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A NEW FUZZY TECHNIQUE TO FIND THE OPTIMAL SOLUTION IN FLOOD MANAGEMENT

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ABSTRACT: Taking account of uncertainty in multicriteria decision-making is crucial, due to the fact that depending on how it is done, ranking of alternatives can be different. This paper uses linguistic variables and triangular fuzzy numbers to take account of uncertainty in obtaining performance measures of both qualitative and quantitative criteria and weights of criteria. Metric sign distance is used to calculate fuzzy distances between performance measures and fuzzy zero to transform performance measures into commensurate units. Ranking of alternatives is done using the combination of metric sign distance and a penalty coefficient. This technique is utilized to solve a flood management example from the literature; this example illustrates the utility of the proposed technique.

Keywords: Multi-criteria Decision-Making; Fuzzy Numbers; Sign Distance; Flood Management.

1. INTRODUCTION

Decision making in water recourses management and planning is a complicated process due to the presence of multiple criteria. In order to satisfy the criteria, usually several alternatives are designed that each one provides different level of satisfaction for each criterion. Multi-criteria decision making techniques have been invented to help decision makers choose the best alternative (Brans et al., 1986; Saaty, 1987; Voogd, 1982; Zeleny, 1982). Multi-Criteria Decision Making (MCDM) techniques used in flood management and planning are categorized into six categories, including fuzzy set analysis, distance to ideal point, pair-wise comparisons, outranking methods, multi-criteria value function, and weighted summation or multiplication (Hajkowicz and Collins, 2007). These techniques have been used in many case studies of water resource management and planning (Abrishamchi et al., 2005; Abutaleb and Mareschal, 1995; Howard, 1991; Eder et al., 1997).

One of the categories of MCDM techniques involves using fuzzy theory by Zadeh (Zadeh, 1965) in order to convert crisp decision making environment to a fuzzy environment (Akter, and Simonovic, 2005; Bender and Simonovic, 2000; Fu, 2008; Makropoulos and Butler, 2006; Simonovic and Akter, 2006). The MCDM techniques using fuzzy theory are able to consider uncertainty and risk, much better than crisp techniques. *Fuzzy techniques have also been widely employed to estimate local hydrological parameters, e.g. daily pan evaporation (E_P) estimates have been achieved by suitable ANFIS (Adaptive Neural-Based Fuzzy Inference System) models for the meteorological data such as temperatures, relative humidity, wind speed and sunny hours from selected weather gauging stations in Isfahan Province, Iran (Eslamian and Amiri, 2011).*

In this paper, a new MCDM technique using fuzzy numbers and fuzzy distance is proposed. The proposed technique utilizes the combination of linguistic variables and triangular fuzzy numbers to obtain the weights of criteria and performance measures of alternatives against qualitative criteria. Performance measures of alternatives against quantitative criteria are determined using triangular fuzzy numbers. Afterwards, fuzzy performance measures are transformed into commensurate units using metric sign distance (Abbasbandy and Asady, 2006). In order to rank alternatives, the combination of metric sign distance and a penalty coefficient is used. The proposed technique is effectively usable in flood management problems. Finally, an example of reservoir flood control operation from the literature is solved using three different values as the penalty coefficient. This example demonstrates how the proposed technique can be utilized to solve problems in flood management discipline.

2. MODELING BACKGROUND

In this section, the notations used in this paper are introduced.

Definition 2.1 (Dubois, and Prade, 2000): A fuzzy number u is a fuzzy subset of the real line with a normal, convex and upper semi continuous membership function of bounded support. The family of fuzzy numbers will be denoted by E. An arbitrary fuzzy number is represented by an ordered pair of functions $\left(\underline{u}(\alpha), \overline{u}(\alpha)\right)$, $0 \le \alpha \le 1$ that satisfies the following requirements:

- $\underline{u}(\alpha)$ is a bounded left continuous non-decreasing function over [0,1], with respect to any α.
- $\overline{u}(\alpha)$ is a bounded left continuous non-increasing function over[0,1], with respect to any α.
- $\underline{u}(\alpha) \le \overline{u}(\alpha)$, $0 \le \alpha \le 1$.

Definition 2.2 (Dubois, and Prade, 2000): A triangular fuzzy number is a fuzzy set U in E that is characterized by an ordered triple $(x_l, x_c, x_r) \in R^3$ with $x_l \le x_c \le x_r$ such that $[U]^0 = [x_l, x_r]$ and $[U]^1 = \{x_c\}$. The α-level set of a triangular fuzzy number U is given by $[U]^\alpha = [x_c - (1-\alpha)(x_c - x_l), x_c + (1-\alpha)(x_r - x_c)]$ for any $\alpha \in I$ (I is a real interval).

3. METHODOLOGY

Details of the proposed technique to solve a multi-criteria decision making problem are presented below.

3.1 Choosing Criteria and Alternatives

In each MCDM problem, first it is necessary to determine criteria and design alternatives to satisfy determined criteria. In the present study, an objective set is shown as follows:

$$O = \{o_1, o_2, ..., o_n\} \tag{1}$$

where o_j is the jth objective j = 1,2,…, n. An alternative set is shown as follows:

$$A = \{a_1, a_2, ..., a_m\} \tag{2}$$

where a_i is the ith candidate alternative i = 1,2,…,m.

3.2 Obtaining Weights of Criteria

Weights of criteria represent the importance of each criterion in comparison with other criteria for a decision maker. Determining weights using crisp methods is in the present study, linguistic variables are used to define the importance of each criterion by decision makers. Linguistic variables and their corresponding fuzzy numbers are shown in Table 1.

3.3 Obtaining Performance Measures

In a MCDM problem, one of the most challenging steps to define the problem is to determine performance measures of alternatives against qualitative criteria. It is clear that a qualitative performance measure can not be defined like quantitative performance measures. Linguistic variables are very helpful to define qualitative performance measures. Table 2 shows linguistic variables and their corresponding fuzzy numbers used to evaluate qualitative performance measures in the present study.

Order to evaluate quantitative performance measures, the following process is utilized. First, primary estimated values are used as center value of fuzzy numbers. Next, maximum and minimum values of each performance measure are determined and utilized respectively as right and left limits of triangular fuzzy numbers.

3.4 Transforming Performance Measures into Commensurate Units

Performance measures, obtained in the prior step, are measured in different units; consequently, they are required to be converted into a dimensionless unit. In order to obtain commensurate units, first fuzzy distance between each performance measure and fuzzy zero is calculated using metric sign distance.

Table 1
Linguistic Variables and Their Corresponding Fuzzy Numbers for Evaluating Weights of Criteria

Linguistic variables	*Fuzzy numbers*
Extremely important	(0.85,0.9,1)
Very important	(0.75,0.8,0.9)
Important	(0.6,0.7,0.8)
Medium	(0.35,0.5,0.65)
Unimportant	(0.2,0.3,0.4)
Very unimportant	(0.1,0.2,0.25)
Extremely unimportant	(0,0.1,0.15)

Distance between two fuzzy numbers $\tilde{u}$ and metric sign distance is calculated by the following formula:

$$d_P(\tilde{u},\tilde{u}_0) = \gamma(\tilde{u}) D_p(\tilde{u},\tilde{u}_0) \tag{3}$$

where $D_p(\tilde{u},\tilde{u}_0)$ and $\gamma(\tilde{u})$ are defined as follows:

$$D_p(\tilde{u},\tilde{u}_0) = \left[\int_0^1 \left(\left|\underline{u}(\alpha)\right|^p + \left|\overline{u}(\alpha)\right|^p\right) d\alpha\right]^{1/p} \quad (p \geq 1) \tag{4}$$

and

$$\gamma(\tilde{u}) = \begin{cases} 1 & \int_0^1 \left(\underline{u}(\alpha) + \overline{u}(\alpha)\right) d\alpha \geq 0, \\ -1 & \int_0^1 \left(\underline{u}(\alpha) + \overline{u}(\alpha)\right) d\alpha < 0 \end{cases} \tag{5}$$

$\tilde{u}_0$ is a fuzzy origin with zero fuzziness on the left and the right. It is defined by the parametric form (0,0), and the following membership function:

$$\tilde{u}_0(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases} \tag{6}$$

In a typical MCDM problem, the values of performance measures and weights are defined using positive either crisp or fuzzy numbers, consequently, $\gamma(\tilde{u})$ is equal to 1. The value of p in each problem is selected depending on its particular conditions. In this section, due to the fact that calculated distances will be divided by each other to be converted to the value of p does not as a result, in order to simplify calculations, in this section, the value of p is chosen equal to 1. Therefore, the distances between performance measures and fuzzy zero using equation (3) is calculated by the following formula:

$$z_{ij} = d\left(\tilde{x}_{ij}, \tilde{u}_0\right) = \int_0^1 \left(\left|\underline{u}(\alpha)\right| + \left|\overline{u}(\alpha)\right|\right) d\alpha \tag{7}$$

After calculating the distances between performance measures and fuzzy zero, the following formula is utilized for benefit criteria, including desirable qualitative criteria, to convert the distances into between 0 and 1:

$$c_{ij} = \frac{z_{ij}}{\max_i z_{ij}} \tag{8}$$

and the following formula is utilized for cost criteria, including undesirable qualitative criteria:

$$c_{ij} = \frac{\min_i z_{ij}}{z_{ij}} \tag{9}$$

3.5 Multiplying Weights by Their Corresponding Dimensionless Performance Measures

Fuzzy weights assigned to each criterion j are multiplied by their corresponding cijs, $i = 1, 2, \ldots, m$, that is calculated in the previous section. Therefore, there will be m, equal to the number of alternatives, fuzzy vectors. Each fuzzy vector includes n, equal to the number of criteria, fuzzy numbers.

Table 2
Linguistic Variables and Their Corresponding Fuzzy Numbers for Evaluating Qualitative Performance Measures

Linguistic variables	*Fuzzy numbers*
Extremely high	(0.85,0.9,1)
Very high	(0.75,0.8,0.9)
High	(0.6,0.7,0.8)
Medium	(0.35,0.5,0.65)
Low	(0.2,0.3,0.4)
Very low	(0.1,0.2,0.25)
Extremely low	(0,0.1,0.15)

3.6 Ranking Alternatives

In order to rank the alternatives, the fuzzy vectors, obtained in the previous section should be ranked. To rank the fuzzy vectors, first the distance between each element of vectors and fuzzy zero is calculated using metric sign distance. The value of p in this step is selected equal to 2. This is done in order to very large distances. After calculating each distance, in order to prevent good performances of an alternative against a few criteria its poor performance against the other criteria, the calculated distances are first increased to the power of k. Afterwards, the resulted values for each alternative are added together and the *k*th root of them are calculated. The resulted numerical values are compared to each other and the alternatives with the higher. Therefore, the following formula is used to obtain final performance assigned to the alternatives to rank them:

$$u_i = \left(\sum_{j=1}^{n} \left(d_2 \left(\tilde{w}_j c_{ij}, 0 \right) \right)^k \right)^{1/k} = \left(\sum_{j=1}^{n} \left(\int_0^1 \left(\left| \underline{w}_j (\alpha) z_{ij} \right|^2 + \left| \overline{w}_j (\alpha) z_{ij} \right|^2 \right) d\alpha \right)^{k/2} \right)^{1/k} \tag{10}$$

In section 4, for solving the example, calculations with *k* equal to 1, 1/2, and 1/10 are done to consider the uncertainty in the value of penalty coefficient.

4. AN EXAMPLE OF FLOOD CONTROL OPERATION

In this section, the proposed technique is used to rank the alternatives for reservoir flood control operation of a case study taken from the literature (Fu, 2008) on Sanmenxia reservoir.

Sanmenxia reservoir in China is located in the middle reach of Yellow River. Flood control desires to diminish the flood peak discharge during the flood season and correspondingly retain the water level of reservoir as low as possible. Therefore, two criteria are defined to consider in the decision-making process; first, flood peak discharge at downstream station and second, the difference between the design flood level and highest water level during the operation. The sediment problem in Yellow River has a great effect on the and its capability of flood control, as a result, another criterion is defined to consider the sediment problem. Sediment load in reservoir area is chosen as third criterion. In order to consider the possible risk for both reservoir and its downstream area, two criteria are defined the risk of flooding in the downstream protected regions as fourth criterion and the risk of failure of the dam and its structures as fifth criterion. Clearly, the performance of alternatives against criteria 4 and 5 can not be evaluated using numerical values, consequently, criteria 1, 2, and 3 are considered as quantitative criteria and criteria 4 and 5 as qualitative criteria (Fu, 2008).

After preliminary screening, three feasible operation alternatives a_1, a_2, and a_3 a major flood. Table 3 shows the performance measures of three alternatives against both qualitative and quantitative criteria. Linguistic variables assigned to criteria extremely important to criterion 1, very important to criteria 2 and 4, and important to criteria 3 and 5.

Table 3
Decision Matrix of the Sanmenxia Reservoir Case Study

Criteria	a_1	a_2	a_3
c_1	(9500,10000,10500)	(8500,9000,9500)	(8000,8500,9000)
c_2	(33.5,33.8,34.5)	(31.5,32.0,32.5)	(29.5,30.0,30.5)
c_3	(6.0,6.5,7.0)	(7.5,8.0,8.5)	(8.5,9.0,9.5)
c_4	H	L	VL
c_5	L	L	M

Table 4
Fuzzy Distances Between Performance Measures and Fuzzy Zero

Criteria	a_1	a_2	a_3
c_1	20000	18000	17000
c_2	67.8	64	60
c_3	13	16	18
c_4	1.4	0.6	0.375
c_5	0.6	0.6	1.0

Now the proposed technique is used to rank alternatives of the case study. In order to transform performance measures to commensurate units, first, the parametric form of triangular fuzzy numbers explained in definition 2.2 and equation (7) are used to calculate fuzzy distances between each performance measure and fuzzy zero. Table 4 shows the calculated distances. Then, equation (8) is used to convert the calculated distances of performance measures criterion 2 into between 0 and 1. Equation (9) is used for criteria 1, 3, 4, and 5. Table 5 shows the resulted cijs. Next step includes multiplying fuzzy weights by calculated cijs, which is done using to assign fuzzy numbers to the determined linguistic weights.

Table 5
Dimensionless Performance Measures

Criteria	a_1	a_2	a_3
c_1	0.85	0.94	1.00
c_2	1.00	0.94	0.88
c_3	1.00	0.81	0.72
c_4	0.27	0.63	1.00
c_5	1.00	1.00	0.60

Table 6 shows the resulted matrix. Subsequently, equation (10) is utilized to calculate final performance of the alternatives to rank them.

Table 6
Results of Multiplying Weights by Dimensionless Performance Measures

Criteria	a_1	a_2	a_3
c_1	(0.72,0.80,0.85)	(0.80,0.85,0.94)	(0.85,0.90,1.00)
c_2	(0.75,0.80,0.90)	(0.71,0.75,0.85)	(0.66,0.70,0.79)
c_3	(0.60,0.70,0.80)	(0.49,0.57,0.65)	(0.43,0.50,0.58)
c_4	(0.20,0.220.24)	(0.47,0.50,0.57)	(0.75,0.80,0.90)
c_5	(0.60,0.70,0.80)	(0.60,0.70,0.80)	(0.36,0.42,0.48)

In order to consider the uncertainty in the value of penalty coefficient, uis are calculated using three values 1, 1/2, and 1/10 as k. The resulted values and their corresponding rankings are as follows:

For $k = 1$: $u_1 = 4.57$, $u_2 = 4.83$, $u_3 = 4.76$; Ranking of alternatives: $a_2 > a_3 > a_1$

For $k = 1/2$: $u_1 = 21.94$, $u_2 = 23.91$, $u_3 = 23.33$; Ranking of alternatives: $a_2 > a_3 > a_1$

For $k = 1/10$: $u_1 = 8.22\times10^6$, $u_2 = 9.27\times10^6$, $u_3 = 8.96\times10^6$; Ranking of alternatives: $a_2 > a_3 > a_1$.

It is observed for penalty coefficient equal to 1, alternative 2 has a quite close performance to alternative 3, but penalty coefficients result better performance by alternative 2 compared to alternative 3. For all of the values of k, alternative 1 has the worse performance compared to the other alternatives. Alternative 2 the best one, because for all of the values of k, it has the best performance. However, the results of MCDM are a guide for decision makers to select their desirable alternative and their decision might be different from the results of MCDM models.

5. CONCLUSION

In this paper, we have proposed a new fuzzy multicriteria decision making technique, which tries to take account of the present uncertainty in decision-making process as much as possible and then rank alternatives. We have utilized linguistic variables and fuzzy numbers to consider uncertainty in the values of performance measures and the weights of criteria. Using fuzzy numbers instead of numerical values enables decision makers to express their views much more comfortably.

The proposed technique uses metric sign distance to defuzzify the decision-making environment. Furthermore, a penalty coefficient is used to prevent neutralization of poor of an alternative against some criteria by its good against the other criteria. As a result, the alternatives with moderate performance against all criteria are higher in rank in comparison with the alternatives with good performance against some criteria and poor performance against the other criteria. The utility of the proposed technique in solving flood management problems is verified using a problem from the literature. The proposed technique is not limited to special conditions and can be utilized to solve any multi-criteria decision making problem with discrete data.

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