβω-open set of (Γ_N, σ_N) because f is Nβω-open mapping. Now since (Γ_N, σ_N) is $T_{N\beta\omega}$-spaces, $f(H_N)$ is N-open set of (Γ_N, σ_N). Therefore $g(f(H_N)) = (gof)(H_N)$ is Nβω-open set of Ω_N because g : $(\Gamma_N,\sigma_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open. Hence gof is Nβω-open mapping.

Theorem 3.12. If f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is NG-open and g : $(\Gamma_N,\sigma_N) \rightarrow (\Omega_N, \phi_N)$is Nβω-open mappings such that (Γ_N, σ_N) is neutrosophic $T_{1/2}$- space then gof : $(\Delta_N, \tau_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open.

Proof. Let H_N be a N-open set of neutrosophic topological space (Δ_N,τ_N). Then $f(H_N)$is NG-open set of (Γ_N, σ_N) because f is NG-open mapping. Now since (Γ_N, σ_N) is neutrosophic $T_{1/2}$-space, $f(H_N)$ is N-open set of (Γ_N, σ_N). Therefore $g(f(H_N)) =(gof)(H_N)$ is Nβω-open set of Ω_N because g : $(\Gamma_N, \sigma_N) \rightarrow (\Omega_N, \phi_N)$ is Nβω-open. Hence gof is Nβω-open mapping.

IV. Neutrosophic Beta Omega Closed Mappings

Definition 4.1. A mapping f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is called neutrosophic beta closed (Nβ-closed) if $f(G_N)$ is Nβ-closed set in (Γ_N,σ_N) for every N-closed set G_N in (Δ_N,τ_N).

Definition 4.2. A mapping f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\phi_N)$ is Nβω-closed if image of every Nclosed set of (Δ_N,τ_N) is Nβω-closed set in (Γ_N,σ_N).

Example 4.1. Let $\Delta_N = \{\lambda_1, \lambda_2\}$, $\Gamma_N = \{\delta_1, \delta_2\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.7}\right),\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.6}\right),\left(\frac{\lambda_1}{0.3},\frac{\lambda_2}{0.2}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.7}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.5}\right),\left(\frac{\delta_1}{0.4},\frac{\delta_2}{0.3}\right)>$. Then τ_N and σ_N are NTs. Define a mapping f : $(\Delta_N,\tau_N)\rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$ and $f(\lambda_2) = \delta_2$. Then f is a Nβω-closed mapping.

Theorem 4.1. Every N-closed mapping is Nβω-closed but the converse may not be true.

Proof. Let f : $(\Delta_N, \tau_N) \rightarrow (\Gamma_N, \phi_N)$ be any N-closed mapping. Let U_N be a N-closed set in (Δ_N, τ_N). Then $f(U_N)$ is N-closed, since f is N-closed. This implies $f(U_N)$ is Nβω-closed. Hence f is a Nβω-closed mapping.

Example 4.2. Let $\Delta_N = \{\lambda_1, \lambda_2\}$, $\Gamma_N = \{\delta_1, \delta_2\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.6},\frac{\lambda_2}{0.7}\right),\left(\frac{\lambda_1}{0.7},\frac{\lambda_2}{0.6}\right),\left(\frac{\lambda_1}{0.3},\frac{\lambda_2}{0.2}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.7}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.5}\right),\left(\frac{\delta_1}{0.4},\frac{\delta_2}{0.3}\right)>$. Then τ_N and σ_N are NTs. Define a mapping f : $(\Delta_N,\tau_N)\rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$ and $f(\lambda_2) = \delta_2$. Then f is a Nβω-closed mapping. But f is not a N-closed mapping, since G_N is N-closed in (Δ_N,τ_N) but $f(G_N)$ is not N-closedin(Γ_N,σ_N).

Theorem 4.2. Every NG^*-closed mapping is a Nβω-closed mapping but not conversely.

Proof. Let U_N be a N-closed set in (Δ_N,τ_N) and f be a NG^*-closed mapping. Then $f(U_N)$ is NG^*-closed. This implies $f(U_N)$ is Nβω-closed. Hence f is a Nβω-closed mapping.

Example 4.3. Let $\Delta_N = \{\lambda_1, \lambda_2\}$, $\Gamma_N = \{\delta_1, \delta_2\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.4},\frac{\lambda_2}{0.3}\right),\left(\frac{\lambda_1}{0.4},\frac{\lambda_2}{0.4}\right),\left(\frac{\lambda_1}{0.8},\frac{\lambda_2}{0.8}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.3},\frac{\delta_2}{0.3}\right),\left(\frac{\delta_1}{0.3},\frac{\delta_2}{0.3}\right),\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.6}\right)>$. Then τ_N and σ_N are NTs. Define a mapping f : $(\Delta_N,\tau_N)\rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$ and $f(\lambda_2) = \delta_2$. Then f is a Nβω-closed mapping. But f is not a NG^*-closed mapping, since G_N is N-closed in (Δ_N,τ_N) but $f(G_N)$ is not NG^*-closed in (Γ_N,σ_N).

Theorem 4.3. Every NP-closed mapping is a Nβω-closed mapping but not conversely.

Proof. Let U_N be a N-closed set in (Δ_N,τ_N) and f be a NP-closed mapping. Then $f(U_N)$ is Nβω-closed. This implies $f(U_N)$ is Nβω-closed. Hence f is a Nβω-closed mapping.

Example 4.4. Let $\Delta_N = \{\lambda_1, \lambda_2\}$, $\Gamma_N = \{\delta_1, \delta_2\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.2},\frac{\lambda_2}{0.3}\right),\left(\frac{\lambda_1}{0.2},\frac{\lambda_2}{0.3}\right),\left(\frac{\lambda_1}{0.6},\frac{\lambda_2}{0.7}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.2}\right),\left(\frac{\delta_1}{0.1},\frac{\delta_2}{0.2}\right),\left(\frac{\delta_1}{0.7},\frac{\delta_2}{0.8}\right)>$. Then τ_N and σ_N are NTs. Define a mapping f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$ and $f(\lambda_2) = \delta_2$. Then f is a Nβω-closed mapping. But f is not a NP-closed mapping, since G_N is N-closed in (Δ_N,τ_N) but $f(G_N)$ is not NP–closed in (Γ_N,σ_N).

Theorem 4.4. Every Nβ-closed mapping is a Nβω-closed mapping but not conversely.

Proof. Let U_N be a N-closed set in (Δ_N, τ_N) and f be a Nβ-closed mapping. Then $f(U_N)$ is Nβ-closed. This implies $f(U_N)$ is Nβω-closed. Hence f is a Nβω-closed mapping.

Example 4.5. Let $\Delta_N = \{\lambda_1, \lambda_2\}$, $\Gamma_N = \{\delta_1, \delta_2\}$, $\tau_N = \{0_N, G_N, 1_N\}$ and $\sigma_N = \{0_N, H_N, 1_N\}$ where $G_N = <\xi,\left(\frac{\lambda_1}{0.1},\frac{\lambda_2}{0.1}\right),\left(\frac{\lambda_1}{0.2},\frac{\lambda_2}{0.2}\right),\left(\frac{\lambda_1}{0.8},\frac{\lambda_2}{0.8}\right)>$ and $H_N = <\xi,\left(\frac{\delta_1}{0.2},\frac{\delta_2}{0.1}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.7}\right),\left(\frac{\delta_1}{0.6},\frac{\delta_2}{0.7}\right)>$. Then τ_N and σ_N are NTs. Define a mapping f : $(\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ by $f(\lambda_1) = \delta_1$ and $f(\lambda_2) = \delta_2$. Then f is a Nβω-closed mapping. But f is not a Nβ-closed mapping, since G_N is N-closed in (Δ_N, τ_N) but $f(G_N)$ is not Nβ-closed in (Γ_N,σ_N).

Remark 4.1. NG-closed mapping and Nβω-closed mapping are independent.

Example 4.6. Consider the example 4.1, Then f is a Nβω-closed mapping. But f is not a NG-closed mapping, since G_N is N-closed in (Δ_N,τ_N) but $f(G_N)$ is not NG-closedin (Γ_N,σ_N).

Example 4.7. Consider the example 3.7, Then f is a NG-closed mapping. But f is nota Nβω-closed mapping, since G_N is N-closed in (Δ_N,τ_N) but $f(G_N)$ is not Nβω-closedin (Γ_N,σ_N).

Theorem 4.5. If $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ is Nβω-closed and (Γ_N,σ_N) is $T_{N\beta\omega}$-space. Then f is N-closed map.

Proof. Let $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ be Nβω-closed map. Let F_N be N-closed set in (Δ_N, τ_N). Then $f^{-1}(F_N)$ is Nβω-closed in (Γ_N, σ_N). Since (Γ_N, σ_N) is $T_{N\beta\omega}$-space, $f(F_N)$ is N-closed in (Γ_N,σ_N). Hence f is N-closed map.

Theorem 4.6. A map $f : (\Delta_N, \tau_N) \rightarrow (\Gamma_N, \sigma_N)$ is Nβω-closed iff for each neutrosophic set G_N of (Γ_N, σ_N) and for each neutrosophic open set U_N such that $f^{-1}(G_N) \subseteq U_N$, there is a Nβω-open set V_N of (Γ_N,σ_N) such that $G_N \subseteq V_N$ and $f^{-1}(V_N) \subseteq U_N$.

Proof. Suppose f is Nβω-closed map. Let G_N be a neutrosophic set of (Γ_N, σ_N) and U_N be a N-open set of (Δ_N,τ_N) such that $f^{-1}(G_N) \subseteq U_N$. Then $V_N = (\Gamma_N,\sigma_N) - f(U_N^C)$ is a Nβω-open set in (Γ_N, σ_N) such that $G_N \subseteq V_N$ and $f^{-1}(V_N) \subseteq U_N$. Conversely, suppose that F_N is a N-closed set of (Δ_N, τ_N). Then $f^{-1}(f(F_N^C)) \subseteq F_N^C$ and F_N^C is N-open. By hypothesis, there is a Nβω-open set V_N of (Γ_N,σ_N) such that $F_N^C \subseteq V_N$ and $f^{-1}(V_N) \subseteq F_N^C$. Therefore $F_N \subseteq f^{-1}(V_N^C)$. Hence $V_N^C \subseteq (F_N) \subseteq f(f^{-1}V_N^C) \subseteq V_N^c$. which implies $f(F_N) = V_N^C$. Since V_N^C is Nβω-closed, $f(F_N)$ is NβωC and thus f is a Nβω-closed map.

Proposition 4.1. If $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ and $g : (\Gamma_N,\sigma_N) \rightarrow (\Omega_N, \Phi_N)$ areNβω-closed maps with (Γ_N, σ_N) is a $T_{N\beta\omega}$-space, then $g : (\Delta_N, \tau_N) \rightarrow (\Omega_N, \Phi_N)$ is also a Nβω-closed map.

Proof. Let F_N be closed set in (Δ_N, τ_N). Since f is Nβω-closed map, $f(F_N)$ is Nβω-closed in (Γ_N,σ_N). Since (Γ_N,σ_N) is a $T_{N\beta\omega}$-space, $f(F_N)$ is closed in (Γ_N,σ_N). Since g is Nβω-closed map, $g(f(F_N)) =(gof)(F_N)$ is Nβω-closed in (Ω_N, Φ_N). Thus gof is a Nβω-closed map.

Theorem 4.7. Let $f : (\Delta_N,\tau_N) \rightarrow (\Gamma_N,\sigma_N)$ be a map from a space (Δ_N,τ_N) to a $T_{N\beta\omega}$-space (Γ_N, σ_N). Then the following are equivalent.

(i). f is Nβω-closed.

(ii). f is closed.

Proof: (i) ⇒ (ii) : Let F_N be closed in (Δ_N,τ_N). By(i) $f(F_N)$ is Nβω-closed in (Γ_N,σ_N). Since (Γ_N,σ_N) is a $T_{N\beta\omega}$-space, $f(F_N)$ is closed in (Γ_N,σ_N). Therefore f is closed.

(ii) ⇒ (i) : Let F_N be closed in (Δ_N,τ_N). By(ii) $f(F_N)$ is closed in (Γ_N,σ_N).Since every closed set is Nβω-closed, $f(F_N)$ is Nβω-closed in (Γ_N, σ_N). Thereforef is Nβω-closed.

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