DOI : https://doi.org/10.56452/7-12-33

ISSN: 0974-5823 Vol. 7 No. 12 December, 2022

International Journal of Mechanical Engineering

Analysis of Practical Applications of Co-ordinate Geometry

S.K. Sahani*[1], K.S. Prasad[2], A.K. Thakur[3]

1*Department of Mathematics, MIT Campus, (T.U), Janakpurdham-45600, Nepal

2 Department of Mathematics, Thakur Ram Multiple Campus, (T.U), Birgunj

3 Department of Mathematics, Dr. C.V. Raman University, Bilaspur (C.G.)

Email:[1]sureshkumarsahani35@gmail.com,[2]kripasindhuchaudhary@gmail.com, [3]drakthakurnath@gmail.com

Abstract

The research has been conducted on practical application of coordinate geometry. Architecture, geolocation (GPS), determining latitude and longitude, locating air transportation, map projections, contemporary technology like robotics and video games, artistic works, etc. are examples of real-world applications of the aforementioned issue. The comprehensive paper includes in-depth research on the practical use of coordinate geometry for calculating distance and its significance in the current gaming industry. A brief historical overview of coordinate geometry and its use in our research field is provided at the outset of the report. The definitions and theorem statements we employed in our research are included in the report's subsequent sections. Two categories best describe our study aims. The first part includes a thorough explanation of how coordinate geometry is used to calculate the distance between two points on a map using their coordinates. Some mathematical examples with graphical representations are included in the description. The usage of coordinate geometry in game creation is covered in the second half of the paper. With the use of suitable graphs and examples, it describes how to use the slope formula, mid-point formula, and formula for equations of lines. Our study findings are mentioned in the report's conclusion, along with some of my own thoughts on the matter.

Keywords: Coordinate geometry, real life, section, applications, etc.

Objectives:

1. To use coordinate geometry to determine distance.
2. To study the use of coordinate geometry in the process of game designing.

Limitations:

1. The research is limited to the bounds of using coordinate geometry to determine the distances between various locations on a map using their coordinates. The map used for this research is a digitalized version of the political world map.
2. The analysis of practical application of coordinate geometry in game designing is limited to adding figures, shapes and paths in a game screen within specified pixels.

Introduction:

Historical Background: The creation of analytic geometry, or geometry with coordinates and equations, was one of the most important events in the history of coordinate geometry. It was created by René Descartes (1596–1650) who introduced rectangular coordinates to locate points and to enable lines and curves to be represented with algebraic equations. The distance formula is a major part of analytic geometry and in mathematics, the Euclidean distance between two points in Euclidean space is the length of a line segment between the two points. It can be calculated from the Cartesian coordinates of the points using the Pythagorean theorem, therefore occasionally being called the Pythagorean distance. These names come from the ancient Greek mathematicians Euclid and Pythagoras, although Euclid did not represent distances as numbers, and the connection from the Pythagorean theorem to distance calculation was not made until the 18th century. Thus, the distance formula was said to be discovered by the Greek mathematician Pythagoras. The distance formula is used to find the distance between two points in a coordinate plane. Since its discovery, the distance formula has been used to determine distances between earthly to celestial bodies. The use of distance formula in map making is a must. Since centuries, Cartographers have used analytic geometry, especially the distance formula to calculate the distance between various cities, countries and locations. Likewise, analytic geometry, also known as coordinate geometry, is used in video game design to create 3D and 2D models and environments. It helps game designers to create geometric objects on the coordinate plane and write linear equations for each side. The foundation of modern game development was laid after the implementation of coordinate geometry in the field. Developers used the basic theorems of coordinate geometry like the mid-point theorem, distance formula, slope formula and equation of a line to create various objects and maps in a game which has become the backbone of modern day gaming industry.

Definitions:

1. **Coordinate geometry:** Coordinate geometry is a branch of geometry that deals with the study of geometric figures using coordinate systems. It is also known as analytic geometry because it involves using algebraic equations to describe geometric objects. In coordinate geometry, geometric objects are represented using a coordinate system, which is a system of numbers used to uniquely identify points in space. The most common coordinate system used in two-dimensional space is the Cartesian coordinate system, which uses two perpendicular axes (x and y) to define a point on the plane.
2. **Distance formula:** The distance formula in coordinate geometry is used to find the distance between two points in an XY plane1. The formula is derived from the Pythagorean theorem and is given by:
   $d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$
   where d is the distance between two points (x_1, y_1) and (x_2, y_2).
3. **Slope:** The slope of a line is a number that measures its "steepness", usually denoted by the letter m. It is the change in y for a unit change in x along the line.
4. **Equation of a line:** The equation of a line in coordinate geometry is an algebraic form of representing the set of points that together form a line in a coordinate system. The numerous points that together form a line in the coordinate axis are represented as a set of variables x, y to form an algebraic equation, which is referred to as an equation of a line. The standard form of equation of a line is $ax + by + c = 0$. Here a, b are the coefficients, x, y are the variables, and c is the constant term. It is an equation of degree one with variables x and y. The values of x and y represent the coordinates of the point on the line represented in the coordinate plane.

Theorems:

1. **Pythagoras Theorem:** The Pythagorean theorem or Pythagoras' theorem is a fundamental relation in Euclidean geometry between the three sides of a right triangle. It states that "the area of the square whose side is the hypotenuse (the side opposite the right angle) is equal to the sum of the areas of the squares on the other two sides." This theorem can be written as an equation relating the lengths of the sides p, b and the hypotenuse h, often called the Pythagorean equation: $h^2 = p^2 + b^2$.

2. **Mid-point Theorem:** In coordinate geometry, The Mid-point theorem states that "the midpoint of the line segment is an average of the endpoints." If (x_1, y_1) and (x_2, y_2) be the two endpoints then the coordinates of the mid-point (x, y) is given by:
   $(x, y) = ((x_1 + x_2)/2, (y_1 + y_2)/2)$

Discussions:

A. **Use of coordinate geometry in measurement of distance between two locations on a map using their coordinates.**

- The following research has been conducted on the given topic:

I. Terms Used:

1. **Flat Map:** A flat map is a two-dimensional representation of a three-dimensional object such as the Earth's surface. Maps are much smaller in size compared to the actual area of the Earth's surface they represent because of the use of scale.
2. **Scale:** Scale is an essential geographic concept that refers to the ratio of distance on a map to the corresponding distance on the ground.
3. **Latitude:** Latitude is a measurement on a globe or map of location north or south of the Equator.
4. **Longitude:** Longitude is a measurement of location east or west of the prime meridian. The prime meridian is an imaginary line that runs from the North Pole to the South Pole through Greenwich, England.
5. **Coordinates:** Coordinates of a location refer to the precise location of a point on Earth. The coordinates are always written with latitude first, followed by a comma and then its longitude.

II. Theory of the research:

In order to determine the distance between two locations on a map, the first order of business would be to determine their coordinates. This can be done through either manual test on a flat map by studying their latitudes and longitudes or through the use of internet tools like the Google Maps. After the coordinates are determined, the distance formula:

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

is used to calculate the distance between the two locations in a specific unit which varies from map to map due to their scaling. After the distance is calculated, it is then converted into the required measurement, i.e. either kilometer, miles, etc. by using the provided scale on the map.

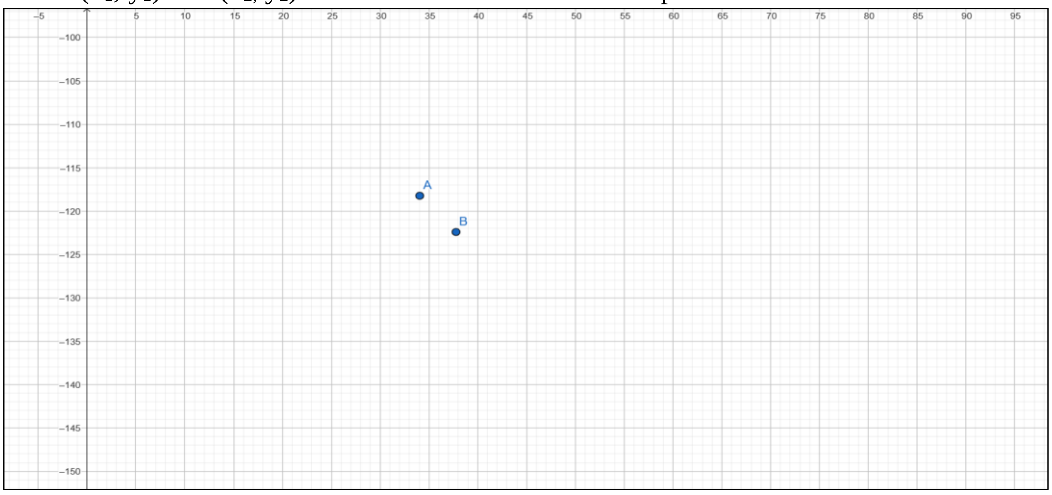
III. Examples:

1. The coordinates of New York city are (40.730610, -73.935242) and that of Florence city is (41.29246,12.5736108). Use coordinate geometry to find the distance between these two cities.

➢ The distance between these two cities can be calculated by initializing their coordinates as points on a coordinate plane and using the distance formula:

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.



Using the coordinates provided by the question, we can find the distance between New York city and Florence as follows:

$x_1 = 40.730610$

$y_1 = -73.935242$

$x_2 = 41.29246$

$y_2 = 12.5736108$

Here, A represents the coordinates of New York while B represents the coordinates of Florence.

$d = \sqrt{[(41.29246 - 40.730610)^2 + \{12.5736108 - (-73.935242)\}^2]}$

$= \sqrt{[(0.56185)^2 + (86.5088528)^2]}$

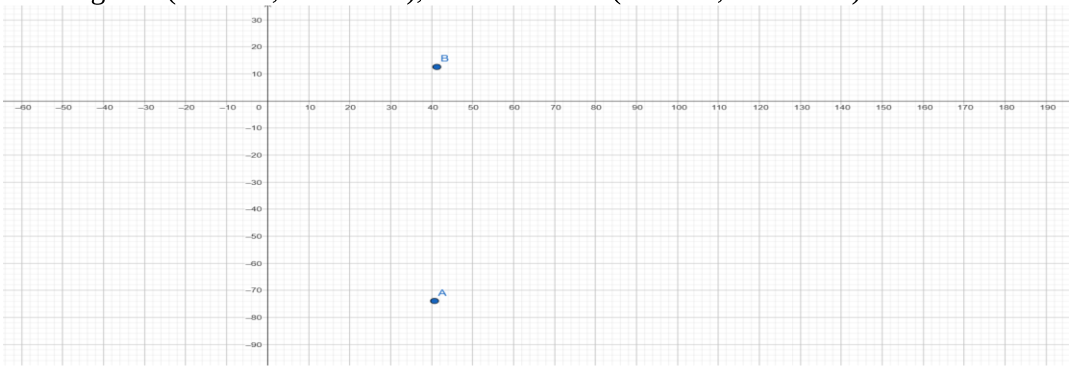
$= \sqrt{(7485.365)}$

≈ 86.5 units.

Therefore, the distance between New York City and Florence is approximately 86.5 units as per the given map coordinates.

2. The coordinates for Los Angeles are (34.0522° N, 118.2437° W) and the coordinates for San Francisco are (37.7749° N, 122.4194° W). Use the distance formula to find the distance between these two cities.s

➢ The distance between these two cities can be calculated by first initializing their coordinates as points on a coordinate plane. For this, we need to convert the coordinates to decimal form as:
Los Angeles: (34.0522, -118.2437), San Francisco: (37.7749, -122.4194).



Now, we use the distance formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
where (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.
Using the coordinates provided by the question, we can find the distance between Los Angeles and San Francisco as follows:
$x_1 = 34.0522$
$y_1 = -118.2437$
$x_2 = 37.7749$
$y_2 = -122.4194$
Here, A represents the coordinates of Los Angeles while B represents the coordinates of San Francisco.
$d = \sqrt{(-122.4194 - (-118.2437))^2 + (37.7749 - 34.0522)^2}$
$= \sqrt{(-4.1757)^2 + 3.7227^2}$
$= \sqrt{17.4225 + 13.8531}$
$= \sqrt{31.2756}$
≈ 5.59 units.

Therefore, the distance between Los Angeles and San Francisco is approximately 5.59 units as per the given map coordinates.

B. Use of coordinate geometry in placing or creating various objects and paths for such objects in a game as a part of designing the game.

➢ The following research has been conducted on the given topic:

I. Terms used:

1. **Pixel:** A pixel is a minute area of illumination on a display screen, one of many from which an image is composed. Pixels are combined to form a complete image, video, text or any visible thing on a computer display. A pixel is also known as a picture element and is the basic logical unit in digital graphics.
2. **Trajectory:** Trajectory refers to the path followed by a projectile flying or an object moving under the action of given forces. Simply put, it is the path travelled by a body.
3. **Projectile:** A projectile is any object thrown into space upon which the only acting force is gravity. The primary force acting on a projectile is gravity. This doesn't necessarily mean that other forces do not act on it, just that their effect is minimal compared to gravity.
4. **Screen:** A screen in a computer is a device with a viewing portion that displays the output of a computer to the user. It is used to view the ongoing as well as completed activities in a computer.

5. **Ramp:** A ramp is a sloping surface that connects two different levels. It is used to move objects from one level to another.

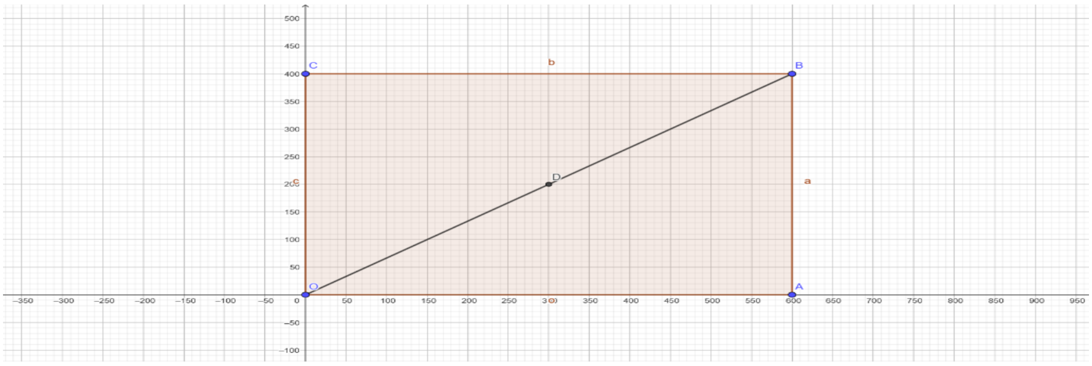
II. Theory of the research:

The use of coordinate geometry in game designing is a vast field with numerous applications. The most common uses of coordinate geometry in this sector are creating objects, shapes, marking the line of motion (trajectory) of such objects and so on. In order to create an object, we first need the information regarding the pixel of the screen and the object we want to create and display on the screen. On a graph, a pixel can represent a point or coordinate. For example, if we say that a rectangle is being created or drawn in a resolution of 500x400 pixels, i.e. the resolution of the screen and one of its vertices (say A) is at the origin, then its other vertex on the x-axis forming a line with A would be at the coordinate (500,0). This will become more clear through the mathematical examples presented after this section. After identifying the pixel (or coordinates) of the object we can use the various formulae included in coordinate geometry such as slope formula, formula for equation of line, mid-point formula, etc. to get the required result.

III. Examples:

1. Ram wants to create a circle of certain radius in the exact center of the screen which is rectangular. The resolution in which he is making the game is 600x400 pixels. Find the coordinates where he should place the center of the circle and draw it.

➢ To find the coordinates of the center of the circle, first of all draw the computer screen and assign the coordinates of it vertices using the given pixels:



Here, O (0,0), A (600,0), B (600,400) and C (0,400) are the coordinates of the screen with D (X, Y) being the center of the screen. Since, the screen is a rectangle the point D passes through one of the diagonals it is thus the mid-point of diagonal OB. Now, we can obtain the coordinates of point D by using the mid-point formula:

Mid-point of a line segment joining two points (x_1, y_1) and (x_2, y_2) is given by:

Midpoint = $\{(x_1 + x_2)/2, (y_1 + y_2)/2\}$

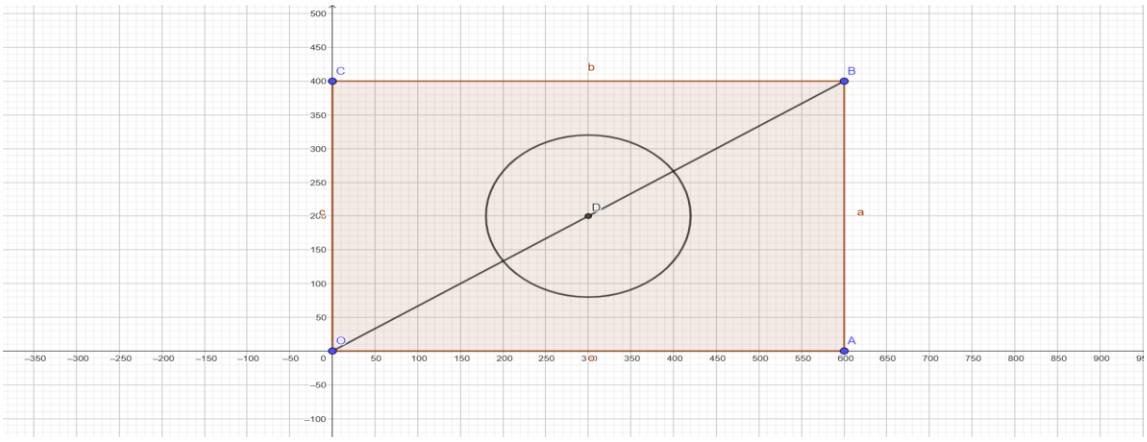
In this case, the two points are (0, 0) and (600, 400). Therefore, we can use the mid-point formula to find the mid-point of the line segment joining these two points:

$(X, Y) = ((0 + 600)/2, (0 + 400)/2)$

$= (300, 200)$

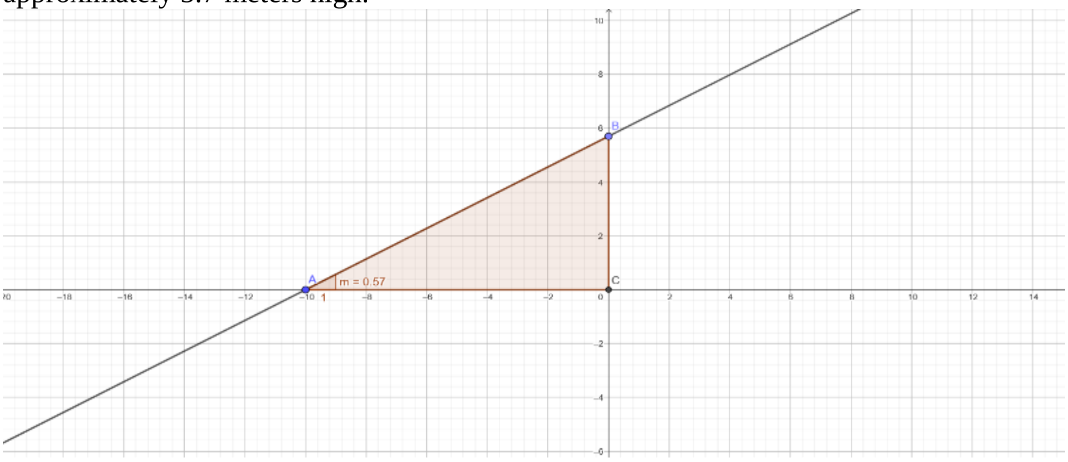
Therefore, the midpoint of the line segment joining the points O (0, 0) and B (600, 400) is D (300, 200). Hence, the coordinates of the center of the circle is (300,200).

DOI : https://doi.org/10.56452/7-12-33



2. Suppose you want to create a ramp in a game that has a slope of 30 degrees and a length of 10 meters. Calculate the height of the ramp.

➢ We know that the relation between slope, height and length of a line (here ramp) is given by:
Slope = rise / run.
Where rise is the height of the ramp and run is the length of the ramp.
Since, we know that the slope is 30 degrees and the length of the ramp is 10 meters, we can use trigonometry to find the height of the ramp:
tan (30) = rise / 10
rise = 10 * tan (30)
rise ≈ 5.7 meters
So, to create a ramp with a slope of 30 degrees and a length of 10 meters, we would need to make it approximately 5.7 meters high.

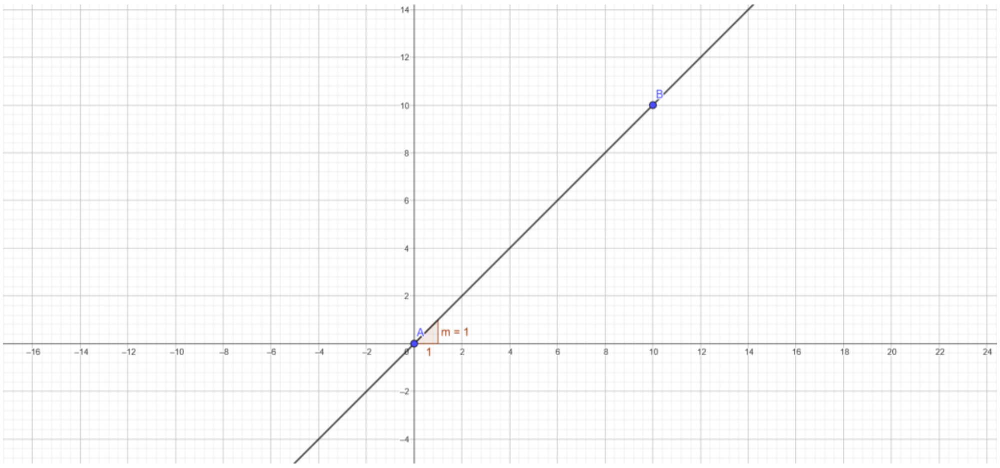


Here, ABC is the required ramp with length 10 meters and height nearly 5.7 meters. The scaling of this graph is presented below:
For x-axis:
5 small boxes = 2 meters
∴ 25 small boxes = 10 meters
For y-axis:
5 small boxes = 2 meters
∴ 14.25 small boxes = 5.7 meters.

3. Rajesh is a game designer and he wants to create a game where an object moves along a straight line path from point A (0,0) to point B (10,10). Use the formula for equation of a line to determine the Trajectory of the object.

➢ The trajectory of the object can be determined by the formula for equation of a straight line of degree one in x and y:
y = mx + c ……………. (i)
where m is the slope of the line and c is the y-intercept.

To find the slope of the line passing through points A and B, we can use:
$m = (y_2 – y_1) / (x_2 – x_1)$ ……………. (ii)
where $(x_1, y_1) = (0,0)$
and, $(x_2, y_2) = (10,10)$
Then, from equation (ii):
$m = (10 - 0) / (10 - 0)$
$\therefore m = 1$ ……………. (iii)
So, from equations (i) and (iii), the equation of the line passing through points A and B is:
$y = x + 0$ [Since, the straight line is passing though the origin.]
or
$y = x$
This means that the path of the object will be along a straight line from point A to point B with a slope of 1.



<u>Conclusion:</u>

Findings: Through the course of this research, we found several new ways to use coordinates and apply the theorems and formulae of analytic geometry. To begin with, the fact that coordinates could be converted directly into decimal points was a pretty shocking revelation (as we always thought there was an elaborate pre-established set of steps to do so). We also found out the way to establish relation between a Cartesian graph and the directions (North, South, East and West). For example, we discovered that the direction North-East represented the first quadrant of a Cartesian graph while South-West represented the third quadrant of the graph. Likewise, the serendipity of scale variation in multiple maps and their effects on the coordinates of a location along with its distance from another location on the same map was quite interesting.

Moreover, the application of coordinate geometry in the process of designing a game was very exhilarating to study. We found new ways to use the slope relation (through the concept of rise and run) and we also learned to derive the trajectory of any object based on only its coordinates on a graph. Beside these, the most interesting finding was the fact that we could actually divide a computer screen (which up until now we just thought to be a collection of VDUs) into several tiny fragments (pixels) and plot them on a graph. This meant that one could actually project an entire screen on a single Cartesian graph.

These findings led me to conclude that the impact of coordinate geometry on modern day technology and developments is quite dramatic.

Remarks:

- The use of distance formula in this project is limited to 2-dimensional plane. But this formula has already expanded to a higher dimension, i.e. 3-dimension. If we are able to somehow insert more axes in this formula, then the calculation of distance between objects in higher dimensions could be possible which could even lead to major breakthroughs in the research of objects like wormhole and warp techniques.

- Likewise, integration of gaming experience in this multi-dimension world has begun in the modern era. But the question remains, till what extent can we integrate the 2D (some 3D) world of gaming with the infinite possible coordinates of our real world. If, by some means, we are successful in integrating the virtual and real worlds through the intermixing of the coordinates of the two worlds, all those science fiction (sci-fi) movies and scenes would no longer remain a fiction.

References:

1. T.L. Health, The works of Archimedes, Cambridge 1897.
2. J.M Child, The Early Mathematical Manuscripts of Leibnitz, Chicago and London 1920.
3. N.J. Lennes, Differential and Integral Calculus, New York,1931.
4. F. Cajori, Bibliography of Fluxions and calculus, Textbooks printed in the United States. United states Bureau of Education, circular of Information, 1890, no. 3, pp. 395-400.
5. ________________________, Grafting of Theory of Limits of the calculus of Leibnitz, American Mathematical Monthly, xxx, 1923, 223-234.
6. _________________, A history of the conceptions of Limits and Fluxions in Great Britain from Newton to wood house, Chicago an London, 1919.
7. ______________, The history of notations of the Calculus, Annals off Mathematics (2), xxv, 1924,146.
8. K. Smith, Precalculus: A Functional Approach to Graphing and Problem Solving, Jones & Bartlett Publishers, p. 8, ISBN 978-0-7637-5177-7, 2013
9. D. Cohen, Precalculus: A Problems-Oriented Approach (6th ed.), Cengage Learning, p. 698, ISBN 978-0-534-40212-9, 2004.
10. R. N. Aufmann, et al., College Trigonometry (6th ed.), Cengage Learning, p. 17, ISBN 978-1-111-80864-8, 2007.
11. D. Judith and P. Sally," Chapter 3: Pythagorean triples". Roots to research: a vertical development of mathematical problems. American Mathematical Society Bookstore. p. 63. ISBN 978-0-8218-4403-8, 2007.
12. D. Benson, The Moment of Proof: Mathematical Epiphanies, pp. 172–173 (Oxford University Press, 1999).
13. Euclid (1956), pp. 351–352
14. Huffman, Carl (23 February 2005). "Pythagoras". In Zalta, Edward N. (ed.). The Stanford Encyclopedia of Philosophy (Winter 2018 Edition)., "It should now be clear that decisions about sources are crucial in addressing the question of whether Pythagoras was a mathematician and scientist. The view of Pythagoras's cosmos sketched in the first five paragraphs of this section, according to which he was neither a mathematician nor a scientist, remains the consensus."