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A case study on Analytic Geometry and its Applications in real life

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ABSTRACT

The Main aim of this report is to use analytical geometry in real life to solve the real world problems. Analytical Geometry has vast applications in our life both directly and indirectly. It has been used in Medicine, Power Generation and in Construction. It has helped us to improve accuracy in medicine field for the betterment of the treatment. In Power Generation it has helped us to create power in large number. This Paper further explores how to solve such real world Problems.

Keywords: Real life problems, parabolic, hyperbolic, nuclear cooling, etc.

I. Introduction to Analytic Geometry

Analytic geometry also known as coordinate geometry was initiated by French philosopher René Descartes in his book, La Geometric (1637) in which h introduced algebra in the study of points and locus of points. Legend has it that Descartes, who liked to stay on the ceiling from his bed. He wondered how to describe the fly's location by deciding one of the corners of the ceiling as a reference point.

He imagined the ceiling as a rectangle drawn on a piece of paper and taking the left bottom corner as the reference point. Then it would b possible to specify the location of fly by measuring the distance between fly and the reference point in the horizontal direction and the distance between fly and the reference point in the vertical direction. These two numbers represent the coordinates of fly. He noted that every pair of coordinates specifies a unique point on ceiling and comes with a unique pair of coordinates. Then this idea extended allowing the axes (the two sides of the room) to become infinitely long in both directions, and y using negative numbers to label the bottom part of the vertical axis and the left part of the horizontal axis. By following these way, all points can be specified on an infinite plane.

1. Some applications of Analytic Geometry in real life

There is extensive use of Analytic Geometry in the field of Physics and Engineering. Some of them are listed as follow:-

- In aviation industry for designing layout of aircraft.
- In space exploration and research.
- In designing, computer graphics and typography.
- In astronomy and astrophysics.
- To plan for the mission in space.
- To find position of 3-Dimensional object.
- In the construction of nuclear cooling tower and in the detection of explosion by microphone.
- In finding the position of the kidney stones.
- In determining the shape of Odeillo solar furnace and depth of parabolic mirror.

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2. Objectives

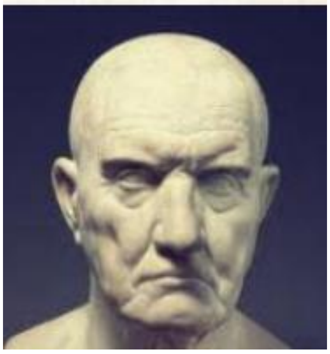
1. To study about the applications of Hyperbola in:
   a. Infrastructural design of Nuclear cooling tower.
   b. Detection of explosion recorded by microphone.
2. To study about the applications of Parabola in:
   a. Finding graph and equation of parabola that model the shape of an odeillo mirrors and depth of parabolic mirror.
   b. Finding the position of stone in kidney from shock wave generator.

3. Limitations

1. This study is limited only up to the construction and infrastructural design. It is not valid in any further chemical process.
2. This case study is limited only for tracing the location or detecting point of explosion. It is not valid for any explosive reactions or blast.
3. This study is not valid for the nature of image formed by odeillo mirror.
4. There is no any relation of my study in surgical operations; it is limited up to the positioning.

II. Introduction

1. Historical background



Menaechmus was a famous mathematician and Platonic philosopher of the 4th century BC from Alopeconnesus. He is remembered for his contributions in geometry. As Proclus puts it "(he) *and his brother Deinostratus made the whole of geometry still more perfect*".

Menaechmus was a student of Eudoxus, a polymath who was in turn a student of Plato in the Academy. He served as the tutor of Alexander the Great in geometry. It is believed that he had founded his own school of mathematics. His most important contributions were in the field of conic sections.

The conic sections, one of the most applicable chapters in analytical geometry were discovered by Menaechmus when he was attempting to solve the doubling of the cube, a famous geometrical problem of antiquity known for its high degree of difficulty. Menaechmus managed to solve the problem with two solutions by using the conic sections, thus paving the way for Apollonius of Perga to develop those years later.

He showed that by cutting a cone with a line that is not parallel to the base, one could obtain different shapes, which he named ellipses, parabolas and hyperboles, depending on the angle. It is asserted that the mathematician used mechanical devices to help him in drawing the shapes, something which Plato disapproved in geometry. His works were translated into Arabic and then to Latin during the late Middle Ages. This contributed significantly to the Renaissance and the revival of mathematics.

Menaechmus endorsed Eudoxus' theory on the heavenly bodies and further developed it based on Theon of Smyrna's writings. Moreover, he studied the structure of mathematics and was involved with astronomy. His work was continued by Apollonius of Perga, Archimedes and by the mathematicians of the Renaissance.

The main objective of this study is to show the use of analytical geometry in our daily life.

It has extensive range of applications in our life. In this case study the designation of nuclear cooling towers, and microphone which is used to detect the location of explosion has been explained using Analytic geometry. These factors are based on hyperbola. The basic idea of analytic geometry is to represent the curves by equations, but this is not the whole idea. If it were, then the Greeks would been considered the first analytic geometers. Analytic geometry is also called as coordinate geometry. The importance of analytic geometry is that it establishes a correspondence between algebraic equations and the geometric curves.

2. **Some index terms:**

2.1. Hyperbola: A hyperbola, in analytic geometry is a conic section that is formed when a plane intersects a double right circular cone at an angle such that both halves of the cone are intersected. This intersection of the plan and cone produces two separate unbounded curves that are mirror images of each other called a hyperbola.

2.2. Parabola: it is a plane curve which is mirror-symmetrical and is approximately U-shaped.

2.3. Nuclear cooling tower: A heat exchanger designed to aid in the cooling of water that was used to cool exhaust steam exiting the turbines of a power plant.

2.4. Microphone: A microphone is a device that translates sound vibrations in the air into electronic signals and scribes them to a recording medium or over a loudspeaker.

2.5. Distance: it is a numerical or occasionally qualitative measurement of how far apart objects or points are..

2.6. Axes: the mutually perpendicular lines drawn from a fixed point are known as axes.

2.7. Major axis: the major axis is a line that passes through the foci, center and vertices of parabola.

2.8. Minor axis: the minor axis is the line that passes through the center and is perpendicular to the major axis.

2.9. Origin: three mutually perpendicular lines intersect at a fixed point which is known as origin.

2.10. Diffusion: it is the net movement of anything generally from a region of higher concentration to a region of lower concentration.

2.11. Aerodynamics: Simply, it describes the way for object moving in the air.

2.12. Explosion: An explosion is a rapid expansion in volume associated with an extreme outward release of energy, usually with the generation of high-pressure gases.

2.13. Reflector: a reflector is an optical device that redirects incident light back to the side of incidence.

2.14. Electrode: a solid electric conductor that carries electric current into non-metallic solids, or liquids, or gases, or plasmas, or vacuums.

2.15. Shockwave generator: it replaces the standard air filled armature with a powder or a solid, which under shock pressure transitions from a dielectric to a metallic state.

2.16. Furnace: a furnace is a structure in which heat is produced with the help of combustion.

2.17. Lithotripter: a noninvasive device that breaks up kidney stones by passing electromagnetic shock waves through a water bath while a patient sits inside.

2.18. Parabolic mirror: a concave mirror whose cross-section is shaped like the tip of a parabola.

2.19. Odeillo mirror: it bounces the sun's rays onto a large concave mirror.

2.20. Steerable mirror: physical systems composed of a reflective mirror, a pair of motion-tracking sensors, a displacement actuator, and a processing unit that aid in receiving an incoming signal beam at the correct angle or redirecting it in a specific direction.

2.21. Anticoagulation: Reduces in risk of blood clotting.

3. **Equation's involved:**

3.1. Equation of hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

3.2. Equation of parabola:

$$y^2 = 4ax$$

III. APPLICATIONS OF HYPERBOLA

1. Hyperboloid Nuclear cooling tower

Nuclear cooling towers are mainly designed under the study of analytic geometry. Most cooling towers from coal-fired power plants are shaped like hyperbolas. As it produces enormously high heat energy that's why it is an optimal shape for letting heat dissipates. The hyperboloid shapes of the tower accelerates the upward air flow, pushing air higher and higher, losing heat as it goes. The hyperboloid shapes also improves the structural strength and reduces the cost and material usage.

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Fig 1: Nuclear Cooling tower

Reason behind the construction and design of Nuclear cooling tower as Hyperboloid shape:

- **Strength factor**
  First and foremost, the shape of cooling tower has direct impacts on the strength of the entire structure. Since cooling towers are supposed to cool the working fluid down to a very low temperature, they release vapors through the opening at the top of the tower into the atmosphere. Therefore, these towers have to be necessarily tall, or else the released vapor may cause fogging or recirculation. To support such a high structure, it is very important that the base is spread over a large area and considerably consolidated so that it can support the tall, heavy structure above it. This is why cooling towers have a circular base that is large.
- **To facilitate aerodynamic lift**
  As hot water evaporates and begins to rise in the concrete structure, the narrowing effect of the tower helps to boost the speed of parallel layers of vapor without any disruption (known as laminar flow). Since hot air is less dense than cool air, it easily rises above in the tower, particularly due to the narrowness of the tower at the middle.
- **Efficiency of diffusion of vapor into the atmosphere**
  A wider top opening enhances the process of diffusion. The top of cooling tower is widened because it is the point where the hot air from inside the tower diffuses and mixes with atmospheric air. Therefore, we should maximize the area through which this diffusion takes place, so that the hotter vapor is quickly mixed and the entire process of cooling is done more efficiently.
  There are also other reasons behind this shape. For example, a wide base will not only provide strength to the whole structure, but also offer ample space for the installation of machinery. From a logical standpoint, this shape is easier to build when compared to others, as it has a lattice of straight beams to erect the tower. Also, the types of structures are more resistant to external natural forces than straight buildings.

Problem 1

The discussion would be clearer by the example solved below:

Suppose a cooling tower stands 179.6 meters tall. The diameter of the top is 72 meters. at their narrowest part, the sides of the tower are 60 meters apart. Find the equation of the hyperbola that obtains the sides of the cooling tower. Assume that the center of the hyperbola- indicated by the intersection of dashed perpendicular lines in the diagram below is the origin of the coordinate plane.

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Solution

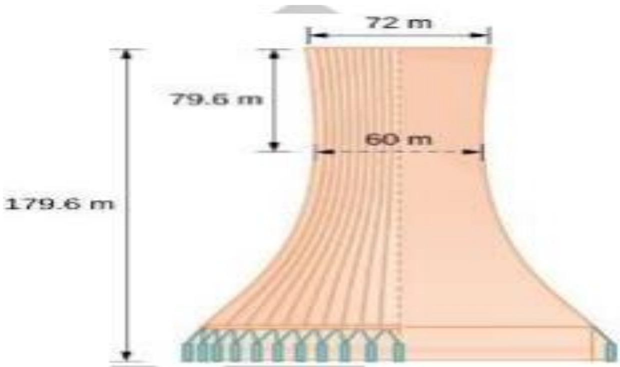


Fig 2: A cooling tower

Given

The design layout of a cooling tower is shown above. The tower stands 179.6meters tall. The diameter of the top is 72 meters. At their narrowest part, the sides of the tower are 60 meters apart.

To find the equation of the hyperbola that models the sides of the cooling tower

Let us consider that the center of the tower is at the origin. Hence, we use the standard form of a horizontal hyperbola centered at the origin, where the branches of the hyperbola form the sides of the cooling tower.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

To find the values of a^2 and b^2

We know that,

Length of the transverse axis of hyperbola is 2a

$$2a = 60$$

$$a = 30$$

Therefore, $a^2 = 900$

Now ,

Substituting value of x and y in above equation using a known point;

Now to find some point (x, y) that lies on the hyperbola we can use the dimensions of the tower.

Consider the top right corner of the tower to represent that point. Since the y-axis bisects the tower, our values can be represented by the radius of the top or 36 meters. The distance from the origin to the top is represented as the y- value which is given as 79.6 meters.

Therefore, standard form of horizontal hyperbola:

$\frac{x2}{a2} - \frac{y2}{b2} = 1$

$b^2 = \frac{y^2}{\frac{x^2}{a^2} - 1}$

Substituting the values of a^2 , x and y

$b^2 = \frac{(79.6)^2}{\frac{(36)^2}{900} - 1}$

=14400.3636

Therefore the sides of the tower can be modeled by the hyperbolic equation,

$$\frac{x^2}{900} - \frac{y^2}{14400.3636} = 1$$

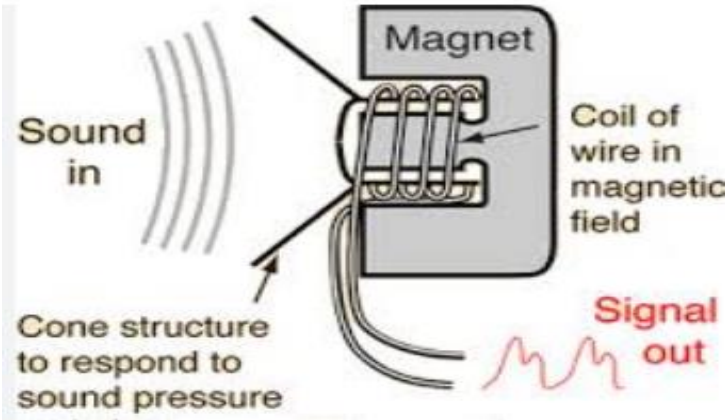
$$\frac{x^2}{30^2} - \frac{y^2}{120.0015^2} = 1$$

Result

Thus purpose of a cooling tower is to cool down the water that gets heated up by industrial equipments and processes. Water comes in the cooling tower at a high temperature (by the industrial process) and goes out of the cooling tower at a cold temperature.

2. Explosion recorded by microphone

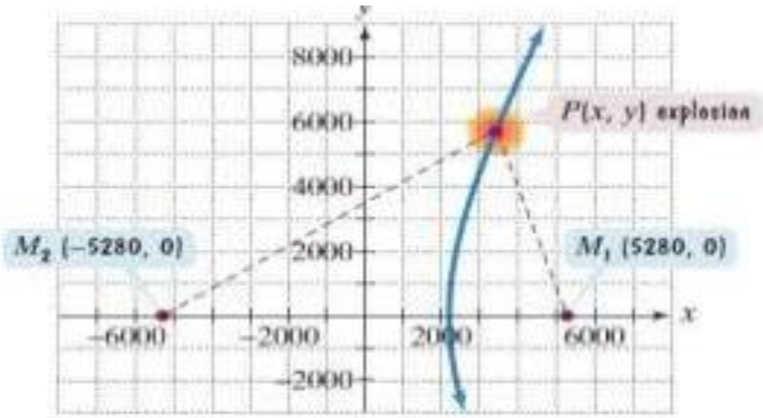
The location of the explosion can be determined by using hyperbola with the help of the sound waves. Microphone works in the mechanism of variations in sound pressure. Changes in sound pressure move the cone which moves the coil of the wire connected to it in the magnetic field. This increases the voltage in the coil wire. Then the coils also start moving in respond to the audio signal, moving the cone and producing sound in the air. Hence it is determined by using hyperbola with the help waves.



Problem 2:

An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 received the sound of the explosion 4 seconds before microphone M_2. Assuming sound travels at 100 feet per second, determine the possible locations of the microphone.

<u>Solution</u>



Given

Here M_1 and M_2 are two microphones placed at distance of 2 miles apart and speed of sound is 100 ft per second. To find t he location of the two microphones we begin by putting the microphone in a coordinate system.

1 mile = 5280 feet

Two microphones are placed 5280 feet from the origin in the horizontal axis in the above graph.

We know that,

M_2 received the sound 4 seconds after M_1. Because the sound travels at 1100 feet per second, the difference between the distance from point P to M_1 and the distance from P to M_2 is

1100*4= 4400

The set of all these points P satisfying these conditions fits the definition of a hyperbola, with microphones M_1 an M_2 at the foci.

The standard form of the hyperbola's equation is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$………………… (1)

By using this, we can find the value of a^2 and b^2

P (x , y) is the explosion point, lies on the hyperbola.

Here, the difference between the distances is 2a which is 4400 feet

We have,

2a = 4400

a= 2200

Substituting value of a in equation (1), we get

$$\frac{x^2}{2200^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{484000} - \frac{y^2}{b^2} = 1$$

The distance from the center to either focus is;

Center: (h , k)=> (0 , 0)

Distance C = 5280

Using $C^2 = a^2 + b^2$

We have,

$(5280)^2 = (2200)^2 + b^2$

$b^2 = (5280)^2 - (2200)^2$

b^2 = 2,30,38,400

Now,

Substituting value of b^2 in equation (1) .we get,

$$\frac{x^2}{484000} - \frac{y^2}{23038400} = 1$$

RESULT

We can conclude that the explosion occurred somewhere on the right side which is closer to the M_1 of the hyperbola.

IV. APPLICATIONS OF PARABOLA

1. **Modeling the shape of an odeillo solar furnace:** Application of parabola is extensive to models the shape of an odeillo solar furnace. This double-mirror solar furnace was constructed in 1949 by engineer Felix Trombe in the citadel of Mont-Louis.it concentrates the sun's rays to melt anything that is within its 3,000 C beam range. It was designed for high- temperature experiments on materials and to show how to fire ceramics without using wood. The odeillo solar furnace is the world's largest solar furnace situated in Font-Romeu-Odeillo-Via, in the department of Pyrenees-Orientale's, in the south of France. it is 48 meters (157 ft) high and 54metrs (177 ft) wide, and includes 63 heliostats. It was built between 1962 and 1968, started operating in 1969, and has power of one megawatt. It serve as a science research sit studying materials at very high temperatures. The principle used is the concentration of rays by reflecting mirrors (9600 of them). The solar rays are picked up by a first set of steerable mirrors located on the slope, and then sent to a second series of mirrors (the concentrators), placed in a parabola and eventually converging on a circular target, 40 cm in diameter, on top of the central tower. Equivalent to concentrating the energy of "10,000 suns", the solar furnace produces a peak power of 1000kW.

Fig: Odeillo solar furnace

Reason behind using parabolic mirror in odeillo solar furnace:

- Temperature above 2,500 ^{o}C (4,530 ^{o}F) can be obtained in a few seconds.
- The obtained or generated energy is free of cost and causes no harm to the environment.
- This kind of furnace provides rapid temperature changes and therefore allows studying the effects of thermal shocks.

Problem 3

The odeillo solar furnace, located in southern France uses a series of 63 flat mirrors, arranged on terraces of a hill side, to reflect the sun's rays on to a large parabolic mirror tilt to track the sun and ensure that its rays are always reflected to the central parabolic mirror. This mirror in turn reflects the sun's rays to the focal point where a furnace is mounted on tower. The concentrated energy generates temperature up to 6870F. If the width of the odeillo mirror is 138feet and furnace is located 58 feet from centre of mirror, how deep is the mirror

a. Find the graph and equation of parabola that models the shape of a odeillo mirrors.
b. Find the depth of the parabolic mirror.

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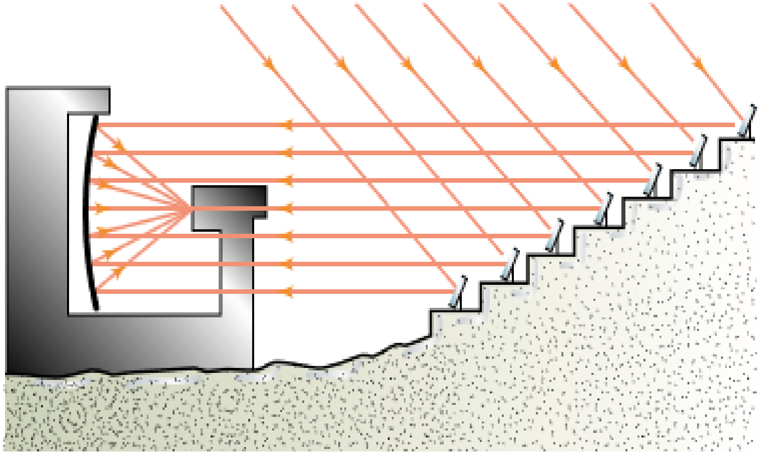


Solution:

GIVEN

Width of the mirror = 138 feet.

Location of furnace from centre of mirror = 58 feet.

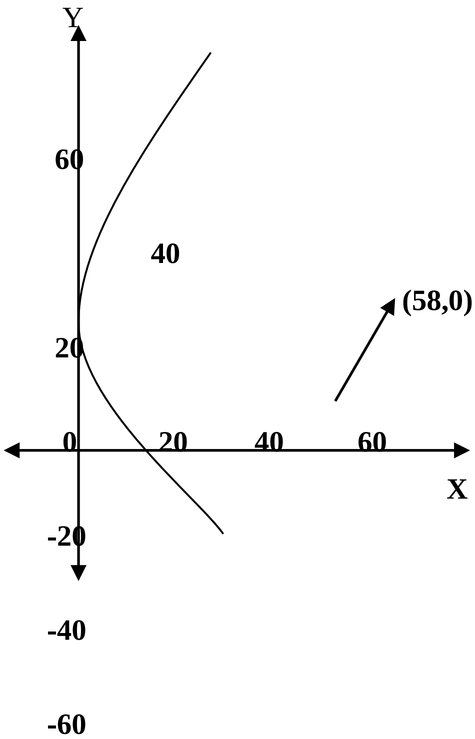
TO FIND

a. Equation of parabola that models odeillo mirror.

b. Depth of the parabolic mirror.

The shape of mirror can be modeled by a parabola with vertex at origin and opening to its right.

The equation is $y^2 = 4ax$.



Let a be focal length

Here, a = 58

$$y^2 = 4ax$$
$$= 4 * 58 * x$$
$$y^2 = 232x$$

Now,

With the mirror's vertex at the origin, the distance from the vertex to one edge of mirror is half the overall width of the mirror.

i.e. (1\2*138)

To find depth 'x' of mirror. When distance from centre is 69 feet.

$$y^2 = 232x$$
$$69^2 = 232 * \text{x}$$
$$\text{x} \quad = 20.5$$

The mirror is about 20.5 feet deep.

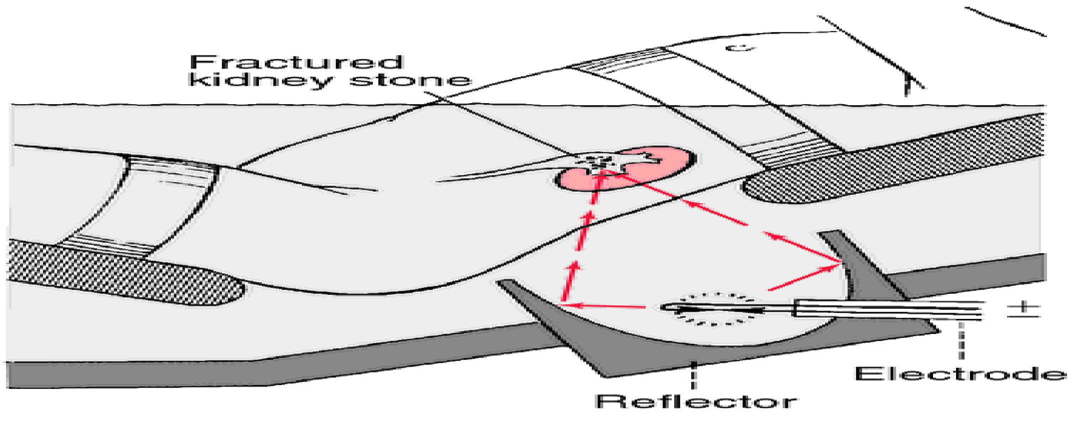
RESULT

We obtained the equation of parabola that models Odeillo mirror as $y^2 = 232x$ and parabolic mirror is 20.5 feet deep from the centre.

2. **Positioning the kidney stone from shock wave generator:** The most commonly performed surgery for kidney stones is shockwave lithotripsy which has mathematical principle of determining the position of stone in kidney. In this procedure, stones in the kidney and uretar are pulverized into small fragments by means of short-duration, high-energy shockwaves that are produced outside the body by a lithotripter. At the university of Michigan, shockwave lithotripsy is performed at the Livonia center for specialty care as an out-patient procedure (patient can go home same day).

   While over 90% of kidney stones can be treated with shockwave lithotripsy, there are certain circumstances when it is not recommended such as a urinary tract infection, urethral obstruction near the stone, in patients on anticoagulation medications or those with bleeding disorders, patient anatomy that makes it difficult to target the stone, and pregnancy.

   Parabola has made this method effective and efficient due to which patient has to suffer less and can get cure. In this process, shockwave is produced due to which stones smashes into various pieces; reflector is made parabolic to locate the stone using electrode and then it can be removed surgically.



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PROBLEM 4

Suppose we have a major axis of length 60 cm and a minor axis of length 53 cm. The Source of high energy shock waves from a lithotripter is placed at one focus.

a) To Smash the Kidney stone of a patient, how far should the stone be positioned from the source of lithotripter?

b) The Shockwave is projected at an angle of 85°, after reflecting from the elliptical surface

At what angle will the shockwave hit the target?

SOLUTION

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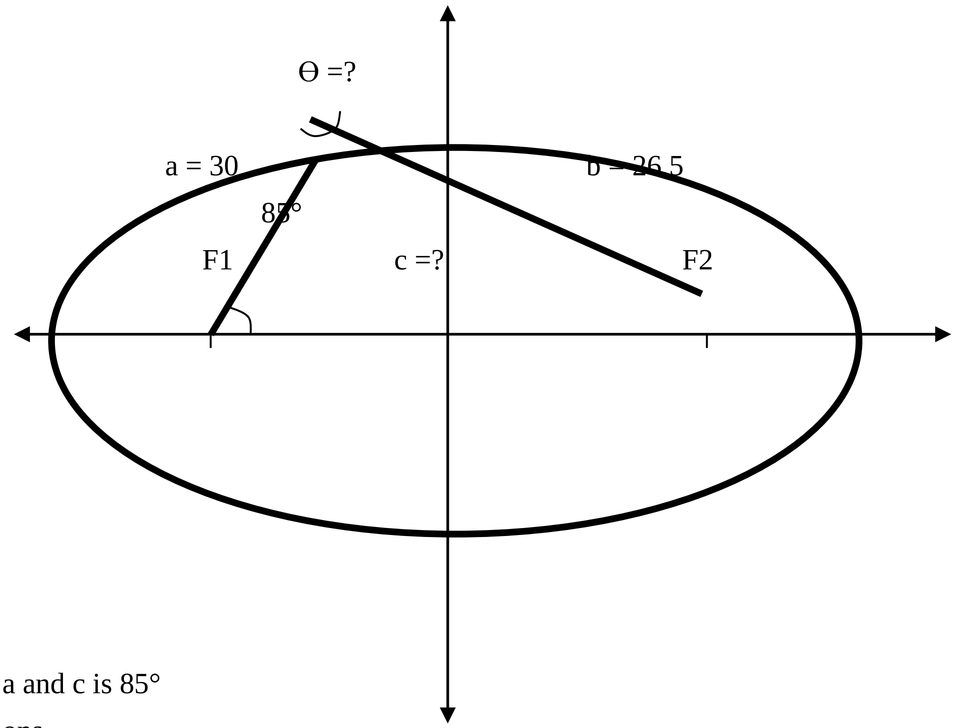
Length of Major Axis = 60 cm

Length of Minor Axis = 53 cm

TO FIND

The Position of Stone from Shock Wave Generator.



We have,

Angle between a and c is 85°

From the questions,

$2a = 60$

$\therefore a = 30$ cm

$2b = 53$

$\therefore b = 26.53$ cm

Using,

$c^2 = a^2 - b^2$

Or, $c^2 = 900 - 703.84$

Or, $c = \sqrt{196.16}\ 2$

Or, $c = 2 \times 14.01\ 2$

$\therefore c = 28.02$ cm

∴The Stone Should be positioned at 28.02 cm from the source.

Now,

Angle of projection = 85°

$$\frac{\text{SinA}}{a} = \frac{\text{SinB}}{b}$$

$$\text{Or, } \frac{\text{Sin } 85°}{30} = \frac{\text{Sin } 85°}{28}$$

$$\text{Or, } \theta = Sin^{-1}\left(\frac{\text{Sin } 85°}{30} \times 28\right)$$

$$\text{Or, } \theta = sin^{-1}(0.9962 \times 0.9333)$$

$$\text{Or, } \theta = sin^{-1}(0.92978)$$

$$\therefore \theta = 68.38°$$

∴The Shockwave will hit the kidney stone at an angle of 68.38°

RESULT

a) The Stone Should be positioned at 28.02 feet from the source of Shockwave Generator.

b) The Shockwave will hit the kidney stone at an angle of 68.38°.

V. CONCLUSION

The infrastrustural design of nuclear cooling tower in hyperbolic shape makes it stable and suitable for practical use as it makes it less costly and eco-friendly and also the use of hyperbola in detection of explosion is illustrated above.The reflction propertyof parabola through its focus helps to gain huge amount of energy, that helps us in meting the demand for the energy. Also, the reflection property of parabola through its focus helps us to break the stones in kidney without harming the tissues of the human beings and it helps us in increasing the accuracyof th treatment.

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VI. REFERENCES

- K.Umamaheswari, Assistant Professor, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJRAR January 2019 Volume 6; 2018 IJSDR Volume 4]

- R.K.Hari Prasath, UG Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJRAR January 2019 Volume 6]

- B.Shruthi, UG Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJRAR January 2019 Volume 6]

- M.Ramya, UG Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJRAR January 2019 Volume 6]

- S.Nivetha, UG Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJRAR January 2019 Volume 6]

- S . Monika, Under Graduate Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJSDR January 2019 Volume 4]

- S.Mohana Priya, Under Graduate Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJSDR January 2019 Volume 4]

- R . Kavya, Under Graduate Scholar, Departments of Mathematics, Sri Krishna Arts and Science College, Coimbatore, India- [A CASE STUDY ON ANALYTIC GEOMETRY AND ITS APPLICATIONS-2018 IJSDR January 2019 Volume 4]