

Ranking of Parabolic Trapezoidal Fuzzy Number Using the Centroids and Focus

A. Thirupathi

Associate Professor, Department of Mathematics, Panimalar Institute of Technology.

C.K. Kirubhashankar

Associate Professor, Department of Mathematics, Sathyabama Institute of Science and Technology.

E. Janaki

Assistant Professor, Department of Mathematics, Panimalar Institute of Technology.

Abstract - Fuzzy set theory has a wide range of application in all fields. Many researchers have developed a different type of fuzzy numbers and its membership function. The fuzzy membership function attains the highest value only between the intervals. The fuzzy numbers are parabolic once they obtain the highest value at the midpoint of an interval and is referred as parabolic fuzzy number. Here a new ranking of centre of centroids and focus of the parabolic ranking trapezoidal fuzzy numbers has been developed. So, the parabolic fuzzy number is transformed into a crisp number using new ranking methods.

Key words: Ranking of parabolic fuzzy number, Trapezoidal-Parabolic Fuzzy Number.

LITERATURE REVIEW

S.F. Mallak [7] defined Trapezoidal parabolic fuzzy numbers and also discussed the comparison between fuzzy numbers. Saed F. Mallak and Duha M. Bedo applied the fuzzy comparison method to K trapezoid-triangular fuzzy number, $K + 1$ trapezoidal fuzzy number, k trapezoidal-parabolic fuzzy number and confidence interval comparison. Bogdan Dorohonceanu and Bogdan Marin reviewed consistent fuzzy number comparison method based on the fuzzy number comparison used in PCM method. The method was implemented in a Java applet that is available online along with documentation and source code. K. Thangavelu, G. Uthra and S. Shunmugapriya [5] developed the parabolic membership functions provided that they attain the highest value at one point in an interval. Dinesh C.S Bisht and Pankaj Kumar Srivastava [4] in the paper fuzzy transportation, first applied Trisectional approach and then newly proposed ranking technique based on in-centre concept applied for conversion to crisp number. In the paper Mag (u), S. Abbasbandy and T. Hajjari [6] found the rank of fuzzy numbers. Amit Kumar, Pushpinder Singh, Amarpreet Kaur, and Parmpreet Kaur [2] proposed non-normal P-norm trapezoidal fuzzy number as a new ranking method.

PARABOLIC TRAPEZOIDAL NUMBER [1][2][5]

Let $\tilde{A} = (p_1, p_2, p_3, p_4)$ be a parabolic trapezoidal fuzzy number where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$ is defined as its membership function is

$$\mu_A(x) = \begin{cases} p \left(\frac{x - p_1}{p_2 - p_1} \right) & \text{for } p_1 \leq x \leq p_2 \\ p + \frac{(1-p)(x-p_2)}{\left(\frac{p_2+p_3}{2} - p_2 \right)} & \text{for } p_2 \leq x \leq \left(\frac{p_2+p_3}{2} \right) \\ 1 + \frac{(1-p) \left(x - \left(\frac{p_2+p_3}{2} \right) \right)}{\left(\left(\frac{p_2+p_3}{2} \right) - p_3 \right)} & \text{for } \left(\frac{p_2+p_3}{2} \right) \leq x \leq p_3 \\ p \left(\frac{x - p_3}{p_4 - p_3} \right) & \text{for } p_3 \leq x \leq p_4 \\ 0 & \text{otherwise} \end{cases}$$

1. $\mu_A(x)$ is straight line from $(p_1, 0)$ to (p_2, p)
2. $\mu_A(x)$ is a parabola $\left(x - \left(\frac{p_2+p_3}{2} \right) \right)^2 = \left[\frac{(p_3-p_2)^2}{(p-1)} \right] (y-1)$ with vertex at $\left(\frac{p_2+p_3}{2}, 1 \right)$,
Focus $\left(\frac{p_2+p_3}{2}, 1 - \frac{(p_3-p_2)^2}{16(1-p)} \right)$
3. $\mu_A(x)$ is line from (p_3, p) to $(p_4, 0)$

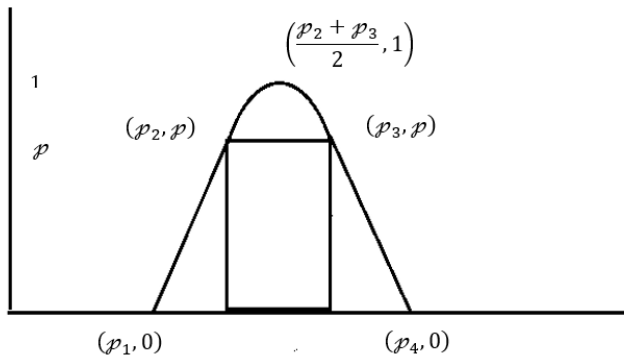


FIGURE 1
DIAGRAM FOR NORMAL PARABOLIC TRAPEZOIDAL FUZZY NUMBER

NEW RANKING OF PARABOLIC TRAPEZOIDAL NUMBER [10]

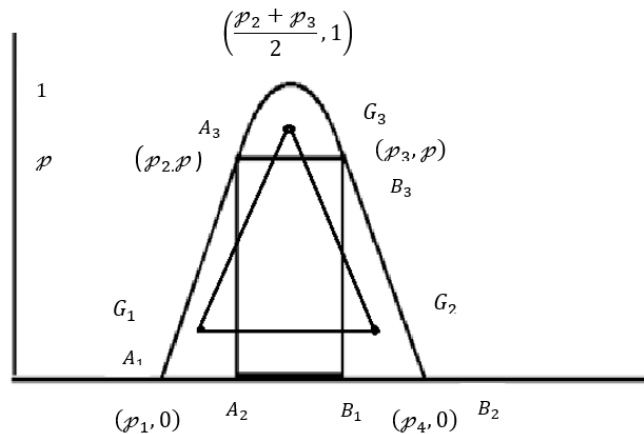


FIGURE 2
DIAGRAM FOR RANKING PARABOLIC TRAPEZOIDAL FUZZY NUMBER

Divide the Parabolic trapezoidal fuzzy number vertically into three figures and divide horizontally at $y = p$ where $0 \leq p \leq 1$. Now, we get two triangles, Triangle A_1, A_2, A_3 Triangle B_1, B_2, B_3 and hyperbola with the point (p_2, p) , (p_3, p) and vertex $(\frac{p_2+p_3}{2}, 1)$. Let G_1, G_2 is the centroid point triangle and focus of the parabola $(\frac{p_2+p_3}{2}, 1 - \frac{(p_3-p_2)^2}{16(1-p)})$.

Let's consider a Parabolic trapezoidal fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4)$ where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$

$$\text{The centroid of triangle } A_1, A_2, A_3 \text{ is } G_1 = \left(\frac{p_1+2p_2}{3}, \frac{p}{3}\right)$$

$$\text{The centroid of triangle } B_1, B_2, B_3 \text{ is } G_2 = \left(\frac{p_4+2p_3}{3}, \frac{p}{3}\right)$$

$$\text{The focus of the parabola is } G_3 = \left(\frac{p_2+p_3}{2}, 1 - \frac{(p_3-p_2)^2}{16(1-p)}\right)$$

The centroid of point G_1 and G_2 is passing through the line $y = \frac{p}{3}$ and focus of the parabola G_3 is passing $y = 1 - \frac{(p_3-p_2)^2}{16(1-p)}$. So focus of the parabola G_3 doesn't lie on the line $\overline{G_1G_2}$. Therefore, join the point G_1, G_2 and G_3 , they form a triangle. Now, the centre of the triangle with vertices of the triangle G_1, G_2 and G_3 of the parabolic trapezoidal fuzzy number is

$$(\bar{x}_{\tilde{A}_H}, \bar{y}_{\tilde{A}_H}) = \left(\frac{2p_1+7p_2+7p_3+2p_4}{18}, \frac{-32p^2-16p+48-3p_3^2-3p_2^2+6p_2p_3}{144(1-p)}\right)$$

Parabolic trapezoidal fuzzy number $R(\tilde{A}_H)$ to a set of real numbers is defined as

$$R(\tilde{A}_H) = \sqrt{\bar{x}_{\tilde{A}_H}^2 + \bar{y}_{\tilde{A}_H}^2}$$

The Mode (M) of the parabolic trapezoidal fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4)$ where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$

$$M = \frac{1}{2} \int_0^p (p_2 + p_3) dx = \frac{p(p_2 + p_3)}{2}$$

The spread (S) of the parabolic trapezoidal fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4)$ where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$.

$$S = \int_0^p (p_4 - p_1) dx = p(p_4 - p_1)$$

The left spread (L_S) of the parabolic trapezoidal fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4)$ where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$.

$$L_S = \int_0^p (p_2 - p_1) dx = p(p_2 - p_1)$$

The right spread (L_R) of the parabolic trapezoidal fuzzy number $\tilde{A} = (p_1, p_2, p_3, p_4)$ where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$.

$$L_R = \int_0^p (p_4 - p_3) dx = p(p_4 - p_3)$$

ORDERING OF PARABOLIC TRAPEZOIDAL FUZZY NUMBER

Step-1

Find $R(\tilde{A}_H)$ and $R(\tilde{B}_H)$ if $R(\tilde{A}_H) = R(\tilde{B}_H) \Rightarrow \tilde{A}_H = \tilde{B}_H$

Find $R(\tilde{A}_H)$ and $R(\tilde{B}_H)$ if $R(\tilde{A}_H) < R(\tilde{B}_H) \Rightarrow \tilde{A}_H < \tilde{B}_H$

Find $R(\tilde{A}_H)$ and $R(\tilde{B}_H)$ if $R(\tilde{A}_H) > R(\tilde{B}_H) \Rightarrow \tilde{A}_H > \tilde{B}_H$

If it is more difficult, continue to the next step.

Step-2

Find $M(\tilde{A}_H)$ and $M(\tilde{B}_H)$ if $M(\tilde{A}_H) = M(\tilde{B}_H) \Rightarrow \tilde{A}_H = \tilde{B}_H$

Find $M(\tilde{A}_H)$ and $M(\tilde{B}_H)$ if $M(\tilde{A}_H) < M(\tilde{B}_H) \Rightarrow \tilde{A}_H < \tilde{B}_H$

Find $M(\tilde{A}_H)$ and $M(\tilde{B}_H)$ if $M(\tilde{A}_H) > M(\tilde{B}_H) \Rightarrow \tilde{A}_H > \tilde{B}_H$

If it is more difficult, continue to the next step

Step-3

Find $S(\tilde{A}_H)$ and $S(\tilde{B}_H)$ if $S(\tilde{A}_H) = S(\tilde{B}_H) \Rightarrow \tilde{A}_H = \tilde{B}_H$

Find $S(\tilde{A}_H)$ and $S(\tilde{B}_H)$ if $S(\tilde{A}_H) < S(\tilde{B}_H) \Rightarrow \tilde{A}_H < \tilde{B}_H$

Find $S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $S(\bar{\mathcal{A}}_{\mathcal{H}}) > S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} > \bar{\mathcal{B}}_{\mathcal{H}}$

If it is more difficult, continue to the next step

Step-4

Find $L_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $L_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $L_S(\bar{\mathcal{A}}_{\mathcal{H}}) = L_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} = \bar{\mathcal{B}}_{\mathcal{H}}$

Find $L_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $L_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $L_S(\bar{\mathcal{A}}_{\mathcal{H}}) < L_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} < \bar{\mathcal{B}}_{\mathcal{H}}$

Find $L_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $L_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $L_S(\bar{\mathcal{A}}_{\mathcal{H}}) > L_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} > \bar{\mathcal{B}}_{\mathcal{H}}$

If it is more difficult, continue to the next step

Step-5

Find $R_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $R_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $R_S(\bar{\mathcal{A}}_{\mathcal{H}}) = R_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} = \bar{\mathcal{B}}_{\mathcal{H}}$

Find $R_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $R_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $R_S(\bar{\mathcal{A}}_{\mathcal{H}}) < R_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} < \bar{\mathcal{B}}_{\mathcal{H}}$

Find $R_S(\bar{\mathcal{A}}_{\mathcal{H}})$ and $R_S(\bar{\mathcal{B}}_{\mathcal{H}})$ if $R_S(\bar{\mathcal{A}}_{\mathcal{H}}) > R_S(\bar{\mathcal{B}}_{\mathcal{H}}) \Rightarrow \bar{\mathcal{A}}_{\mathcal{H}} > \bar{\mathcal{B}}_{\mathcal{H}}$

NUMERICAL EXAMPLE

In this section, the proposed ranking method explained with some parabolic trapezoidal fuzzy number $\tilde{\mathcal{A}} =$

(p_1, p_2, p_3, p_4) be a parabolic trapezoidal fuzzy number where p_1, p_2, p_3, p_4 and p are real number and $0 < p < 1$

Using T.C. Chu and C.T. Tsao [9] centroid of area method

Let $\tilde{\mathcal{A}} = (5, 8, 11, 14)$ $p = 0.25$

$$\begin{aligned} \text{Rank}(\tilde{\mathcal{A}}) &= \frac{\frac{1}{12} \int_5^8 x(x-5)dx + \frac{1}{4} \int_8^{9.5} xdx + \frac{3}{6} \int_8^{9.5} x(x-8)dx + \int_{9.5}^{11} xdx - \frac{3}{6} \int_{9.5}^{11} x(x-9.5)dx + \frac{1}{12} \int_{11}^{14} x(x-11)dx}{\frac{1}{12} \int_5^8 (x-5)dx + \frac{1}{4} \int_8^{9.5} dx + \frac{3}{6} \int_8^{9.5} (x-8)dx + \int_{9.5}^{11} dx - \frac{3}{6} \int_{9.5}^{11} (x-9.5)dx + \frac{1}{12} \int_{11}^{14} (x-11)dx} \\ &= \frac{\frac{21}{8} + \frac{105}{32} + \frac{81}{16} + \frac{123}{8} - \frac{189}{32} + \frac{39}{8}}{\frac{3}{8} + \frac{3}{8} + \frac{9}{16} + \frac{3}{2} - \frac{9}{16} + \frac{3}{8}} \\ &= \frac{25.3125}{2.625} \\ &= 9.64285 \end{aligned}$$

Our new ranking method

$$\begin{aligned} (\bar{x}_{\tilde{\mathcal{A}}_{\mathcal{H}}}, \bar{y}_{\tilde{\mathcal{A}}_{\mathcal{H}}}) &= \left(\frac{2p_1+7p_2+7p_3+2p_4}{18}, \frac{-32p^2-16p+48-3p_3^2-3p_2^2+6p_2p_3}{144(1-p)} \right) \\ \tilde{\mathcal{A}} &= (5, 8, 11, 14) \quad p = 0.25 \\ (\bar{x}_{\tilde{\mathcal{A}}_{\mathcal{H}}}, \bar{y}_{\tilde{\mathcal{A}}_{\mathcal{H}}}) &= \left(\frac{2 \times 5 + 7 \times 8 + 7 \times 11 + 2 \times 14}{18}, \frac{-32(0.25)^2 - 16(0.25) + 48 - 3 \times (8)^2 - 3 \times (11)^2 + 6 \times 8 \times 11}{144(1-0.25)} \right) \\ (\bar{x}_{\tilde{\mathcal{A}}_{\mathcal{H}}}, \bar{y}_{\tilde{\mathcal{A}}_{\mathcal{H}}}) &= (9.5, 0.1389) \\ \text{Rank}(\tilde{\mathcal{A}}) &= \sqrt{(9.5)^2 + (0.1389)^2} \\ &= 9.501015 \end{aligned}$$

Using T.C. Chu and C.T. Tsao centroid of area method

Let $\tilde{\mathcal{A}} = (5, 8, 11, 14)$ $p = 0.5$

$$\begin{aligned} \text{Rank}(\tilde{\mathcal{A}}) &= \frac{\frac{1}{6} \int_5^8 x(x-5)dx + \frac{1}{2} \int_8^{9.5} xdx + \frac{1}{3} \int_8^{9.5} x(x-8)dx + \int_{9.5}^{11} xdx - \frac{1}{3} \int_{9.5}^{11} x(x-9.5)dx + \frac{1}{6} \int_{11}^{14} x(x-11)dx}{\frac{1}{6} \int_5^8 (x-5)dx + \frac{1}{2} \int_8^{9.5} dx + \frac{1}{3} \int_8^{9.5} (x-8)dx + \int_{9.5}^{11} dx - \frac{1}{3} \int_{9.5}^{11} (x-9.5)dx + \frac{1}{6} \int_{11}^{14} (x-11)dx} \\ &= \frac{\frac{1}{6} \left[\frac{63}{2} \right] + \frac{1}{2} \left[\frac{105}{8} \right] + \frac{1}{3} \left[\frac{81}{8} \right] + \frac{123}{8} - \frac{1}{3} \left[\frac{189}{16} \right] + \frac{1}{6} \left[\frac{117}{2} \right]}{\frac{1}{6} \left[\frac{9}{2} \right] + \frac{1}{2} \left[\frac{3}{2} \right] + \frac{1}{3} \left[\frac{9}{8} \right] + \frac{3}{2} - \frac{1}{3} \left[\frac{9}{8} \right] + \frac{1}{6} \left[\frac{9}{2} \right]} \\ &= \frac{36.375}{3.75} \\ &= 9.7 \end{aligned}$$

Our new ranking method

$$\begin{aligned} (\bar{x}_{\tilde{\mathcal{A}}_{\mathcal{H}}}, \bar{y}_{\tilde{\mathcal{A}}_{\mathcal{H}}}) &= \left(\frac{2p_1+7p_2+7p_3+2p_4}{18}, \frac{-32p^2-16p+48-3p_3^2-3p_2^2+6p_2p_3}{144(1-p)} \right) \\ \tilde{\mathcal{A}} &= (5, 8, 11, 14) \quad p = 0.5 \\ (\bar{x}_{\tilde{\mathcal{A}}_{\mathcal{H}}}, \bar{y}_{\tilde{\mathcal{A}}_{\mathcal{H}}}) &= \left(\frac{2 \times 5 + 7 \times 8 + 7 \times 11 + 2 \times 14}{18}, \frac{-32(0.5)^2 - 16(0.5) + 48 - 3 \times (8)^2 - 3 \times (11)^2 + 6 \times 8 \times 11}{144(1-0.5)} \right) \end{aligned}$$

$$(\bar{x}_{\bar{A}_{3f}}, \bar{y}_{\bar{A}_{3f}}) = (9.5, 0.06944)$$

$$\text{Rank}(\bar{A}) = \sqrt{(9.5)^2 + (0.06944)^2}$$

$$= 9.500$$

Using T.C. Chu and C.T. Tsao [9] centroid of area method

$$\text{Let } \bar{A} = (5,8,11,14) \quad p = 0.75$$

$$\text{Rank}(\bar{A})$$

$$= \frac{\frac{3}{12} \int_5^8 x(x-5)dx + \frac{3}{4} \int_8^{9.5} xdx + \frac{1}{6} \int_8^{9.5} x(x-8)dx + \int_{9.5}^{11} xdx - \frac{1}{6} \int_{9.5}^{11} x(x-9.5)dx + \frac{3}{12} \int_{11}^{14} x(x-11)dx}{\frac{3}{12} \int_5^8 (x-5)dx + \frac{3}{4} \int_8^{9.5} dx + \frac{1}{6} \int_8^{9.5} (x-8)dx + \int_{9.5}^{11} dx - \frac{1}{6} \int_{9.5}^{11} (x-9.5)dx + \frac{3}{12} \int_{11}^{14} (x-11)dx}$$

$$= \frac{\frac{63}{8} + \frac{315}{32} + \frac{27}{16} + \frac{123}{8} - \frac{63}{32} + \frac{117}{8}}{\frac{9}{8} + \frac{9}{8} + \frac{3}{16} + \frac{3}{2} - \frac{3}{16} + \frac{9}{8}}$$

$$= \frac{47.4375}{4.675}$$

$$= 9.730769$$

Our new ranking method

$$(\bar{x}_{\bar{A}_{3f}}, \bar{y}_{\bar{A}_{3f}}) = \left(\frac{2p_1+7p_2+7p_3+2p_4}{18}, \frac{-32p^2-16p+48-3p_3^2-3p_2^2+6p_2p_3}{144(1-p)} \right)$$

$$\bar{A} = (5,8,11,14) \quad p = 0.75$$

$$(\bar{x}_{\bar{A}_{3f}}, \bar{y}_{\bar{A}_{3f}}) = \left(\frac{2 \times 5 + 7 \times 8 + 7 \times 11 + 2 \times 14}{18}, \frac{-32(0.75)^2 - 16(0.75) + 48 - 3 \times (8)^2 - 3 \times (11)^2 + 6 \times 8 \times 11}{144(1-0.75)} \right)$$

$$(\bar{x}_{\bar{A}_{3f}}, \bar{y}_{\bar{A}_{3f}}) = (9.5, -0.25)$$

$$\text{Rank}(\bar{A}) = \sqrt{(9.5)^2 + (-0.25)^2}$$

$$= 9.503289$$

Comparison of ranking methods,

	Centroid of area method	Our new ranking method
	$p = 0.25$	$p = 0.25$
(5,8,11,14)	9.64285	9.501015
(3,5,6,8)	4.027778	5.511842
(4,6,7,9)	6.648148	6.510023
(2,4,5,7)	4.6487	4.514466
(6,8,10,12)	9.09524	9.004286
(2,5,6,9)	5.7727	5.511842
(5,7,10,12)	8.46972	8.501135
(9,10,11,12)	10.544763	10.50621
(1,3,5,7)	4.09523	4.009633
(7,9,11,13)	10.09523	10.00386

	Centroid of area method	Our new ranking method
	$p = 0.50$	$p = 0.50$
(5,8,11,14)	9.7	9.500254
(3,5,6,8)	5.690	5.514728
(4,6,7,9)	6.690	6.512467
(2,4,5,7)	4.690	4.51799
(6,8,10,12)	9.133	9.004286
(2,5,6,9)	5.833	5.514728
(5,7,10,12)	8.602	8.500284
(9,10,11,12)	10.566	10.50772
(1,3,5,7)	4.132	4.009633
(7,9,11,13)	10.074	10.00386

	Centroid of area method	Our new ranking method
	$p = 0.75$	$p = 0.75$
(5,8,11,14)	9.730769	9.503289
(3,5,6,8)	5.710526	5.51576
(4,6,7,9)	6.710	6.513341
(2,4,5,7)	4.7105	4.519249
(6,8,10,12)	9.1538	9.001543
(2,5,6,9)	5.86	5.51576
(5,7,10,12)	8.44623	8.503676
(9,10,11,12)	10.5769	10.50826
(1,3,5,7)	4.1538	4.003471
(7,9,11,13)	10.153846	10.00139

CONCLUSION

In this paper, a new ranking of parabolic trapezoidal fuzzy number has been developed. The in-centre of centroids and focus of the parabolic ranking trapezoidal fuzzy numbers has been introduced. This method is simple and easy to implement. Application of this ranking procedure in various decision making problems such as fuzzy risk analysis and in fuzzy optimization like network analysis, decision-making, optimization, forecasting etc. additionally offers the precise order.

REFERENCES

- [1] Gani, A.N., & Mohamed, V.N. (2013). Solution of a fuzzy assignment problem by using a new ranking method. *International Journal of Fuzzy Mathematical Archive*, 2, 8-16.
- [2] Kumar, A., Singh, P., Kaur, A., & Kaur, P. (2011). A new approach for ranking nonnormal p-norm trapezoidal fuzzy numbers. *Computers & Mathematics with Applications*, 61(4), 881-887.
- [3] Dorohonceanu, B., & Marin, B. (2002). A simple method for comparing fuzzy numbers. *CiteSeerX Scientific Literature Digital Library and Search Engine*. Eu-rope PubMed Central. <http://europepmc.org>
- [4] Bisht, D.C., & Srivastava, P.K. (2020). Trisectional fuzzy trapezoidal approach to optimize interval data based transportation problem. *Journal of King Saud University-Science*, 32(1), 195-199.
- [5] Thangavelu, K., Uthra, G., & Shunmugapriya, S. (2017). A new approach on the membership functions of fuzzy numbers. *International Journal of Pure and Applied Mathematics*, 114(6), 145-152.
- [6] Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & mathematics with applications*, 57(3), 413-419.
- [7] Mallak, S.F., & Bedo, D. (2013). Particular fuzzy numbers and a fuzzy comparison method between them. *International Journal of Fuzzy Mathematics and Systems (IJFMS)*, 3(2), 113-123.
- [8] F Mallak, S.A.E.D., & M BEDO, D.U.H.A. (2013). A Fuzzy comparison method for particular fuzzy numbers. *Journal of Mahani Mathematical Research Center*, 2(1), 1-14.
- [9] Chu, T.C., & Tsao, C.T. (2002). Ranking fuzzy numbers with an area between the centroid point and original point. *Computers & Mathematics with Applications*, 43(1-2), 111-117.
- [10] Thirupathi, A., & CK, K.D. (2020). New Ranking of Generalized Hexagonal Fuzzy Number Using Centroids of Centroided Method. *Advances in Mathematics: Scientific Journal*, 9(8), 6229-6240.
- [11] Thirupathi, A., & Kirubhashankar, C. K. (2021, March). Novel Fuzzy Assignment Problem Using Hexagonal Fuzzy Numbers. *In Journal of Physics: Conference Series*, Vol. 1770, No. 1.