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A Novel Iterative Scheme for Dynamical Expansion of Logistic Map

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Abstract

One-dimensional maps and their various generalized forms are considered most deliberated discrete chaotic maps in nonlinear dynamical systems. In spite of this, still there are some superior orbits in which the maps may be characterized and examined using fixed point theory. Therefore, this article deals with the modified Mann procedure in which the newly added parameter elaborates more interesting behavior and may have applications various branches of science such as information technology, control systems, transportation problems, cryptography, and security systems. The analytical as well as computational study, is carried out using graphical representations. Further, a comparison study versus modified Mann procedure, standard Mann procedure and Picard procedure is also demonstrated. Moreover, the maximum Lyapunov exponent property is also discussed to present the sensitive dependence in the orbits of modified Mann procedure.

Keywords: Chaos, Modified Mann procedure, bifurcation plot, Lyapunov exponent

1. INTRODUCTION

The beginning of difference equations may be traced to the discovery of logistic map $\lambda\vartheta(1-\vartheta)$ by Robert May [22] in 1896. It is a one-dimensional nonlinear map which depends on the population growth rate parameter λ. Also, it is the humblest map amid all nontrivial nonlinear maps. During the past several decades there have looked many revisions of the logistic map. As per requirement, most of these studies were done numerically. Even a deep understanding has been yielded in [2, 11, 12] and [23]. The dynamical properties in nonlinear systems are studied using discrete difference maps in the various fields of science and engineering. But the aperiodicity behaviour plays an important role in science and technology such as electrical engineering, traffic transportation problems, ups and downs in share market, biology, economics, physics, chemistry, etc. The books on the dynamics of nonlinear system such as Devaney [13], Holgrem [17], Ausloos et. al. [7], and Alligood et. al. [1], are refereed for the elementary study of chaos theory.

In the last two decades, an experimental as well as analytical study for one-dimensional logistic maps showing dynamic nature like fixed, periodic, aperiodic, chaos and maximum Lyapunov exponent using Mann procedure with applications in transportation problems have been examined by

Ashish et. al. [3, 5]. In 2020, Kumar et. al. [20] again worked on the dynamics of one-dimensional equations and examined the chaos control in modified one-dimensional systems using iterative procedure with strong convergence. If a growth rate parameter is modulated by using adjustable constant parameter of another logistic map, the complexity of such dynamical system is discussed by Elhadj and Sprott [14]. In 1987, Harikrishanan and Nandakumaran [15] studied that a less chaotic behavior of logistic map can be found by a definite nonlinear variation. In 2017, Sanz et. al. [26] determined that it depends on starting value of independent variable that the trajectories diverge or not for smaller value of logistic parameter when endowed with memory and fixing geometric decay of past iterations. Further, the developments using newly added parameter α on irregular behaviour of logistic like nonlinear difference equation in Mann iterative procedure is given in [8]. Afterward, for an accretive operator the strong convergence for the mappings which are non-expensive in a uniform Banach space was proposed by Kim and Xu [18] in 2005, using modified Mann procedure. In 2008, by using asymptotically pseudo-contractions they also examined the convergence of the modified Mann system in [19]. Some assumptions were made in boundary point methods by He and Zhu [16] to introduce minimum norm stationary states in the non-expensive equations using modified Mann system. A new algorithm for modified Mann process in Hilbert spaces for having strong convergence by imposing conditions on control sequences of non-self-Pseudo contractive mapping was examined by Tian and Jin [27]. For more results on the dynamics of one-dimensional maps one may also read [4, 6, 9, 10, 24, 25].

In this article the author examines the dynamics of one-dimensional map through advanced iterative method depending on the parameters α, k and λ. Further, a comparative study versus modified Mann procedure, standard Mann procedure and Picard procedure is also described using bifurcation plotting. Section 1 gives elementary knowledge about the one-dimensional chaotic maps and chaos theory. Section 2 presents the major results related to dynamical behaviour of the logistic map using modified Mann procedure. Section 3 contains the comparison analysis versus modified Mann procedure, standard Mann and Picard procedure. Further, the Lyapunov exponent property is also discussed in the Section 4. Finally, Section 5 gives the overall conclusion of the paper.

2. DYNAMICS IN MODIFIED MANN ORBIT

Here, we determines the various properties of the conventional logistic map $f(\vartheta, \lambda) = \lambda\vartheta(1-\vartheta)$, where $\lambda \in [0, 4]$ and $\vartheta \in [0, 1]$ are studied using modified Mann orbit. Therefore, the modified Mann algorithm is described as follows:

$$\vartheta_{n+1} = \frac{(1-\alpha)}{k}\vartheta_n + \left(1 - \frac{(1-\alpha)}{k}\right) f(\vartheta_n, \lambda), \quad (1)$$

where $\alpha \in (0, 1)$ and k, n are natural numbers. Then, for an initiator $\vartheta_0 \in [0, 1]$, we obtain the following modified outcome

$$\vartheta_1 = \frac{(1-\alpha)}{k}\vartheta_0 + \left(1 - \frac{(1-\alpha)}{k}\right) f(\vartheta_0, \lambda)$$

where $f(\vartheta_0, \lambda) = \lambda\vartheta(1 - \vartheta)$. Inductively, we may write as

$$MO_{\alpha,\lambda,k}(\vartheta_n) = \frac{(1-\alpha)}{k}\vartheta_n + \left(1 - \frac{(1-\alpha)}{k}\right) f(\vartheta_n, \lambda) \quad (2)$$

where $n \in N$, $\vartheta_n \in [0, 1]$ and $\alpha \in (0, 1)$. Since the equation (2) depends on the control parameters α, k and λ, therefore, at $k = 1$ it reduces into standard Mann procedure and at $\alpha = 0$, it reduces into conventional Picard procedure of recursion. Therefore, throughout the analysis of system (2) we take some particular values of $\alpha \in (0, 1)$ and $k > 1$. Therefore, the fixed point for the original one-dimensional logistic map in modified Mann procedure is given by:

$$MO_{\alpha,\lambda,k}(\vartheta^*) = \vartheta^*$$

$$\frac{(1-\alpha)}{k}\vartheta^* + \left(1 - \frac{(1-\alpha)}{k}\right) f(\vartheta^*, \lambda) = \vartheta^*$$

$$\frac{(1-\alpha)}{k}\vartheta^* + \left(1 - \frac{(1-\alpha)}{k}\right) \lambda\vartheta^*(1-\vartheta^*) - \vartheta^* = 0$$

$$\frac{(1-\alpha)}{k}\vartheta^* - \vartheta^* + \left(1 - \frac{(1-\alpha)}{k}\right) \lambda\vartheta^*(1-\vartheta^*) = 0$$

$$\left(\frac{(1-\alpha)}{k} - 1\right)\vartheta^* + \left(1 - \frac{(1-\alpha)}{k}\right) \lambda\vartheta^*(1-\vartheta^*) = 0$$

$$\left(\frac{(1-\alpha)}{k} - 1\right)(\vartheta^* - \lambda\vartheta^*(1-\vartheta^*)) = 0$$

$$\vartheta^* - \lambda\vartheta^*(1-\vartheta^*) = 0$$

$$\vartheta^*(1 - \lambda(1-\vartheta^*)) = 0 \quad (3)$$

Then, solving (3) we obtain $\vartheta^* = 0$ and $\vartheta^* = 1 - 1/\lambda$ as the trivial fixed point in the modified Mann procedure. The stabilization in the fixed-point state is determined using Devaney's definition for attracting and repelling fixed points. For this let us consider

$$\left|MO'_{\alpha,\lambda,k}(\vartheta^*)\right| = \left|\frac{(1-\alpha)}{k} + \left(1 - \frac{(1-\alpha)}{k}\right) f'(\vartheta^*,\lambda)\right|$$

$$= \left|\frac{(1-\alpha)}{k} + \left(1 - \frac{(1-\alpha)}{k}\right) (\lambda - 2\lambda\vartheta^*)\right|$$

$$= \left|\frac{(1-\alpha)}{k} + \left(1 - \frac{(1-\alpha)}{k}\right) (\lambda - 2\lambda(1 - 1/\lambda))\right|$$

$$= \left|\frac{(1-\alpha)}{k} + \left(1 - \frac{(1-\alpha)}{k}\right) (2 - \lambda))\right| < 1$$

Thus, $\left|MO'_{\alpha,\lambda,k}(\vartheta^*)\right| < 1$ depending on the range of the control parameter λ, k and $\alpha \in [0,1]$. Hence the given fixed point $\vartheta^* = 1 - 1/\lambda$ remains stable for a particular range of the control parameter λ depending on k and $\alpha \in [0, 1]$. Similarly, the stability of fixed points of periodicity of orders 2^n may be also determined on the same lines as discussed above. The stability in higher order periodic states, beauty of chaos and the dynamical nature of the logistic map using modified Mann procedure is demonstrated using the bifurcation plotting in "Matlab" which depends on the growth rate parameter λ and the additional Mann parameters $\alpha \in [0, 1]$ and $k > 1$. The following cases arises:

2.1. When k = 2 and 0 < α < 1

Taking $k = 2$ and $\alpha = 0.9$, it is found that the dynamical system (2) admits all the dynamical properties such as stability region in fixed and a periodic state, periodic windows of higher orders, and period three window with full chaos. The first bifurcation in the system occurs at $\lambda = 3.11$ and the second bifurcation take place at $\lambda = 3.58$ which approaches for $3.58 < \lambda \leq 3.68$. The period–3 window appears in the chaotic regime when the parameter λ lies between 3.978 and 3.995. Finally, the period–doubling bifurcation dynamics with the periodicity of order 2^n approaches to $\lambda = 4.10$. Figure 1 shows the complete bifurcation plotting for the system (2).

Similarly, at $k = 2$, and $\alpha = 0.6$ the dynamical properties like stability, period doubling and chaos is drawn in the Figure 2 for the full range of logistic parameter $0 \leq \lambda \leq 4.48$. But in case of parameter $\alpha = 0.3$, it shows that the period-doubling occurs in the system for almost each degree and aperiodicity for a small range of the parameter λ while the overall regime of the logistic parameter λ lies between 0 and 5. Figure 3 gives the complete presentation of the dynamical nature of the system for $0 \leq \lambda \leq 5$. Further, the Figure 4 describes the dynamical nature for the system (2) for $\alpha = 0.1$ and $0 \leq \lambda \leq 5.5$. It shows that no chaos occurs in the system for the full range of parameter λ.

Remark 2.1. *It's amazing to see that as the parameter α decreases from 1 to 0 the system (2) first admits full chaos and then starts to get the stability in chaos. Thus, the system becomes fully stable as α approaches to 0.1, that means, there exists no chaos. While the corresponding logistic parameter λ increases continuously as α decreases from 1 to 0. Table 1 gives overall details of the dynamical behaviour versus parameters k, α and λ.*

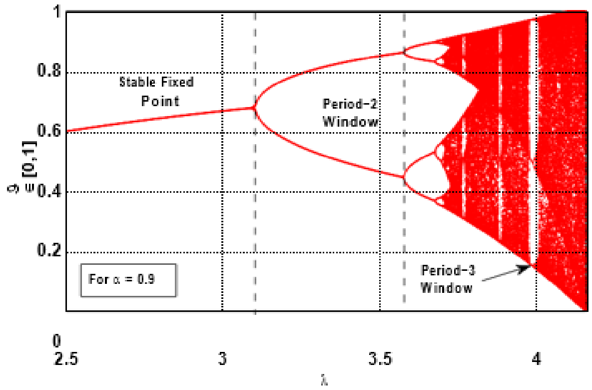

Figure 1: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 4.10$, $\alpha = 0.9$

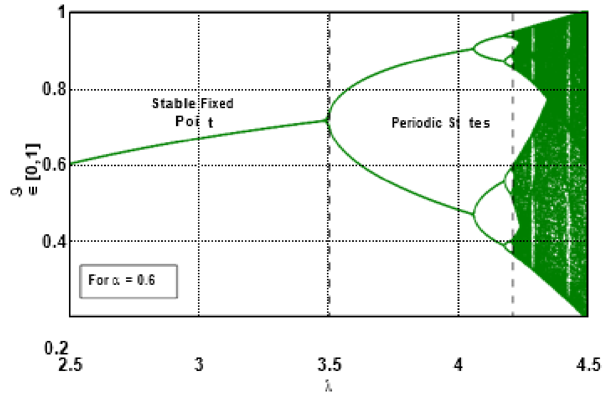

Figure 2: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 4.48$, $\alpha = 0.6$

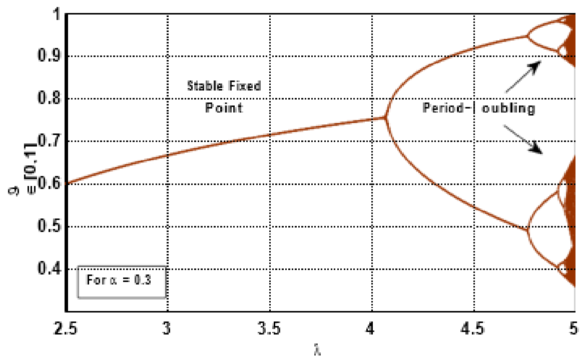

Figure 3: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 5$, $\alpha = 0.3$

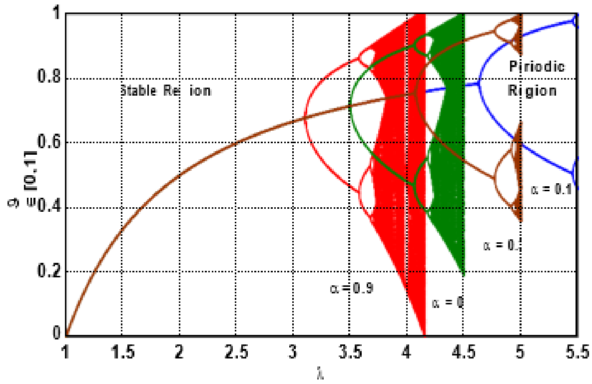

Figure 4: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $\alpha = 0.1, 0.3, 0.6, 0.9$

$\alpha \in [0,1]$	$k > 1$	λ	Dynamical States
0.9	2	$0 \le \lambda \le 4.10$	Fixed, Periodic, Fully Chaotic
0.6	2	$0 \le \lambda \le 4.5$	Fixed, Periodic, Chaotic
0.3	2	$0 \le \lambda \le 5.00$	Fixed, Periodic
0.1	2	$0 \le \lambda \le 5.5$	Fixed, Period-2

Table 1: Dynamical states for different parameter ranges of the parameters α, k, and λ

2.2 When k = 3 and 0 < α < 1

By fixing $k = 3$ and $\alpha = 0.9$, it is studied from the Figure 5 that the modified Mann system generates stable fixed-point regime, period–doubling bifurcations, period-3 window and chaotic regime. The complete dynamical behaviour of the modified system (2) take place for the logistic parameter $0 \le \lambda \le 4$ as shown in Figure 5. Similarly, at $k = 3$ and $\alpha = 0.6$ the dynamical properties such as stable fixed point, periodicity of order 2^n and the chaotic state are determined which exists for the logistic parameter $0 \le \lambda \le 4.30$. Figure 6 shows the complete dynamics for the system at $\alpha = 0.6$.

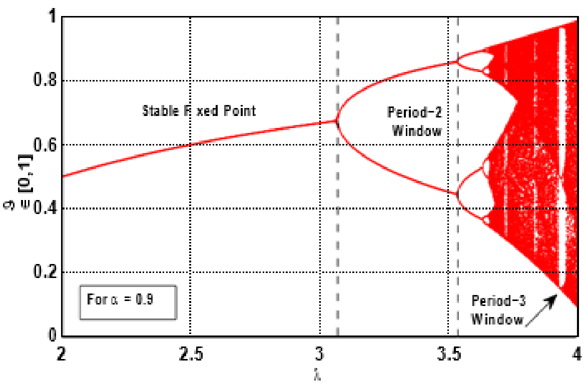

Figure 5: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \le \lambda \le 4$, $\alpha = 0.9$

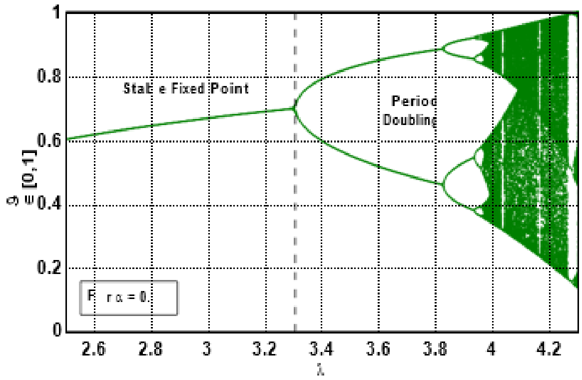

Figure 6: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \le \lambda \le 4.3$, $\alpha = 0.6$

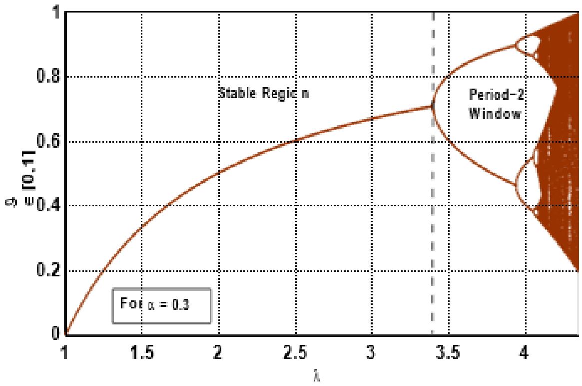

Figure 7: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \le \lambda \le 4.55$, $\alpha = 0.3$

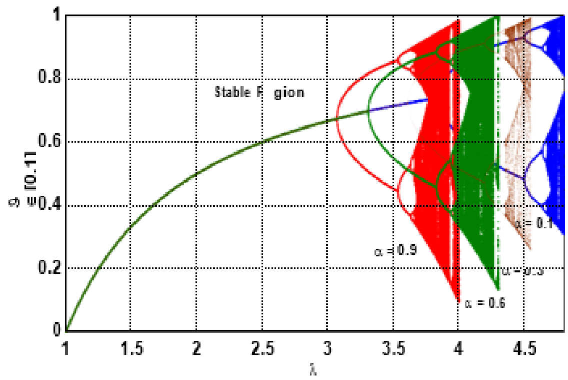

Figure 8: Chaotic region of the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $\alpha = 0.1, 0.3, 0.6, 0.9$

For $k = 3$ and $\alpha = 0.3$ the chaotic region possessing period–doubling bifurcations but not period-3 window approaches to $\lambda = 4.55$ as shown in Figure 7 and for $\alpha = 0.1$ the range of logistic parameter λ approaches to 4.80 but no period-3 window exists here. Figure 8 shows a comparative analysis for the complete dynamics at $\alpha = 0.1, 0.3, 0.6, 0.9$.

Remark 2.2. *Here, it is examined that at k = 3 and α = 0.9, 0.6 the system shows dynamical behaviour with period-3 window in chaotic regime and then the chaotic regime starts to decrease continuously as shown in Figure 8. But the logistic parameter range increases continuously as α approaches from 1 to 0.*

2.3. When $k = 4$ and $0 < \alpha < 1$

In this section, we deal with the dynamics of the logistic map using modified Mann procedure (2) for the parameter value k = 4 and α = 0.9, 0.5, and 0.1. For α = 0.9 the system shows all the dynamical properties and the logistic parameter λ approaches to 4 as shown in Figure 9. At α = 0.5 and 0.1 it is observed that the system admits fixed point states, periodic states and chaotic states for the logistic growth rate parameter regime $0 \leq \lambda \leq 4.26$, and $0 \leq \lambda \leq 4.55$, respectively. Figure 10 shows a fully comparative dynamical behaviour for α = 0.9, 0.5 and 0.1. Further, it is observed that as the parameter α decreases from 1 to 0 the growth rate parameter range increases simultaneously.

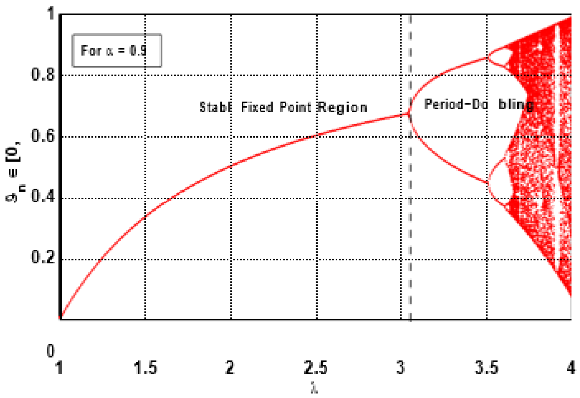

Figure 9: Bifurcation plot for the system $MO_{\alpha,\lambda,k}(\vartheta)$ when k = 4 and α = 0.9

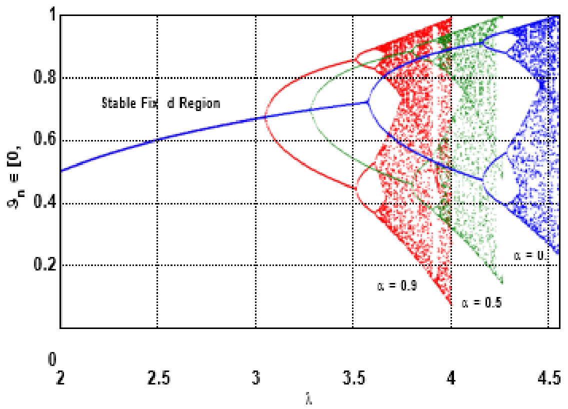

Figure 10: Bifurcation plot for the system $MO_{\alpha,\lambda,k}(\vartheta)$ when k = 4 and α = 0.1, 0.5, 0.9

Since the logistic growth rate parameter range in modified Mann procedure changes depending on the parameters k and α, therefore, in the next section a comparison versus standard Mann procedure and Picard procedure is described.

Remark 2.3. *It is noticed that as the value of the parameter k increase through 2, the growth rate parameter attains maximum 5.5 at k = 2 and then starts to decrease for k = 3, 4, 5 and so on as shown in Figures 1-10.*

3. MODIFIED MANN VERSUS STANDARD MANN AND PICARD ORBIT

In this section, we deal with the comparative analysis versus modified Mann, standard Mann, and Picard iterative orbit. As we know that at k = 1 the modified Mann orbit reduces to the following standard Mann orbit [21]:

$$MO_{\alpha,\lambda,k}(\vartheta) = \vartheta_{n+1} = (1-\alpha)\vartheta_n + \alpha f(\vartheta_n, \lambda)$$

The dynamical behaviour of the standard logistic map using Mann iterative procedure was studied by Ashish et. al. [3, 4] in 2018 and 2019. The chaotic nature of the system is extended as the control parameter α changes in the closed interval [0, 1]. Due to an extra degree of freedom of the control parameter $k > 1$ the modified Mann procedure compel us to make this comparison. It is observed that when we take α = 0.1 and k = 2, the modified Mann system presents the stability in the system for $0 \leq \lambda \leq 5.5$ and the stability is seen the chaotic region of the bifurcation plot as shown in Figure 4. While in Mann procedure the logistic parameter λ attains maximum value 4.22 when α = 0.9 and the bifurcation diagram so obtained consists of all the dynamical properties as shown in Figure 11. But in modified Mann iteration the growth rate parameter λ also approximately approaches to 4.22 for some different parameter values of α and k.

Similarly, at α = 1 the modified Mann procedure (2) reduces to Picard iterative procedure $\vartheta_{n+1} = f(\vartheta_n, \lambda)$, where the logistic parameter λ converges to 4. Therefore, in the modified Mann procedure the logistic parameter λ converges to 4 for various values of parameters k and α, and is encountered first time at k = 3 when α = 0.9. The bifurcation diagram so obtained consists of all the dynamical characteristics as shown in Figure 12.

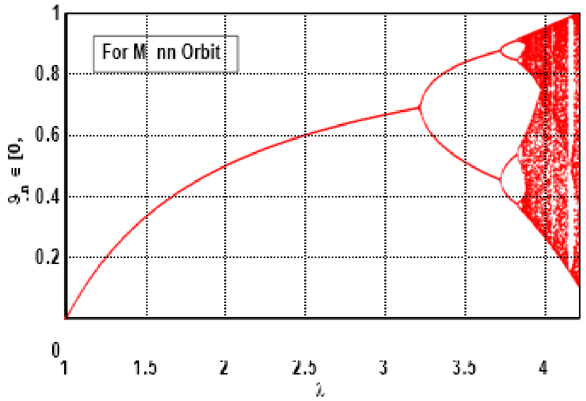

Figure 11: Bifurcation plot in Mann orbit for $0 \leq \lambda \leq 4.22$ and $\alpha = 0.9$

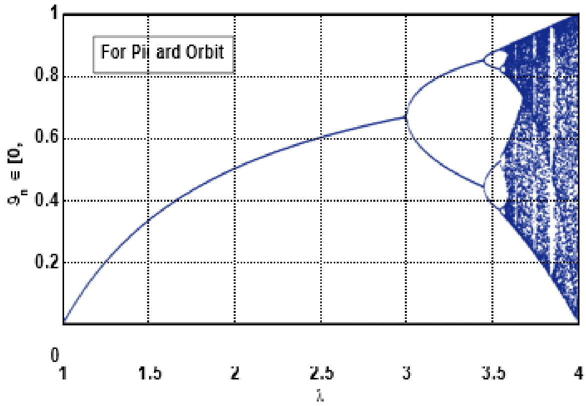

Figure 12: Bifurcation plot in Picard orbit for $k = 3$, $0 \leq \lambda \leq 4$ and $\alpha = 0.9$

4. LYAPUNOV EXPONENT IN MODIFIED MANN ORBIT

In the earlier section, we have discussed the dynamical behavior of the logistic map for different values of parameters k, α, and λ using modified Mann iterative orbit. Now, to measure the rate of convergence, divergence and sensitive dependence of two orbits there exists another important property, that is, Lyapunov exponent (ρ). Here, the negative value of the Lyapunov exponent represents the stability in the modified Mann orbits and the positive Lyapunov exponent gives instability in the modified Mann orbits. That means for the stable fixed and periodic orbits the Lyapunov exponent is always negative and for aperiodic orbits the Lyapunov exponent is always positive. Therefore, we deal with the Lyapunov exponent property of logistic map using modified Mann orbit for various parameter values of k, α, and λ. Thus, we start with the following modified Mann system (2):

$$MO_{\alpha,\lambda,k}(\vartheta_n) = \frac{(1-\alpha)}{k}\vartheta_n + \left(1 - \frac{(1-\alpha)}{k}\right) f(\vartheta_n, \lambda)$$

where $\alpha \in [0, 1]$, $k > 1$ and $\vartheta_n \in [0, 1]$. Therefore, by the definition of Lyapunov exponent for the map $f(\vartheta, \lambda)$ under the modified Mann iterative procedure is given by:

$$\rho = \lim_{p\to\infty} \frac{1}{p} \sum_{i=0}^{p} log\left|MO'_{\alpha,\lambda,k}(\vartheta_i)\right|$$

where p denotes the periodicity of the orbit. It is noticed that for the fixed point of periodicity one it reduces into $\rho = log\left|MO'_{\alpha,\lambda,k}(\vartheta_i)\right|$.

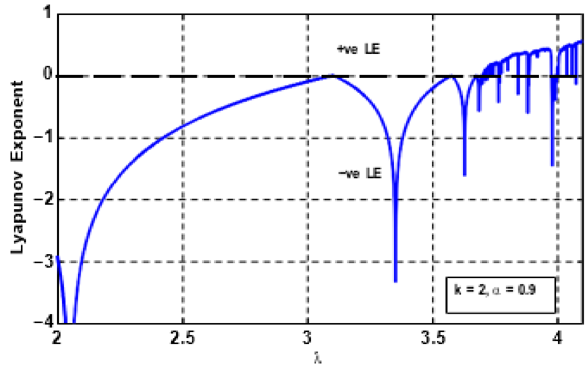

Figure 13: Lyapunov spectrum for the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 4.10$, $\alpha = 0.9$

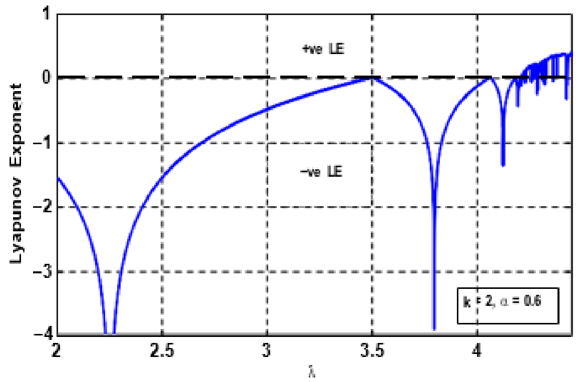

Figure 14: Lyapunov spectrum for the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 4.48$, $\alpha = 0.6$

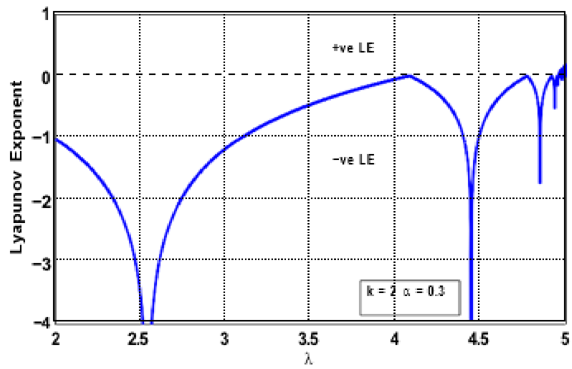

Figure 15: Lyapunov spectrum for the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 5$, $\alpha = 0.3$

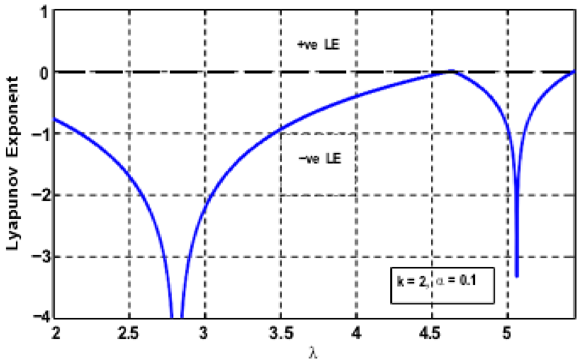

Figure 16: Lyapunov spectrum for the system $MO_{\alpha,\lambda,k}(\vartheta)$ for $0 \leq \lambda \leq 5.5$, $\alpha = 0.1$

In particular, the maximum Lyapunov exponent value for fixed, periodic and chaotic states is determined using the modified Mann system (2) at $\alpha = 0.9$ and $k = 2$ and the growth parameter λ is determined. Therefore, using (3) and taking $\alpha = 0.9$, $k = 2$, $0 \leq \lambda \leq 3.08$ and the fixed point $\vartheta^* = 0.6666$, we get

$$\rho = log\left|MO'_{0.9,\ 3,\ 2}(0.6666)\right|. \quad (4)$$

Also, from (2) we obtain

$$MO'_{\alpha,\lambda,k}(\vartheta^*) = \frac{(1-\alpha)}{k} + \left(1 - \frac{(1-\alpha)}{k}\right)\ (\lambda - 2\lambda\vartheta^*) \quad (5)$$

Then, from (4) and (5), we obtain

$$\rho = log|-0.8996| = -0.0459 < 0. \quad (6)$$

Similarly, taking $\alpha = 0.9$, $k = 2$, $3.08 < \lambda \leq 3.5728$and the periodic points $\vartheta_1^* = 0.4653$ and $\vartheta_2^* = 0.8505$, of order-2, we get

$$\rho = \frac{1}{2} log\left|MO'_{0.9,\ 3,\ 2}(0.4653)\right| + log\left|MO'_{0.9,\ 3,\ 2}(0.8505)\right|] \quad (7)$$

Then, from (5) and (7), we have

$$\rho = -0.09678 < 0. \quad (8)$$

Thus, in all the above cases no chaos occurs for some given values of α, k and λ, because, the Lyapunov exponent attains negative Lyapunov exponent which is an indication of no chaos. Figure 13-16 shows that the Lyapunov spectrum in the negative quadrant approaches to the stable states and the Lyapunov spectrum in the positive quadrant approaches to the irregularity in the system, that is, chaos.

5. CONCLUSION

Using the modified Mann orbit the dynamics for the standard one-dimensional logistic map is carried out. The analytical as well as computational study is described through the modified control parameters α, k, and the logistic parameter λ. Since the whole dynamics in the system depends on the parameters α and k, therefore, the following results are concluded:

1. In the Section 2, the modified Mann system is introduced using the one-dimensional logistic map and the modified Mann iterative procedure. Then, the corresponding fixed points in the system are determined and their stability is studied. It is observed that the fixed states of order-1 generated in the modified Mann system are similar to the trivial fixed points in the Mann orbit.
2. Due to the dependency on the parameters α and k, some special cases are derived for different values of $k > 1$ and $0 \leq \alpha \leq 1$. The complete dynamical behaviour is analysed using bifurcation plot.
3. Therefore, it is clear from the bifurcation plots that at $k = 2$ and $\alpha \in [0, 1]$ the growth rate parameter (λ) range increases continuously while the system approaches to stability in chaos. But as the value of k increases the system approaches to fully chaos, that means, fixed, periodic and chaotic behaviour. The Lyapunov spectrum is drawn and the Lyapunov value is determined for different parameter values of α.

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