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STUDY OF THERMAL INSTABILITY OF A MICROPOLAR FLUID WITH COUPLE-STRESS HEATED FROM BELOW

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ABSTRACT

This paper discusses the issue of thermal convection in a micropolar fluid with couple-stress that saturates a porous medium. We perform a linear stability analysis and normal mode analysis to determine the dispersion relation. Using analytical and graphical methods, they determine the critical Rayleigh number for instability onset. We find that, for stationary convection, various factors such as couple-stress, coupling, medium permeability, and heat flux can have stabilizing or destabilizing effects depending on specific conditions. We also observe the principle of exchange of stabilities to be valid in certain scenarios

KEYWORDS: thermal instability, micropolar fluid, couple-stress fluid

INTRODUCTION

The concept of micropolar fluids was introduced by Eringen in 1964 to account for systems that cannot be accurately described by the Navier-Stokes equation. Micropolar fluids belong to the category of microfluids, which exhibit micro-rotational inertia. Examples of such fluids include colloidal solutions, liquid crystals, and animal blood. In addition to the velocity vector, micropolar fluids have two extra variables: the micro-inertia tensor and the spin vector. The spin vector is responsible for micro-rotation, while the micro-inertia tensor creates disturbances in the molecules within the fluid elements. The theory of micropolar fluids was further developed by Kazakia and Ariman in 1971.

Micropolar fluid instability problems have gained significant attention in current research. Shukla and Isa (1975) and Chandrashekhar (1981) are prominent scholars in the field of micropolar theory. Rayleigh-Benard instability in a horizontal thin layer of micropolar fluid has become a well-known stability problem. Chandrashekhar (1981) extensively studied thermal convection in a horizontal thin layer of Newtonian fluid, taking into account the effects of hydrodynamics and hydromagnetics. The book by Gezegorz (1999) provides a wide range of applications and modeling factors of micropolar fluids. Ariman's (1973) review paper offers further research and important discussions on micropolar fluids with microstructures.

Researchers have studied the impact of microstructures on Rayleigh-Benard instability, and they have found that the principle of exchange of instabilities remains valid when there is no coupling between thermal and micropolar effects. However, Perez and other researchers have shown that this principle may not hold true when there is coupling between these effects. Bradley and Lekkerkerker in 1978 have demonstrated the existence of oscillatory motions in micropolar fluids in dielectric fluid and liquid crystal, respectively.

Micropolar fluids have emerged as a crucial area of study not only in engineering, but also in various industries such as food processing, chemical processing, and metal solidification. As a result, the understanding and exploration of micropolar fluids has become an essential component of modern technology and industrial development. Researchers have emphasized the significance of investigating the behavior of these fluids in the presence of porous mediums. Moreover, the micropolar theory has proven to be highly valuable in the field of engineering and various branches of mathematics.

The study of micropolar fluid stability has been a subject of interest for numerous scholars and authors, such as Kaloni and Qin (1992). To analyze the behavior of fluids in porous media with very low permeability, a generalized Darcy's Walter model has been used, which takes into account the impact of inertial forces.

The concept of couple-stress fluid, introduced by Stokes in 1966, has found numerous applications in understanding the mechanics of synovial joints, which is a prominent area of research. Synovial joints present in the human body, such as those in the knee, hip, shoulder, and ankle, contain normal synovial fluid that is viscous and non-Newtonian in nature. As a result of its negligible wear and low friction coefficient, the use of couple-stress fluid has been proposed as a viable model for synovial fluid in these joints. This proposal was first suggested by Walicki and Walicka in 1999.

The impact of couple-stress fluid has been a topic of investigation by various researchers. For instance, Kumar et al. (2010) delved into the behavior of a couple-stress fluid heated from below in hydromagnetics, while Stokes et al. (1969) studied the effects of couple-stresses on heat transfer in fluids.

The exploration of couple-stress fluids in various conditions continues to be an active area of research. For example, Kumar et al. (2020) investigated the thermal instability of a rotating couple-stress ferromagnetic fluid in the presence of a variable gravity field, while Nadian et al. (2021) examined double-diffusive convection in a rotating couple-stress ferromagnetic fluid subjected to varying gravitational and horizontal magnetic fields, with a porous medium present.

To date, the issue of thermal convection in a couple-stress micropolar fluid that is saturating in a porous medium has not been explored, based on our current knowledge. As a result, this paper aims to address this instability problem.

FORMULATION OF PROBLEM

Our study examines a layer of incompressible couple-stress micropolar fluid with a thickness d that is confined between two horizontal planes located at z=0 and z=d in a porous medium. The layer is initially at rest and heated from below. Convection occurs when a critical temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is exceeded, which depends on the boundary conditions and bulk properties of the fluid. Prior to reaching the critical gradient, the fluid remains stationary.

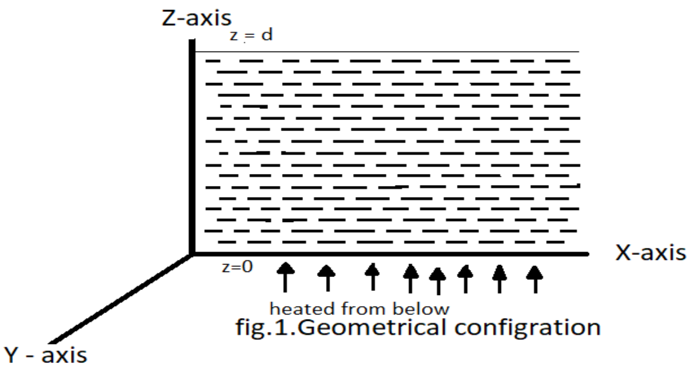

fig.1.Geometrical configration

Governing equations for the motion of couple-stress micropolar fluid situated in porous medium following Boussinesq approximation are as follows:

Continuity equation

$$\nabla \cdot v = 0 , \tag{1}$$

Momentum equation

$$\frac{dv}{dt} = \varepsilon\left(-\nabla\frac{p}{\varrho_0} + \frac{\kappa}{\varrho_0}\nabla\times\vartheta - \frac{1}{\varrho_0 k_1}(\mu+\kappa)v - \frac{\varrho g\widehat{e_z}}{\varrho_0} + \frac{1}{\varepsilon}\left(\nu - \frac{\mu^{|}}{\varrho_0}\nabla^2\right)\nabla^2 v\right) \quad (2)$$

Angular momentum equation

$$\varrho_0 j\frac{d\vartheta}{dt} = (\varepsilon^{|}+\beta^{|})\nabla(\nabla\cdot\vartheta) + \frac{\kappa}{\varepsilon}\nabla\times v + \gamma\nabla^2\vartheta - 2\kappa\vartheta \quad (3)$$

Temperature equation

$$\left[\varrho_0 C_v\varepsilon + \varrho_s C_s(1-\varepsilon)\right]\frac{dT}{dt} + \varrho_0 C_v(v\cdot\nabla)T = \delta(\nabla\times v).\nabla T + k_T\nabla^2 T \quad (4)$$

Where v and T are used for velocity and temperature ; ϑ and ϱ are used for spin and total density ; ϱ_s and ϱ_0 are used for solid matrix density and reference density ; κ_1 and ε are used for medium permeability and medium porosity ; g and μ are used for gravity acceleration and viscosity coefficient ; p and $\hat{e}_z$ are used for thermodynamic pressure and unit vector in z-direction ; j and $\varepsilon^{|}$ are used for microinertia constant and bulk spin viscosity constant ; γ and $\beta^{|}$ are used micropolar viscosity constant and shear spin viscosity constant ; κ and ν are used for coupling viscosity constant and kinematics viscosity ; $\mu^{|}$ is used for couple stress viscosity. Also, in temperature equation,

c_s, c_v are used for heat of solid (porous matrix) material and specific heat at constant volume; κ_T , δ are used for thermal conductivity and micropolar heat conduction constant.

Basic state equation is,

$$\varrho = \varrho_0\left[1-\alpha\left(T-T_0\right)\right], \quad (5)$$

Where α is used for constant of thermal expansion and T_0 is used for average temperature as T=$\frac{(T_1+T_2)}{2}$, where T_1 , T_2 are constant temperatures at lower and upper boundary of fluid layer.

In the basic state, when fluid is at rest,

v = v_b = (0,0,0), $\varrho = \varrho_b$ = (0,0,ϱ_0) , T = T_b = (0,0,0) , p = p_b = (0,0,0) , $\vartheta = \vartheta_b$ = (0,0,0)

By (2)

$$-\nabla p - \varrho g\widehat{e_z} = 0$$

$$\Rightarrow \frac{d\varrho_b}{dz} + \varrho_b g = 0 \quad (6)$$

By (4)

$$\nabla^2 T = 0$$

$$\Rightarrow T = T_0 - \beta z \quad (7)$$

Where $\beta = \frac{(T_2-T_1)}{d}$

By (5)

$$\varrho = \varrho_0(1+\alpha\beta z) \quad (8)$$

PERTURBATION EQUATIONS

We consider small perturbation for stability around basic state. The basic state is

v = $v_b + u$, $\vartheta = \vartheta_b + \omega$, p = $p_b(z) + \delta p$, $\varrho = \varrho_b(z) + \delta\varrho$, T = $T_b(z) + \theta$

where $u(u_x, u_y, u_z)$, $\omega(\omega_x, \omega_y, \omega_z)$, $\delta\varrho$, δp , θ are used for perturbation in velocity v, spin ϑ , pressure p, density ϱ , temperature T respectively. Equation (1) – (5) after perturbation are,

$$\nabla.u = 0, \quad (9)$$

$$\frac{du}{dt} = \varepsilon\left(-\nabla\frac{\delta p}{\varrho_0} + \frac{\kappa}{\varrho_0}\nabla\times\omega - \frac{1}{\varrho_0 k_1}(\mu+\kappa)u + g\alpha\theta\widehat{e_z} + \frac{1}{\varepsilon}\left(\nu - \frac{\mu^{|}}{\varrho_0}\nabla^2\right)\nabla^2 u\right) \tag{10}$$

$$\varrho_o j\frac{d\omega}{dt} = (\varepsilon^{|}+\beta^{|})\nabla(\nabla.\omega) + \frac{\kappa}{\varepsilon}\nabla\times u + \gamma\nabla^2\omega - 2\kappa\omega \tag{11}$$

$$\left[\varrho_0 C_v\varepsilon + \varrho_s C_s(1-\varepsilon)\right]\frac{d\theta}{dt} = k_T\nabla^2\theta + \delta(\nabla\times\omega)\cdot\nabla\theta - \delta(\nabla\times\omega)_z\beta + \varrho_0 C_v\beta u_z \tag{12}$$

$$\delta\varrho = -\alpha\varrho_b\theta \tag{13}$$

On putting, $t = \frac{\varrho_0 d^2}{\mu}t^*$, $z = z^* d$, $\theta = \theta^*\beta d$, u = $\frac{\kappa_T}{d}u^*$, $\omega = \frac{\kappa_T}{d^2}\omega^*$, p = $\frac{\mu\kappa_T}{d^2}p^*$ in equation (9) - (13) and removing stars for convenience. We get non-dimensional equations as,

$\nabla\cdot u = 0$
(14)

$$\frac{du}{dt} = \varepsilon\left(-\nabla\delta p + K(\nabla\times\omega) - \frac{1}{\overline{\kappa}_1}(1+K)u + R\theta e_z + F\left(\nu - \frac{\mu^{|}}{\varrho_0}\nabla^2\right)u\right) \tag{15}$$

$\bar{j}\frac{d\omega}{dt} = C_1\nabla(\nabla\cdot\omega) + K\left(\frac{1}{\varepsilon}\nabla\times u - 2\omega\right) - C_0\nabla\times\nabla\times\omega$
(16)

$$EP\frac{d\theta}{dt} = \nabla^2\theta + \bar{\delta}\left(\nabla\theta\cdot\nabla\times\omega - (\nabla\times\omega)_z\right) + u_z \tag{17}$$

where new non-dimensional coefficient are,

$$\bar{\delta} = \frac{\delta}{\varrho_0 C_v d^2}\ ,\ \bar{j} = \frac{j}{d^2}\ ,\ K = \frac{\kappa}{\mu}\ ,\ \overline{k_1} = \frac{k_1}{d^2}\ ,\ C_1 = \frac{\varepsilon^{|}+\beta^{|}}{\mu d^2}$$

$$C_0 = \frac{\gamma}{\mu d^2}\ ,\ F = \frac{\varrho_0 d^2}{\mu\varepsilon}\ ,\ E = \varepsilon + \frac{(1-\varepsilon)\varrho_s C_s}{\varrho_0 C_v}$$

$$P = \frac{\mu}{\varrho_0\kappa_T}\ ,\ R = \frac{g\alpha\beta\varrho_0 d^4}{\mu\kappa_T}$$

Here R = Rayleigh number and P = Prandtl number.

LINEAR THEORY AND DISPERSION RELATION

In linear stability we only consider the linear term and due to very small disturbance the non-linear term in (14) – (17) namely $(u\cdot\nabla)\omega$, $\nabla\theta\cdot\nabla\times\omega$, $(u\cdot\nabla)u$, $(u.\nabla)\theta$ may be neglected.

Apply curl operator twice on (15) and take only z-component, then linearized form of (15) is

$$\varepsilon^{-1}\frac{\partial}{\partial t}\left(\nabla^2 u_z\right) = R\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + K\nabla^2\xi - \frac{1}{\overline{k}_1}(1+K)\nabla^2 u_z + F\left(L - \frac{\mu^{|}}{\varrho_0}\nabla^2\right)\nabla^2 u_z \tag{18}$$

Where $\xi = \left(\frac{\partial\omega_y}{\partial x} - \frac{\partial\omega_x}{\partial y}\right) = (\nabla\times\omega)_z$ consider only z-components

Apply curl operator once on (16) and take only z-component, then linearized form of (16) is

$$\bar{j}\frac{\partial\xi}{\partial t} = C_0\nabla^2\xi - K\left(\frac{1}{\varepsilon}\nabla^2 u_z + 2\xi\right)$$

Where K use for coupling between vorticity and spin effects and C_0 use for spin diffusion.

Now, linearized form of (17) is

$$EP\frac{\partial\theta}{\partial t} = \nabla^2\theta + u_z - \bar{\delta}\xi \tag{20}$$

NORMAL MODE ANALYSIS

By normal mode method we assume solution of equation (18) - (20) are

$$\left[u_z , \xi , \theta \right] = \left[U(z) , G(z) , \odot(z) \right] exp\left(ik_x x + ik_y y + \sigma t\right) \tag{21}$$

Now we consider $k = \left(k_x^2 + k_y^2\right)^{\frac{1}{2}}$, k is wave number and σ(complex number in general) is stability parameter

After use of (21) equation (18) – (20) become,

$$(D^2 - k^2)\left[\varepsilon^{-1}\sigma - M + B(D^2 - k^2) + \frac{1}{\overline{k}_1}(1+K)\right]U = K(D^2 - K^2)G - R\theta K^2 \tag{22}$$

$$\left[l\sigma + 2A - \left(D^2 - k^2\right)\right]G = -A\varepsilon^{-1}\left(D^2 - k^2\right)U \tag{23}$$

$$\left[EP\sigma - \left(D^2 - k^2\right)\right]\odot = -\overline{\delta}G + U \tag{24}$$

Where $A = \frac{K}{C_0}$, $l = \bar{j}\frac{A}{K}$, $D = \frac{d}{dz}$, $M = Fv$, $B = F\frac{\mu^{l}}{\varrho_0}$

Now on eliminating G and $\odot$ between (22) – (24), we get,

$$\begin{aligned}(D^2 - k^2)&\left[\varepsilon^{-1}\sigma - M + \frac{1}{\overline{k}_1}(1+K) + B(D^2 - k^2)\right]\left[l\sigma - (D^2 - k^2) + 2A\right]\left[EP\sigma - (D^2 - k^2)\right]U \\ &= -Rk^2\left[l\sigma + (D^2 - k^2)(\overline{\delta}\varepsilon^{-1}A - 1) + 2A\right]U - KA\sigma\varepsilon^{-1}PE(D^2 - k^2)^2U + KA\varepsilon^{-1}(D^2 - k^2)^3U\end{aligned} \tag{25}$$

Now boundary condition transforms to

$$U = D^2U = 0 , G = 0 = \odot \text{ at } z = 0 \text{ and } 1 \tag{26}$$

For proper solution of U characterizing lowest mode is

$$U = U_0 \sin \pi z \tag{27}$$

where U_0 is constant.

Put value of U by (27) in (25) and also put $\pi^2 + k^2 = b$ then we get

$$\begin{aligned}b&\left[\varepsilon^{-1}\sigma - M - Bb + \frac{1}{\overline{k}_1}(1+K)\right]\left[l\sigma + b + 2A\right]\left[EP\sigma + b\right] \\ &= Rk^2\left[l\sigma + b\left(1 - \overline{\delta}\varepsilon^{-1}A\right) + 2A\right] + KA\sigma\varepsilon^{-1}PEb^2 + KA\varepsilon^{-1}b^3\end{aligned} \tag{28}$$

ANALYTICAL DISCUSSION

(i)Stationary convection

At stationary convection, when stability sets, the marginal state will be characterized by $\sigma = 0$. So, put $\sigma = 0$ in (28), we get

$$b^2\left[\frac{(1+K)}{\overline{k}_1} - M - Bb\right]\left[2A + b\right] = Rk^2\left[2A + b\left(1 - \overline{\delta}\varepsilon^{-1}A\right)\right] + KA\varepsilon^{-1}b^3$$

$$R = \frac{\left[\left(\frac{1+K}{\overline{k}_1}\right) - M - 2BA - KA\varepsilon^{-1}\right]b^3 + \left[\frac{2A(1+K)}{\overline{k}_1} - 2AM\right]b^2 - Bb^4}{k^2\left[2A + b\left(1 - \overline{\delta}\varepsilon^{-1}A\right)\right]} \tag{29}$$

This relation is in Rayleigh Number R as a function of parameters K(coupling), B(couple-stress), $\overline{k}_1$ (permeability) and $\overline{\delta}$(heat flux).

For behavior of couple stress(B), we find $\frac{dR}{dB}$

$$\frac{dR}{dB} = \frac{-2Ab^3 - b^4}{k^2\left[2A + b\left(1 - \overline{\delta}\varepsilon^{-1}A\right)\right]} \tag{30}$$

So, Couple-stress has a destabilizing effect on the system under the condition

$\bar{\delta}\varepsilon^{-1}A < 1$.

For behavior of coupling(K), we find $\frac{dR}{dK}$

$$\frac{dR}{dK} = \frac{\left(\frac{b^3}{\bar{k}_1} - A\varepsilon^{-1}b^3 + \frac{2A}{\bar{k}_1}b^2\right)}{k^2\left(2A + b\left(1-\bar{\delta}\varepsilon^{-1}A\right)\right)} \quad (31)$$

So, Coupling has a stabilizing effect on the system under the condition

$\frac{1}{\bar{k}_1} - A\varepsilon^{-1} > 0$ and $\bar{\delta}\varepsilon^{-1}A < 1$.

On the other hand, coupling has a destabilizing effect on system under condition

$\bar{k}_1 < 0$ and A=0.

For behavior of permeability ($\bar{k}_1$) we find $\frac{dR}{d\bar{k}_1}$

$$\frac{dR}{d\bar{k}_1} = \frac{-\frac{(1+K)b^3}{\bar{k}_1^{\,2}} - \frac{2A(1+K)}{\bar{k}_1^{\,2}}b^2}{k^2\left(2A + b\left(1-\bar{\delta}\varepsilon^{-1}A\right)\right)} \quad (32)$$

So, Permeability has a stabilizing effect on the system under the condition

A=0 and $1+K<0$.

On the other hand, permeability has a destabilizing effect under the condition

$0 < 1-\bar{\delta}\varepsilon^{-1}A$.

For behavior of heat flux($\bar{\delta}$) we find $\frac{dR}{d\bar{\delta}}$

$$\frac{dR}{d\bar{\delta}} = \frac{\left(\left(\left(\frac{1+K}{\bar{k}_1}\right) - M - 2BA - KA\varepsilon^{-1}\right)b^3 + \left(\frac{2A(1+K)}{\bar{k}_1} - 2AM\right)b^2 - Bb^4\right)\left(-k^2b\varepsilon^{-1}A\right)}{k^4\left(2A + b\left(1-\bar{\delta}\varepsilon^{-1}A\right)\right)^2} \quad (33)$$

So, heat flux has a stabilizing effect on the system under the condition

$\frac{1+K}{\bar{k}_1} < 0$ and $0 < 1-\bar{\delta}\varepsilon^{-1}A$.

(ii)Oscillatory mode

Equation (22)$\times U^*$(conjugate of U) and integrate with the range of z after that use equation (23)$\times G^*$ and (24) $\times \odot^*$ with boundary conditions (26), we get

$$\left(A\left(\left(M+Rk^2\right) - \frac{\sigma}{\varepsilon} + \left(\frac{1+K}{\bar{k}_1}\right)\right) + \varepsilon K\right)I_1 - ABI_2 + \varepsilon K\left(l\sigma^* + 2A\right) - RAk^2EP\sigma^*I_3 = 0 \quad (34)$$

Where

$I_1 = \int\left(\left|D^2U\right|^2 + 2k^2\left|D^2U\right| + k^4|U|^2\right)dz$

$I_2 = \int\left(\left|D^2U\right|^2 + 3k^2\left|D^2U\right|^2 + 3k^4|DU|^2 + k^6|U|^2\right)dz$

$I_3 = \int\left(\left|D^2U\right| + k^2|U|\right)dz$

All these integral from I_1 to I_3 are positive definite.

Now put $\sigma = \sigma_r + i\sigma_i$ in equation (34) and on taking imaginary part , we get

$\frac{A\sigma}{\varepsilon}\left(D^2-k^2\right)^2 - \varepsilon Kl\sigma + RAk^2EP\sigma\left(D^2-k^2\right) = 0$

$$\Rightarrow \sigma\left(\frac{A}{\varepsilon}\left|I_1\right| - K\varepsilon l + RAk^2EP\left|I_2\right|\right) = 0 \quad (35)$$

We have taken (35) positive definite in the absence of coupling between spin and heat flux because $|I_1|$ and $|I_2|$ are all positive definite. So, $\sigma = 0$.Therefore oscillatory modes are not allowed and principle of exchange of stabilities is satisfied for the problem.

NUMERICAL COMPUTATIONS

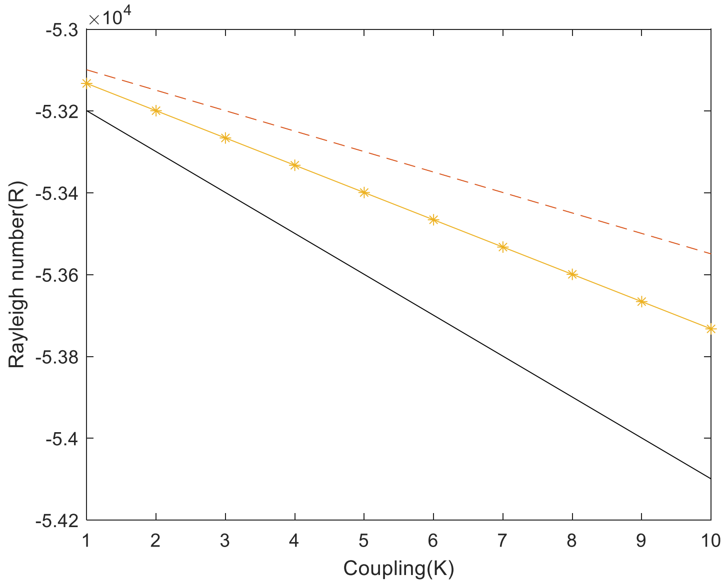

Fig. No. : 1

Variation of Rayleigh number and couple-stress(B) for

A=-60,$\bar{\delta}$=50,k=4, $\bar{k}_1$=4, M=45, K=23 (ε=-125,-100,-107)

(Destabilizing effect)

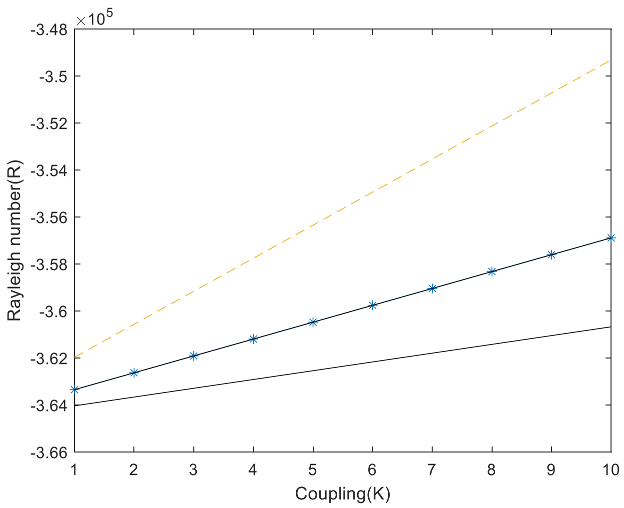

Fig. No. : 2

Variation of Rayleigh number and coupling(K) for

A=60, $\bar{\delta}$=40, ε=-110, k=2, M=60, B=35 ($\bar{k}_1$ = 0.4 , 3 , 4)

(Stabilizing effect)

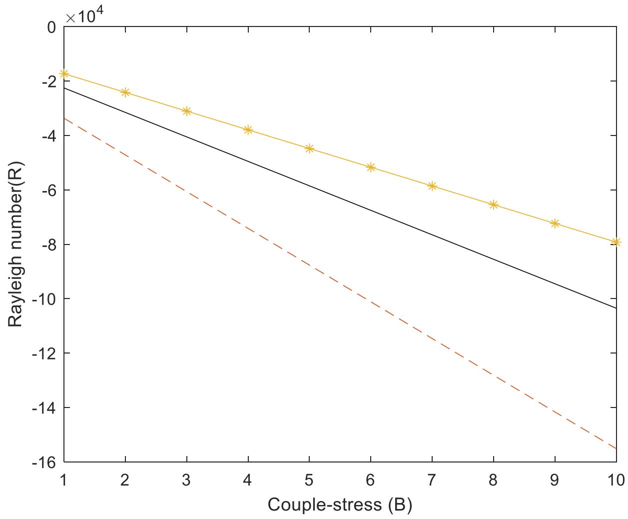

Fig. no. : 3

Variation of Rayleigh number and coupling(K) for

A=0, $\bar{\delta}$=-30, ε=130, k=3, M=20, B=40 ($\overline{k_1}$ = -2 , -2.5 , -4)

(Destabilizing effect)

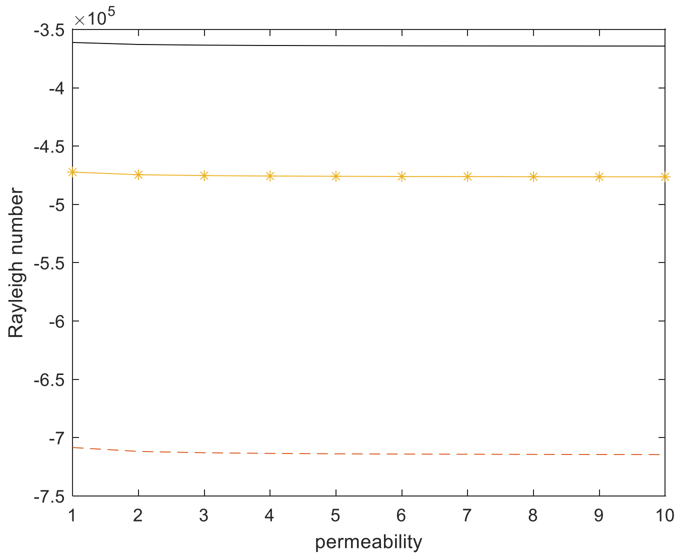

Fig. no. : 4

Variation of Rayleigh number and permeability($\overline{k_1}$)for

A=-40, $\bar{\delta}$=-20, k=1, M=20, K=4,B=80(ε=-120,-110,-128)

(Destabilizing effect)

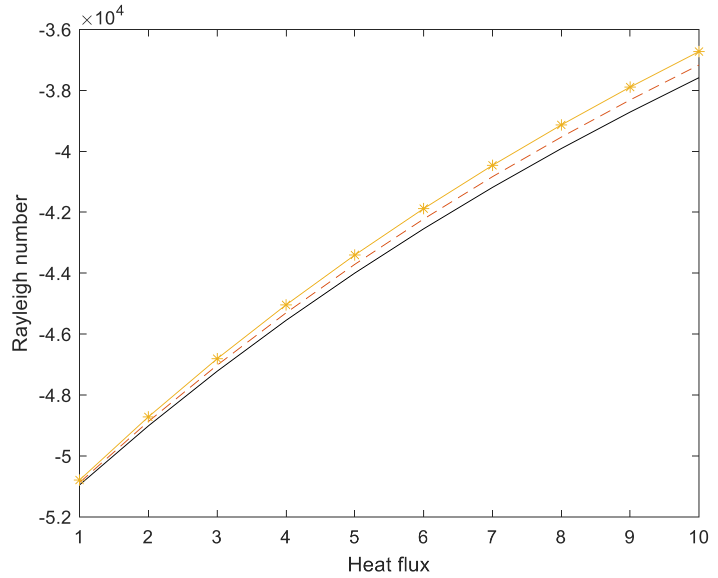

Fig. no. : 5

Variation of Rayleigh number and permeability($\overline{k_1}$) for

A=0, $\overline{\delta}$=-30, k=1, M=50, B=60, ε=-110 (K= -3, -2.7, -6)

(Stabilizing effect)

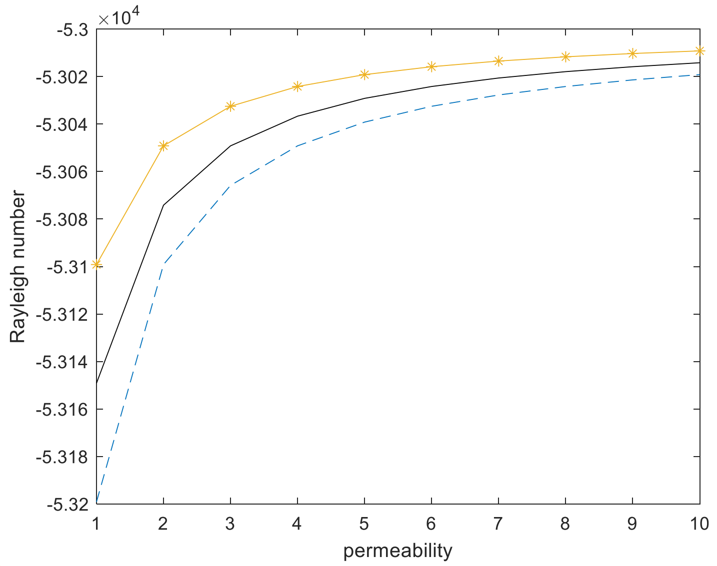

Fig. No. : 6

Variation of Rayleigh number and heat flux($\overline{\delta}$) for

A=-60, k=, M=60, B=70, K=0, U=-2 (ε = -122 , -110 , -145)

(Stabilizing effect)

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