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The B(E2) transition Probabilities for $^{114-118}$Cd Isotopes with the help of Cubic terms from Casimir Invariant Operators and IBM-1

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Abstract-

The interacting boson model-1 has been used to calculate the reduced electric transition probability B(E2) ↓ of even-even $^{114-118}$Cd nuclei with even neutrons from N = 66 to 70. The three-three boson interactions are also formed in the Hamiltonian from Casimir invariant operators. The parameters of best fit to measure the data is used from the experimental value of B (E2; $2_1^+ \rightarrow 0_1^+$). The theoretical values are good in agreement especially with the experimental ones. The branching ratios B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) is less than 2 represents U (5) symmetry in $^{114-118}$Cd nuclei.

Key words: Interacting Boson Model-1, reduce electric transition probabilities, three-three interactions.

INTRODUCTION

The IBM is explained in detail, with a focus on the variant of the model that involves higher-order interactions between the bosons. A full account of the IBM is given by Iachello and Arima (1974) [1]-[6]. The nucleus is represented in the IBM in terms of interacting s and d bosons. The vibrational U (5) limit, the rotational SU (3) limit, and the γ-unstable SO (6) limit are three separate types of analytical solutions or limits for such a device.
While the vibrational and rotational limits were well-known aspects of the nuclear environment at the time of the IBM introduction in 1974, the third limit was not. Its predictions were found to match the empirical structure of some Pt nuclei very closely. Supersymmetry is the IBM second major contribution to nuclear physics [7].

The main objective of this research is to investigate $^{114-118}$Cd nuclei transitional nuclei for the calculations of B(E2) reduced transition probabilities within framework of IBM-1 with the help of Casimir invariant operators.

THEORETICAL CONSIDERATION

One assumes that low-lying collective quadrupole states can be generated as states of a system of N bosons able to occupy two levels, one with the angular momentum J = 0, called s, and one with angular momentum J = 2, called d boson [8-9]. The number N is the total number of active nucleon pairs.
If the bosons were independent of one another, a system of n_s s-bosons and n_d d-bosons would have the energy $n_s\varepsilon_s + n_d\varepsilon_d$.

For the interaction between the active bosons take two boson operators for the Hamiltonian such as
$\frac{1}{2}\sum_{1=i<j}^{N} W(ij) = W$

$$W = \frac{1}{2}\sum_{l_f\, l_g\, l_p\, l_q,\, J=even} < l_f\, l_g\, J \mid w \mid l_p\, l_q\, J > \sqrt{(2J+1)}.$$

$[[b^{\dagger}_{f} \times b^{\dagger}_{g}]^{(J)} \times [b^{\sim}_{p} \times b^{\sim}_{q}]^{(J)}]^{(0)}$ (1)

Take three boson operators for the Hamiltonian for the interaction between the active bosons, such as

$D = \frac{1}{2} \sum_{lf, lg, lp, lq, lh, li, J=even} < l_f\, l_g\, l_h\, J \mid D \mid l_p\, l_q\, l_i > \sqrt{2J+1}. \quad \{[[b^{\dagger}_{f} \times b^{\dagger}_{g}]^{(J0)} \times b^{\dagger}_{h}]^{(J)} \times [[b^{\sim}_{p} \times b^{\sim}_{q}]^{(J0)} \times b^{\sim}_{i}]^{(J)}]\}^{(0)}$ (2)

A Hamiltonian that conserves the total number of bosons is of the generic form [10]

$\hat{H} = E_0 + \hat{H}^{(1)} + \hat{H}^{(2)} + \hat{H}^{(3)} + \ldots$ (3)

Where the index refers to the order of the interaction in the generators of U (6). The first term E_0 is a constant which represents the binding energy of the core.

The most general Hamiltonian containing one-, two-, three-body terms can be written as

$H = \varepsilon_s N + (\varepsilon_d - \varepsilon_s)\, n_d + \frac{1}{2} \sum_{J=0,2,4} C_J \sqrt{(2J+1)}$

$[[d^{\dagger} \times d^{\dagger}]^{(J)} \times [d^{\sim} \times d^{\sim}]^{(J)}]^{(0)} +$

$\sqrt{(1/2)}\, v_2\, ([[d^{\dagger} \times d^{\dagger}]^{(2)} \times d^{\sim} s]^{(0)} +$

$[s^{\dagger} d^{\dagger} \times [d^{\sim} \times d^{\sim}]^{(2)}]^{(0)}) +$

$\frac{1}{2}\, v_0\, ([[d^{\dagger} \times d^{\dagger}]^{(0)} \times ss]^{(0)} +$

$[s^{\dagger} s^{\dagger} \times [d^{\sim} \times d^{\sim}]^{(0)}]^{(0)}) +$

$u_2\, [d^{\dagger} s^{\dagger} \times d^{\sim} s]^{(0)} + \frac{1}{2}\, u_0\, [s^{\dagger} s^{\dagger} \times ss]^{(0)} +$

$\sqrt{5}/2\, A_2\, (\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [d^{\sim} \times d^{\sim}]^{(2)}\}^{(0)} \times$

$\{[d^{\dagger} \times d^{\sim}]^{(2)}\}^{(0)}) +$

$\sqrt{5}/2\, B_2\, (\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [d^{\sim} \times d^{\sim}]^{(2)}\}^{(0)}$

$\times \{[s^{\dagger} s + s s^{\dagger}]^{(2)}\}^{(0)}) +$

$p_2\, (\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [d^{\sim} \times d^{\sim}]^{(2)}\}^{(0)} \times$

$\{[d^{\dagger} s + s^{\dagger} d^{\sim}]^{(2)}\}^{(0)}) +$

$1/2\, p_0\, [(\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [s^{\sim} \times s^{\sim}]^{(2)}\}^{(0)} \times$

$\{[d^{\dagger} s]^{(2)}\}^{(0)}) + (\{[s^{\dagger} \times s^{\dagger}]^{(2)} \times$

$[d^{\sim} \times d^{\sim}]^{(2)}\}^{(0)} \times \{[d^{\sim} s^{\dagger}]^{(2)}\}^{(0)})] +$

$q_2\, (\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [d^{\sim} \times d^{\sim}]^{(2)}\}^{(0)} \times \{[s^{\dagger} s]^{(2)}\}^{(0)}) +$

$\sqrt{5}\, D_2\, (\{[s^{\dagger} \times s^{\dagger}]^{(2)} \times [s^{\sim} \times s^{\sim}]^{(2)}\}^{(0)} \times$

$\{[d^{\dagger} \times d^{\sim}]^{(2)}\}^{(0)}) +$

$1/2\, q_0\, \{[s^{\dagger} s^{\dagger} s^{\dagger} \times s^{\sim} s^{\sim} s^{\sim}]^{(2)}\}^{(0)} +$

$r_0\, (\{[s^{\dagger} s^{\dagger} \times s^{\sim} s^{\sim}]^{(2)}\}^{(0)} \times \{[s^{\dagger} d^{\sim} + d^{\dagger} s^{\sim}]^{(2)}\}^{(0)}) +$

$r_2\, [(\{[d^{\dagger} \times d^{\dagger}]^{(2)} \times [s^{\sim} \times s^{\sim}]^{(2)}\}^{(0)} \times$

$\{[s^\dagger \tilde{d}]^{(2)}\}^{(0)}) + (\{[s^\dagger \times s^\dagger]^{(2)} \times$

$[\tilde{d} \times \tilde{d}]^{(2)}\}^{(0)} \times \{[d^\dagger \tilde{s}]^{(2)}\}^{(0)})]$ (4)

The IBM-1 Hamiltonian (4) can be expressed as a linear combination of the U (6) [11] and its subgroups linear and quadratic Casimir operators.

$H = a_1 C_{1,U(5)} + a_1' C_{2,U(5)} + a_2 C_{1,U(6)} + a_2' C_{2,U(6)} + a_3 C_{1,U(6)} C_{1,U(5)} + a_4 C_{2,SO(5)} + a_5 C_{2,SO(3)} + a_6 C_{2,SO(6)} + a_7 C_{2,SU(3)} + b_1[C_{1,U(5)}]^3 + b_2 C_{2,SO(5)} C_{1,U(5)} + b_2' C_{2,SO(3)} C_{1,U(5)} + b_3 C_{2,U(6)} C_{1,U(6)} + b_4 C_{1,U(6)} C_{2,U(5)} + b_5 C_{2,SO(5)} C_{1,U(6)} + b_6 C_{2,SO(3)} C_{1,U(6)} + b_7 [C_{1,U(6)}]^3$ (5)

The Casimir invariant operators of U (6) and its subgroups in the pattern are given below:

$C_{1,\,U(6)} = N$, $C_{1,\,U(5)} = n_d$, $C_{2,\,U(5)} = n_d (n_d + 4)$, $C_{2,\,U(6)} = N (N + 5)$,

$C_{2,\,SO(6)} = N (N + 4) - \{\sqrt{5}\,[d^\dagger \times d^\dagger]^{(0)} - s^\dagger s^\dagger\}\ \{\sqrt{5}\,[\tilde{d} \times \tilde{d}]^{(0)} - ss\}$

$C_{2,\,SO(5)} = n_d (n_d + 3) - 5\ \{[d^\dagger \times d^\dagger]^{(0)}\ [\tilde{d} \times \tilde{d}]^{(0)}\}$

$C_{2,\,SO(3)} = -10\sqrt{3}\ \{[d^\dagger \times \tilde{d}]^{(1)} \times [d^\dagger \times \tilde{d}]^{(1)}\}$

$C_{2,\,SU(3)} = \sum_\mu (-1)^\mu Q_\mu Q_{-\mu}$, Where $Q_\mu = \{d_\mu^\dagger \tilde{s} + s^\dagger \tilde{d}_\mu - \sqrt{7}/2\ [d^\dagger \times \tilde{d}]_\mu^{(2)}\}$

Transition operators are associated with the IBM-calculated collective states. The number of bosons must be conserved because the B(E2) transition operator must be a Hermitian tensor of rank two. Because there are only two operators that can be used in the lowest order with these constraints, the general E2 operator can be written as [12]

$T_m(E2) = \alpha_2\ [s^\dagger \tilde{d} + d^\dagger s]^{(2)}_m + \beta_2\ [d^\dagger \times \tilde{d}]^{(2)}_m$ (6)

where α_2 plays the role of the effective boson charge and $\beta_2 = \sqrt{7}/2\ \alpha_2$. The BE (2) strength for the E2 transitions is given by

$B\ (E2;\ L_i \rightarrow L_f) = 1/\ (2L_i + 1)^{1/2}\ |\langle L_f\ ||\ T_m(E2)\ ||L_i\rangle|^2$ (7)

The reduced transition probabilities in IBM-1 are given for the limit U (5)-O (6) [13].

$B\ (E2;\ L + 2 \rightarrow L) \downarrow = 1/4\ \alpha_2^2\ (L + 2)\ (2N - L)$ (8)

where L is the state that nucleus translate to and N is the boson number, which is equal to half the number of valence nucleons. From the given experimental value of transition ($2^+ \rightarrow 0^+$), one can calculate the parameter α_2^2 for each isotope, where α_2^2 indicates the square of the effective charge. This value is used to calculate the transition 8^+ to 6^+, 6^+ to 4^+, 4^+ to 2^+ and 2^+ to 0^+. The value of B(E2) in units of $e^2\,b^2$, is related to B(E2) in units of Weisskopf single particle transition (w.u) [14].

$1\ \text{w.u} = 5.94 \times 10^{-6} \times A^{4/3} \times B(E2)\ e^2\, b^2$ (9)

Here e is the charge of electron and b (1 barn = 10^{-28} square meters) is the unit of area.

RESULTS AND DISCUSSION

Even-even nuclei with Z = 48 and N = 66-70 offer good chances to analyse the behaviour of total low-lying E2 strengths in the transitional region between deformed and spherical nuclei [15]. To determine the reduced transition probabilities strengths B(E2), the calculated absolute strengths B(E2) of the transitions within the ground state band can be fitted to the experimental ones.

Table-3: Electric transition probabilities for $^{114-118}$Cd in $e^2\,b^2$ units.

Spin Parity $J_i^+ \rightarrow J_f^+$	$_{48}Cd^{114}$		$_{48}Cd^{116}$		$_{48}Cd^{118}$	
	Experimental ($e^2\,b^2$)	This work ($e^2\,b^2$)	Experimental ($e^2\,b^2$)	This work ($e^2\,b^2$)	Experimental ($e^2\,b^2$)	This work ($e^2\,b^2$)
$2_1^+ \rightarrow 0_1^+$	0.102	0.101	0.113	0.108	0.113	0.116
$4_1^+ \rightarrow 2_1^+$	0.204	0.176	0.192	0.188	0.219	0.194
$6_1^+ \rightarrow 4_1^+$	0.145	0.226	0.212	0.253	0.233	0.243

The value of effective charge (α_2) of IBM-1 was determined by normalizing the experimental data B (E2; $2_1^+ \rightarrow 0_1^+$) of each isotope. From the given experimental value of transitions ($2_1^+ \rightarrow 0_1^+$), we have calculated the value of the parameter α_2^2 for each isotope and used this value to calculate the transitions from $4^+ \rightarrow 2^+$, $6^+ \rightarrow 4^+$.

Various E2 reduced transition probabilities are examined experimentally [17]-[22]. Theoretical and experimental data for proton charge are compared. The theoretical B(E2) values agree with the experimental results within the specified errors.

We have compared the ratio R = B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) of IBM-1 and the experimental values in the ground state bands as a function of angular momentum L.

The branching ratios B (E2; $4_1^+ \rightarrow 2_1^+$) / B (E2; $2_1^+ \rightarrow 0_1^+$) is less than 2 represents U (5) symmetry, less than 1.42 for O (6) symmetry and zero for SU (3) symmetry. We investigate U (5) symmetry in $^{114-118}$Cd isotopes.

CONCLUSION

One can show the two phonon excitations by close the energy level 0_2^+ from the twice value of the energy level 2_1^+ and the closest of the energy levels (4_1^+, 2_2^+ and 0_2^+). Also, the reduced transition probabilities between $4^+ \rightarrow 2^+$, $6^+ \rightarrow 4^+$ of even-even Cd (Z = 48, N = 66 to 70) have been studied within the framework of interacting boson model-1. It is found that electric quadrupole reduced transition probability are in good agreement with the experimental results for $^{114-118}$Cd isotopes. These results are extremely valuable for generating nuclear data tables, makes it a good resource.

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