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Mistakesin Simplifying Algebraic Expressions, TheirRepetition Patterns and Thinking Strategies Associated with These Mistakes in Nine Graders

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Abstract

The study aims to identify common errors made by nine graders in algebraic subjects and their repetition patterns. It also aims to identify thinking strategies associated with these mistakes. It follows the quantitative and qualitative approaches, and the study cohort consisted of (100) learners from several Arab schools in Israel. Two tools were used in this study: in the first stage, a test built on algebraic concepts was given to the cohort, and in the second stage individual interviews were conducted with five learners, who were randomly selected from those who answered incorrectly to many test items. The interviews were designed to gain insight in thinking strategies associated with common errors learners make. The outcome of the study showed the presence of a large number of common mistakes among nine graders in algebraic concepts. The rate of generalization mistakes was 41.2%, followed by conceptual mistakes at a rate of 37.1%; procedural mistakes came in third place with a rate of 32.1%. Finally, other miscellaneous mistakes occurred at a rate of 26.3%.

Keywords -Algebraic Expressions, Thinking Strategies, Mistakes, 9th grade.

Introduction

Modern mathematics is distinguished by its tight structures that are closely related to each other creating a solid structure. Its basic principles are to emphasize learners' acquisition of mathematical skills in a balanced manner, depending on mathematical concepts (Hady, 2005).

Mathematical knowledge (mathematical structure) has been classified into four main components: Concepts, terms, principles and generalizations, algorithms and skills, issues and applications, so that the concept is considered the cornerstone of the mathematical cognitive structure, and generalization comes next in importance, which defines the relationship between two or more concepts, and the concepts and procedures are not formed separately fromSome of them are related to other concepts and procedures, just as each structure includes what comes before it and is based on it. Mathematical skills contain concepts, generalizations and principles (Badawi, 2008).

Algebra is an essential component of the mathematics curriculum, and learning how to solve equations is considered essential in the study of mathematics in postgraduate studies (NCTM, 2000).

One of the difficulties that occur when transitioning from computational thinking to algebraic thinking is the concept of the variable, which is an obstacle to learners' success in algebra,especially in high-achievers in math, as it requires them to make more adjustmentswhen studying algebra (Kieran2004).

Another difficulty is fractions, one of the important topics in algebra, as fractions are basic mathematical concepts and they play an important role in understanding algebraic operations in the upcoming school stages.

Teachers' job is to help learners understand the specific procedures of mathematics in ways that are conceptually consistent with generalized algebra procedures, and to give them networks of interconnections that they can draw upon when they begin the formal study of algebra. Algebra is the language of mathematics, and understanding this language involves understanding the concept of the variable, the phrases of the variable, and the meanings of the solutions. Often, Algebra is viewed as a tool for studying correlations and mathematical modeling which in turn constitutes contexts for the application of algebraic ideas (Badawi, 2008).

Literature review

Teaching mathematical skills is important for many reasons, including the acquisition of the skills and its mastery which helps to understand mathematical concepts and guide the learner's thinking to solve problems in a sound scientific manner. The implementation of the skills increases the learners' knowledge and familiarity with the characteristics of numbers and the various operations applied them, and deepens their understanding of the numerical system and mathematical structure in general (Abu Zina, 2010).

With regard to algebraic skills, the National Council of Mathematics Teachers in the United States of America (NCTM) identified a number of skills and competencies that are necessary for every learner, including (NCTM, 2000): Forming mathematical expressions from verbal problems, solving linear equations and analyzing algebraic expressions and simplifying them. However, many learners find great difficulties in learning mathematics in its various branches.

Challenges or difficulties are defined as everything that prevents the learner from reaching a correct solution when solving a problem. A common error, which is repeated by 25% or more of learners, shows a challenge or a difficulty (Musleh, 2017). The International Dictionary of Education defines learning difficulty as the degree to which learning is difficult for learners, and it is used to indicate something that prevents learning (Al-Saadi, 2011).

Of the many types of learning difficulties that are common in mathematics, some of them can be summarized as follows: Verbal learning difficulty: where the learners finds it difficult to use mathematical vocabulary, they cannot verbally express the steps of the solution in mathematical problems and exercises, and they finds it difficult to formulate them, and find difficulty in understanding facts or mathematical problems when presented orally and find the symbolic learning difficulty represented by their inability to deal with perceptions in a symbolic manner due to difficulties in understanding the symbols and expressing them in writing. This type of difficulty is one of the most common types of mathematical learning difficulties prevalent among learners, due to the lack of using tangible and semi-tangible means in the activities presented to them (Al-Saadi, 2011).

- Idiomatic learning difficulties: when the learners finds themselves unable to read mathematical symbols such as numbers, algebraic symbols, signs of mathematical operations, and to be able to understand mathematical terms.
- Conceptual learning difficulties: where the learners find it difficult to translate mathematical terms or concepts into their meanings, perceive mathematical relationships, understand ideas and perform mental mathematics operations (Musleh, 2017).

Practical or procedural learning difficultiesare represented by difficulties in basic mathematical skills related to performing the four mathematical operations: addition, subtraction, multiplication, and division (Al-Saadi, 2011).

Mathematics has three main aspects: geometry, mathematics, and algebra. These aspects overlap to a large extent. Geometry is a branch of mathematics concerned with studying the sizes and positions of geometric shapes. As for mathematics, it includes the study of integers or fractions and operations on related to them. However, mathematics and algebra overlap, for example, if the sum of two numbers is 20 and one of them is 4, then what is the other number? The correct answer is 16, but the foundations of finding it a basic technique of algebra techniques. In algebra we represent the unknown number with the letter a, and we say we have $a+ 4 = 20$, and the equation can be simplified by subtracting 4 from both sides to make a represents the unknown quantity.

Algebra carries both procedural and conceptual aspects, where procedures refer to mathematics operations such as substitution in the algebraic expression $6\ x + 2y$: $x = 3$, $y = 2$, so that the result is $6 \times 3 + 2 \times 2 = 22$, while the conceptual aspect includes topics such as abbreviation, simplification, and factorization to solve equations and operations on algebraic expressions, as in simplifying the expression $4a + 2b + 2a = 2\ (3a + b)$.And because algebra exposes learners to abstraction and symbols, it is a source of negative problems and its study constitutes a source f difficulty and failure for many learners, mostly in secondary school education(Cockroft, 1982).

A - Mistakes in the concept of the variable

Among the common misconceptions that learners fall into and which were mentioned in the research are those related to concepts of variables and symbols in algebra, failure to distinguish between letters used as units of measurement and variables in algebraic terms, and learners' lack of awareness of the significance of the interconnectedness of both algebra and mathematics (Gleason, 2001).

Heinze and Reiss (2007) indicated that in mathematics, the adjacent is used to write numbers adjacent to each other without commas to denote the value of the number digits, but in algebra it has a different meaning as it denotes the multiplication process, the symbol (xy) means x * y, which causes learners believe that the solution to the equation ($6x = 63$) is ($x = 3$) because they transfer the meaning of the proximity in the calculation to algebra.The NCC-National Curriculum Council for Great Britain has detailed common mistakes, and they are as follows:

B - Mistakes in the meanings of the variables (letters)

The belief that mathematical symbols are nothing but an abbreviation of sensorial, not numerical, things; e.g.:a - apple and b - banana). Learners failing to realize that letters in algebra are symbols of numbers, which leads to learners not being convinced of the possibility of multiplying variables, such as multiplying x by x + 1: x (x + 1), unless x is known.
In mathematics, the proximity is used to write numbers side by side without commas or spaces such as 34 to denote the value of the number (number 34 means 4 + 30), or an indication of the state of addition included in fractional numbers $2\frac{1}{3}$means $2+\frac{1}{3}$, while the adjacent in algebra has a different meaning, as it denotes the multiplication process, the symbol ab means a × b, and 3a means 3 × a, which causes learners to think that 06a = 6, a = 0.

C - Mistakes associated with the equal sign

The equal sign is considered one of the keys to developing algebraic mathematical thinking, and the limited understanding of the equal sign is one of the main mistakes that stand in the way of learning algebra.All treatments on equations require an understanding of the meaning of the equalsign. Learners who view the equal sign as a symbol of an equivalence relationship are more successful in solving Algebraic equations compared to their peers who did not fully comprehend it(Stephens, 2006).

Many studies have found that learners make mistakes about open sentences such as (4y + 5) or (x + 3) as their final correct answers, believing that the equal sign should give one result, in other words without signs such as “+” or “-”. Like in mathematics, that is, they do not have the ability to recognize the written formula in the simplest form of a given algebraic magnitude (Abu Awad, 2006).

D- Mistakes in basic algebraic structures and their analysis

Learners find it difficult to decipher brackets. When using the law of distributing multiplication over addition, the following incorrect rule is used: x (y + z) = xy + z.
When applying the distribution to terms and algebraic expressions, we notice, for example, that learners' mistakes in multiplying algebraic expressions do not appear in calculating the middle term when multiplying two binary expressions, so the two expressions (2x + 3y) (4x + 5y) are multiplied incorrectly like this: (8x2 + 15y2).
Some learners consider the subtraction process as additive, when the algebraic form x- (y-z) is simplified thus (x-y-z), and the mistake in adding and subtracting algebraic expressions is one of the most common mistakes of tenth grade learners (Dababat, 1999).
Abu Awad (2006) pointed out the multiplicity of learners' mistakes in this type of mistakes, as it appears that there are mistakes in subtracting an algebraic coefficient from a constant, and it appears through the following example (6-3y = 3). (6x-3x) wrongly like this (9x), while the incidence of this kind of mistakes among seventh-grade learners decreased to 10%.
Erbas (2004) pointed out that there is a wrong use of abbreviations, and this appears when the similar things are summarized when simplifying algebraic fractions, including:$\frac{3}{x}+\frac{3}{y}=\frac{3}{x+y}$ and learners make generalizations mistakes when simplifying algebraic formulas where the rule generalizes incorrectly from ($\frac{x}{y}\div\frac{z}{a}$) = ($\frac{xa}{yz}$) to ($\frac{x}{y}+\frac{z}{a}$) = ($\frac{xa}{yz}$).
More errors are: addition and multiplication are used wrongly when solving equations, there is a wrong call to the general law for solving quadratic equations in one variable, errors are made in finding the value of a variable when solving a system of equations in two linear or quadratic variables and value of the other variable is not found (Yunus, 2004).

Study Problem

The study focuses on this issue, since ninth gradersstill have difficulties in solving algebraic problems, and they lack a positive orientation towards algebra. Therefore, the relevance of the study lies in identifying common mistakes made by ninth graders in simplifying algebraic expressions and describing the thinking methods that learners practicewhen they try to solve algebraic problems, analyzing any developments or changes in these mistakes.

Study questions

1. What are the misconceptions that learners display through mistakes they make when solving algebraic problems?
2. What are the categories of mistakes that learners display when solving algebraic problems?
3. What are the thinking strategies associated with these mistakes?

Objectives

The study aims to identify thinking patterns used by learners when they have difficulties in solving algebraic problems. Its relevance lies in shedding light on the analysis and classification of errors that learners make in simplifying algebraic expressions and thinking strategies and seeing how learners can be helped to avoid these mistakes in the future.

Terminology

The common mistake: it is the mistake that is repeated again and againby many learners, and is common to them, and it has been identified by many researchers, as it occurs in 15% of tests or more. The common mistake during this study was defined as the wrong answer made by 25% of the cohort.
Conceptual mistakes: Mistakes that cannot be corrected by a quick audit; becausethey result from a lack of understanding of the required mathematical rules, procedures, or equations (Abu Awad, 2006).
Algebraic concepts: concepts that include brief operations on algebraic expressions, analysis of algebraic expressions in terms of their prime factors, solving quadratic equations in one variable, and multiplying and dividing algebraic terms.
Mathematics is a common universal language for all civilizations. It is one of the most important educational activities at different levels of education because of its role in the current scientific renaissance. Algebraic skills are characterized by having motor and cognitive components, unlike the rest of the mathematical skills that contain one of the two components (Afaneh, 2012).

Methodology

Two tools were used in this study. In the first stage, we used a quantitative approach to gather information and to identify the common mistakes that nine grade learners make when studying the topics of algebraic expressions. The prior was achieved by a test built on algebraic concepts that was given to the cohort. In the second stage, we used a qualitative approach to find the most important causes and factors leading to the emergence of these mistakes, by individual interviews conducted with five learners, who were randomly selected from those who answered incorrectly to many test items.

Study community

The cohort of the study consisted of 100 male and female learners (15-16 years old) whose knowledge in algebra was put to test. They were chosen by the simple random method, and the following table shows the demographic characteristics of the ***cohort*** members:

Table1. Characteristics of the study cohort individuals by gender

Variable	Variable levels	the number	The ratio %
	Male	44	44.0
Gender	Female	56	56.0
	Total	100	100.0

Study tools

This study relied on two different tools to collect data, namely: the personal test, in order to obtain the largest possible knowledge of learners in solving algebraic problems, and individual interviews, to reveal the thinking strategies associated with falling into common mistakes and to know the extent of their stability among learners. Here is a breakdown of each tool:

Quantitative method

Test building
Depending on the network of concepts and skills related to algebraic topics and solving equations, the paragraphs of this test were prepared as open-end questions. It was given to ninth graders and included (15) questions to determine the mistakesmade by them and to determine the percentage of learners who have difficulties in simplifying algebraic expressions, to get to know the incorrect concepts and difficulties in learning the topic in order to discover or predict learners' mistakes or a specific learning difficulty.
Test application
The test was distributed to the study population consisting of 100 learners from several Arab schools in Israel.The purpose of this test is to identify the mistakes common to ninth graders in algebraic concepts, and to effectively diagnose learners'performance. The test paragraphs are built as open-end questions that give the learners the opportunity to express their answers, instead of choosing the answer from several answers that are already available to them.

Qualitative method

Interviews were conducted with some learners of the research ***cohort*** after completing the test, in order to identify and find out the reasons that led to the existence of mistakes shown by the research results.
The interviews in this study were based on asking questions and examining learners'opinions. The questions focused on the identificationof ways of thinking used by the ninth graders, which lead to errors while solving algebraic problems, and the extent to which these learners adhere to these strategies.
Statistical processing

Percentages and repetitions were counted for each type of mistake (conceptual mistakes, generalization mistakes, procedural mistakes, and miscellaneousmistakes) that learners made in the test, after which the mistakesrating higher than 25% were identified and approved as common mistakes.
After the statistical processing, learners were interviewed to hear them describe how they worked on the problems, especially those who answered incorrectly many of the test items. For this purpose, questions were directed to reveal the strategies that were used in answering the test questions in a wrong way, and the interviews consisted of algebraic tasks for each learner.

Resultsanalysis and presentation

In an attempt to identify the mistakes that learners make in solving algebraic problems and the methods of thinking associated with them, it becomes clear through interviews conducted with learners that the nature of the mistakes is due to the lack of understanding of the basic facts of mathematical operations, and some of them are due to the wrong application of algorithms. Learnersstart off by solving the problem in a correct manner, and then they resort to a different process that they were previously taught which leads to the confusion.In the following, we will discuss 13 categories of mistakes that were repeated in the data that were collected through learners' solutions to the test.

Table2. The percentages of common mistakes among ninth graders in the Algebraic Concepts test

No.	Items	percentage
1.	Mistake in the distributive property of multiplication on addition and subtraction (decodingbrackets)	33.33%
2.	Mistake in performing addition and subtraction on terms and expressions	18.92%
3.	Mistake in multiplication operations in algebraic expressions	16.32%
4.	Mistake in division operations in algebraic expressions	25.1%
5.	Mistake in finding the square of a term and an expression (prime factors)*how to find the square root by prime factorization*?	57.1%
6.	Signal mistakes	40.00%
7.	Mistakes in finding the greatest common denominator in algebraic equations	53.33%
8.	Misuse of reduction	52.3%
9.	Mistake in the analysis of prime factors	48.1%
10.	Mistake in performing operations on algebraic equations	26.5%
11.	Failure to apply algebraic rules and laws	35.24%
12.	A mistake in performing operations within algebraic limits by dividing the exponents	30.2%
13.	Mistake in the distributive property of multiplication on addition and subtraction (decodingbrackets)	33.33%

Through the data provided in the table, it is noted that the highest percentage of common mistakes in algebraic expressions among ninth graderare the mistake in finding the square of a term and an expression (prime factors) *how to find the square root by prime factorization*?(57.1%), while the lowest percentage of common mistakes in the ninth grade was in applying rules and laws (25.0%). The mistakes that learners made were classified in basic algebraic concepts, which came in the form of four types: conceptual mistakes, generalizations mistakes, procedural mistakes, and various other mistakes.

Table3. The percentages of the types of algebraic conceptual mistakes common among ninth graders in solving the test

No.	Type of mistake	percentage
1.	Conceptual mistakes	%37.1
2.	Generalizations mistakes	%41.2
3.	Procedural mistakes	%32.1
4.	Miscellaneous other mistakes	%26.3

It is evident from the table that the most common mistakes among ninth graders were generalization mistakes that occurred at a rate of 41.2%, followed by conceptual mistakes in second place at a rate of 37.1%, and then procedural mistakes came in third place at a rate of 32.1%, and finally other miscellaneous mistakes at 26.3%.

The following is a detailed presentation of each of the mistakes learners made when taking the test:

1. Mistakes in distributing multiplication over addition or subtraction of two algebraic terms:

The frequency of the mistake in the first question of multiplying the algebraic expression ax + b by a number reached 15.5%, and the second question, which represented the multiplication of the algebraic expression ax-b by a number, reached 17.83%. The prior may be due to the failure of learners to understand the previous algebraic expression consisting of two parts, that both of them are algebraic terms, and that each term must be multiplied by a number, as most of the answers related to multiplying the first term only by the number with mistakes in the multiplication process, especially with regard to signs, and the reason for this may be automatic memorization without understanding that rule, which led to forgetting it later.

$$(\mathbf{2a - a})^{\mathbf{2}} - \mathbf{3}(\mathbf{a}+\mathbf{5})(\mathbf{a}-\mathbf{5}) =$$

$\rightarrow 4a^2 - 4a + 1 - 3a^2 + 25 \rightarrow a^2 - 4a + 26$
*The equation above represents a mistake in the distribution of multiplication on two algebraic expressions.

2. Mistakes in performing addition and subtraction in algebraic expressions:

The frequency of the mistake represented in the first and second questions in adding and subtracting algebraic terms reached 18.92%. The prior may be due to learners regarding the processes of adding and subtracting algebraic terms as the operations of adding and subtracting numbers, as most of them added or subtracted the variables as though they were numbers. The prior could be explained by the lack of rule-illustration to learners.

$$(\mathbf{2a - a})^{\mathbf{2}} - \mathbf{3}(\mathbf{a}+\mathbf{5})(\mathbf{a}-\mathbf{5}) =$$

$\rightarrow 1\text{a} - a^2 - 5a =$

$$(\mathbf{3}\boldsymbol{x} + \mathbf{4})^{\mathbf{2}} - (\mathbf{3} - \mathbf{2}\boldsymbol{x})^{\mathbf{2}} =$$

$\rightarrow 7x^2 - 1x^2 = 6x^2$

*The equation above represents a mistake in addition and subtraction algebraic expressions.

3. Mistakes in performing operations in algebraic expressions.

The results of the test responses revealed that 26.5% of the learners misused the signs (+, -, ×, ÷). In solving Item the following item of the test.Learners misused the multiplication sign and replaced it with the division sign, and then solved the problem as a process of dividing algebraic expressions.

$$\frac{\mathbf{b^2-5b+6}}{\mathbf{b^2-4}} \times \frac{\mathbf{b+2}}{\mathbf{b-3}} =$$

$$\rightarrow \frac{b^2-5b+6}{b^2-4} \div \frac{b+2}{b-3} = \frac{(b-3)(b-2)}{(b-2)\cancel{(b+2)}}\, x\, \frac{\cancel{b-3}}{b+2} = \frac{(b-3)^2}{(b+2)^2}$$

*The equation above represents a mistake in the multiplication of algebraic expressions by changing multiplication into division.

$$\frac{\mathbf{a}}{\mathbf{a+b}} - \frac{\mathbf{a^2+b^2}}{\mathbf{b^2-a^2}} = \frac{\mathbf{2a+b}}{\mathbf{a+b}}$$

$$\rightarrow \frac{a}{a+b} - \frac{a^2b^2}{b^2a^2} = \frac{2ab}{ab} \rightarrow \frac{a(a) - 1(a^2b^2) + 2ab}{ab}$$

$$\rightarrow \frac{a^2 - 1a^2b^2 + 2ab}{ab} - \frac{a^2 + 1ab^2}{ab} = 2a^2b^2$$

*The equation above represents a repetition of the same mistake

4. Mistake in performing division in algebraic expressions.

These mistakes appeared in the process of dividing an algebraic expression on the other, and this is evident through the solutions of some learners where the learner did not remember completing the division task thus, thenominator was not inverted in the second algebraic fraction.

$$\frac{\mathbf{a^3 - a^2}}{\mathbf{a^2 - 7a + 6}} \div \frac{\mathbf{a - 1}}{\mathbf{5a^2 - 5a}} =$$

$\rightarrow \frac{a^2(a-1)}{(a-3)(a-1)} \; x \; \frac{a-1}{5a(a-1)} = \frac{a^2}{(a-3)} \; x \; \frac{1}{5a} = \frac{a}{(a-3)5}$
*Nominator is not inverted in the second algebraic fraction.

Certain mistakes in algorithms were observed where some learners flip the dividend instead of the divisor, or both, when performing division.The learner's knowledge may be incorrect or based on a specific misunderstanding, which becomes a source of incorrect solutions. When performing the division process, learners consider that commutative division is like multiplication. As in the learner's solution to the following item in the test, where learners considered the division process as the cross multiplication, and applied the law of cross multiplication a × b = b × a and then began to solve the problem.

$$\frac{\mathbf{a^3 - a^2}}{\mathbf{a^2 - 7a + 6}} \div \frac{\mathbf{a - 1}}{\mathbf{5a^2 - 5a}} =$$

$\rightarrow \frac{a-1}{5a^2-5a} \div \frac{a^3-a^2}{a^2-7a+6} \rightarrow \frac{a-1}{5a(a-1)} \, x \, \frac{(a-1)(a-3)}{a^2(a-1)} \rightarrow \frac{1}{5a} \, x \, \frac{(a-3)}{a^2} = \frac{(a-3)}{5a^3}$
*The equation above represents a wrong use of division in algebraic expressions.

5. Mistake finding the square of a term and an expression

This mistake also appeared in the following item of the test, where the algebraic expression (4m^2 n) ^{2}) / (4m ^ 3 n) was simplified incorrectly thus (16m ^ 2 n) / (4m ^ 3 n) explaining that: "That square means multiplying a number", or another wrong way :(4m ^ 2 n) ^ 2) / (4m ^ 3 n) is (4m ^ 4 n ^ 2) / (4m ^ 3 n).

$$\frac{(\boldsymbol{4m^2n})^2}{\boldsymbol{4m^3n}} =$$

$\rightarrow \frac{4m^4n^2}{4m \cdot 4m \cdot 4m \cdot n} = 4m \cdot n$

$$\frac{(\boldsymbol{4m^2n})^2}{\boldsymbol{4m^3n}} =$$
$\rightarrow \frac{16m^2n^2}{4m^3n} = \frac{4n}{m}$

6. Signal-relatedmistakes

Some learners face difficulty in performing the process of simplifying the algebraic expressions, which leads them to mistakes. This mistake appears in the learner's solution, Item No. 1 of the test, where the expression is simplified (2a-1) ^ 2-3 (a + 5) (a-5) as follows: a2-4a-74 = (4a2 -4a + 1) - (3a2-75).The learnercame to a wrong conclusion in simplifying the second section, and ignoring the sign (-).

$$(\mathbf{2a - a})^2 - \mathbf{3}(\mathbf{a} + \mathbf{5})(\mathbf{a} - \mathbf{5}) =$$

$\rightarrow 4a^2 - 4a + 1 - 3a^2 - 75 = a^2 - 4a - 74$

*The equation above shows a signal mistake.

7. Mistakes in finding the greatest common denominator in algebraic equations

These mistakes are the most common that learners make. It is evident that the learner is unable to find the greatest common denominator and thus, takes a random common denominator for the algebraic equation in order to make it easier to handle. In solving the following item of the test, it becomes clear that the learner is unable to determine the appropriate common denominator in the algebraic equation, due to lack of analysis into factors in the denominators of the first equation, and the use of the strategy of unifying denominators without analyzing them, then subtracting the first numerator from the second numerator. Through this solution we conclude that there is a problem of extracting the greatest common denominator in an equation that needs to be analyzed.

$$\frac{\mathbf{2}}{\boldsymbol{x^2 + 8x + 16}} - \frac{\mathbf{2}}{\mathbf{2x - 8}} = \frac{\mathbf{1}}{\mathbf{x + 4}}$$

$\rightarrow \frac{2-2}{(x^2+8x+16)(2x-8)(x+4)} \rightarrow \frac{1}{(x+4)(x^2+8x+16)(2x-8)}$

*The equation above shows mistakes in finding the greatest common denominator in algebraic equations.

8. Misuse of reduction

Usually,learners cancel similar terms that exist in the numerator and denominator, and this is oneof the study findings. It became clear from the learner's solution to the following item of the test, that the learner applied the cancellation mistake and remained with a-1 in the denominator in the first algebraic fraction, Another cancellation was made in the second algebraic fraction to obtain a zero in the denominator of the second algebraic fraction, then the learner ends with the solution with an unknown answer.

$$\frac{\mathbf{a^3 - a^2}}{\mathbf{a^2 - 7a + 6}} \div \frac{\mathbf{a - 1}}{\mathbf{5a^2 - 5a}} =$$

$\rightarrow \frac{a^3-a^2}{a^2-\cancel{7a}+6}$ x $\frac{5a^2-5a}{a-1} \rightarrow \frac{a^2-a}{a-7+6}$ x $\frac{5a-5}{1-1} \rightarrow \frac{a^2-a}{a-7+6}$ x $\frac{5a-5}{0}$ = unknown

*The equation above shows a misuse of reduction

9. Partial reductionmistakes

$$\frac{\mathbf{-8 + 2b}}{\mathbf{2}} =$$

$\rightarrow \frac{-8+\cancel{2}b}{\cancel{2}} = -8 + b$

The equation above shows a partial reduction mistake

10. Mistakes in prime factorization

When some learners have to factor in the algebraic expression to simplify the expression, their difficulty appears, as in the example of the learner's solution to the equation below. We find a weakness in the analysis into factors in the first algebraic fraction and use another analysis of the factors of the denominator of the algebraic fraction, where x2- 8x + 12 to x-4) (x + 4)) and then canceled.

$$\frac{\boldsymbol{x^2 + 7x + 12}}{\boldsymbol{x^2 - 8x + 12}} \div \frac{\boldsymbol{9 - x^2}}{\boldsymbol{4x - 2x^2}} =$$

$\rightarrow \frac{(x+3)(x+4)}{(x-4)(x-4)} \div \frac{(3-x)(3+x)}{2x(x-2)} \rightarrow \frac{(x+4)}{(x-4)(x-4)}\ x\ \frac{2x(x-2)}{(3-x)} \rightarrow \frac{2x(x-2)(x+4)}{(x-4)(x-4)(3-x)} \frac{(x+3)(x+4)}{(x-4)(x-4)}\ x\ \frac{2x(x-2)}{(3-x)(3+x)}$

*The equation above shows a mistake in prime factorization

11. Mistakes in performing operations on algebraic equations

These mistakes appeared through the learner's solutions to the equation below, and the common mistake was followed by adding the two algebraic terms that are not the same, and this is shown through the solutions and then following the method of eliminating the similar terms.

$$\frac{\mathbf{-8 + 2b}}{\mathbf{2}} =$$

$\rightarrow \frac{2(-4+b)}{2} = -4 + b$

12. Failure to applying algebraic laws

Some learners make the mistake of negligence, as they have the knowledge and capacity to conduct operations and apply laws, but they make mistakes due to lack of attention.
Learners lack a deep understanding of the basics of mathematics, so their learning is not conceptual. These mistakes are based on the learners'prior knowledge. The learner's knowledge may be incorrect or based on a specific understanding, forming a source of incorrect concepts in their answer. They apply the law (A + B) 2 = A 2 + B 2 + 2AB as follows (A + B) 2 = A 2 + B 2 and this appears in the solutions of many learners as in Item No. 2 of the test.

$(\mathbf{3x} + \mathbf{4})^{\mathbf{2}} - (\mathbf{3} - \mathbf{2x})^{\mathbf{2}} =$

$\rightarrow 7x^2 - 1x^2 = 6x^2$

13. Mistakes in dividing algebraic terms by dividing exponents

Learners showed to be confused about the relevance and function of brackets. A learner raised the denominator to the power of two, in line with what he did to numerator.

$\frac{(\mathbf{4m^2n})^{\mathbf{2}}}{\mathbf{4m^3n}} =$

$\rightarrow \frac{16m^4n^2}{16m^6n^2} = \frac{1}{m^2}$

$\frac{(\mathbf{4m^2n})^{\mathbf{2}}}{\mathbf{4m^3n}} =$

$\rightarrow \frac{8m^4n^2}{4m^3n} = 2mn$

14. Mistakes in the distributive property of multiplication on addition and subtraction (decodingbrackets).

The second question: What thinking strategies do ninth graders use that accompany their mistakes in simplifying algebraic expressions?

This question was answered by interviewing learners and listening to them describing how they came to solve the questions, thus revealing the thinking strategy that each of them used. Each strategy was addressed based on the interview, which aimed to describe thinking strategies (mental processes) in some algebraic tasks. A group of (5) nine graders answered incorrectly many of the test items, and each of them was asked to explain the solution method he or she used in the test (where they made the mistake) and how they thought about it.

In general, the diversity of thinking strategies associated with learners'mistakes in simplifying algebraic expressions was observed.Among the most important of the priors is the use of incorrect properties of the algebraic system, such as the property of distributing multiplication over addition, applying it incorrectly, confusing concepts, or using incorrect algebraic generalizations such as omitting parts or resorting to modifying other generalizations and using them in inappropriate position.

Thinking strategies

Brackets Decoding Strategies

The learners' beliefs centered through the interviews as follows: The distribution of multiplication over addition is by multiplying one of the terms by the algebraic expression. The interviews aimed to address the learners 'ability to decode the brackets. we found that 5 learners simplified the algebraic expression (5 + a) 3 incorrectly, thus -3a + 5, and the learners' belief was: "The distribution is done by multiplying what is on the outside of the brackets by the first term and then adding it to the second term". Multiplying two algebraic expressions is done by multiplying each term in the first expression by its counterpart by the second expression, such as:

(a + b) (c-d) = ac-bd.

When finding the result of (3-2x) -2x (3) in Item 2 of the test, four of the nine graders, during the interview, multiplied the corresponding elements, and recorded the result 9 + 4x2 and were unable to decode the brackets correctly, and this shows their understanding of the distribution of multiplication in the combination is limited and imprecise.

Strategies in adding algebraic terms

Adding numbers and multiplying the variables are different strategies. The learners have a special strategy related to adding two algebraic terms, as all the learners who were interviewed (5) nine graders simplified the expression 3x + 4 (^ 2), and that was done by adding the terms (4,3x), and squared Result 7 x (^ 2).

Strategies for subtracting terms and algebraic expressions

Subtracting two algebraic expressions inside the brackets was raised to the exponent (2) by subtracting what is inside the brackets without looking at the exponent. (4) Learners out of (5) simplified the algebraic expression, (2a-1) ^ 2-3 (a + 5) (a-5) as follows: (a ^ 2-3 (a ^ 2 -25).

Learners notice misconceptions about mathematical concepts and procedures in subtraction

The subtraction of two different algebraic terms in the exponents is done by subtracting the exponents. During the interview, (4) Learners out of (5) subtracted the two algebraic terms that are similar in the basic and different in exponents: (9x4-9x2) so that the result is 9x 2, and the subtraction was between the exponents only in the process of subtracting algebraic expressions. Subtracting two expressions inside the bracketsrose to the exponent (2) by subtracting what is inside the brackets without considering the exponent.

Strategies for dividing two algebraic terms by dividing exponents

Most learners have a wrong concept related to finding the expansion of the square of an algebraic term as three out of five ninth graders simplified the algebraic expression (2s ^ 3) / t) ^ 2 incorrectly like (4S ^ 3) / t ^ 2during the test.One student explained "the square means multiplying the number by itself twice", while another learner indicated that the product is expanded (2s ^ 3) / t) ^ 2 is (2S ^ 6) / t ^ 2 explaining that "the square is done by multiplying the letter by itself twice".

This strategy also appeared in the test item: (4 m ^ 2 n) ^ 2) / (4m ^ 3 n), where the algebraic expression was simplified wrongly (16m ^ 2 n) / (4m ^ 3 n) with this explanation: "The square means multiplying a number, or in another wrong way (4m ^ 2 n) ^ 2) / (4m ^ 3 n) is (4m) ^ 4 n ^ 2) / (4m ^ 3 n) (multiply the letter by itself twice).

Reduction strategies

Learners believe that reduction when simplifying algebraic fractions is done by abbreviating similar expressions without taking into account the mathematical operation. Three learners simplified the expression (10 m ^ 2 + 15m) / (4 ^ 2 + 12m + 9) where the outcome was (1), justifying this in short (m ^ 2, m) from the numerator and denominator because it is common. We also see that some learners also followed this strategy when they simplified (-8 + 2b) / 2 by reducing the number (2) from the numerator and denominator to get the score -8 + b. It all relies on an incorrect rule of reducing terms and similar expressions, by canceling the common factor between the numerator and the denominator.

Strategies in simplifying fractional algebraic expressions

Mistakes and misconceptions in fractional algebraic expressions lead learners to solve problems based on prior information.For example, the simplification of a mathematical fraction 2/3 + 1/4 can be understood differently from the simplification of the algebraic expression 1X / 4 + 2X / 3. Here, learners are unable to comprehend new knowledge because they see it as a completely separate element of their existing knowledge of the subject.

Strategies in operations on algebraic fractions

The learners' weakness appeared in performing the multiplication process on algebraic fractions, as 5 out of (6) learners reached a simplification of algebraic fractions, and this is evident through the test item where the algebraic expression was simplified (b ^ 2-5b + 6) / (b ^ 2-4) × (b + 2) / (b-3) as follows: b ^ 2-5b + 6) (b-3) = (b + 2) (b ^ 2-4).
Cross multiplication was performed, and the answer of the majority of learners indicated that there was amistake in the fraction multiplication algorithm. They have explicitly expressed that multiplication is performed by performing a cross-multiplication, which in turn reflects their lack of understanding of proper multiplication of algebraic fractions. This is also evidenced by the test iem where the algebraic expression is simplified (81-y ^ 2) / 7 × (-49) / (y-9): 81-y ^ 2) (y-9) = - 49 x 7)

Analysis strategies of primary factors

There were many wrong strategies in this section. For example, learners believed that a2-7a + 6 analyzes prime factors depending on the distribution law a (bc) = ab-ac. During the interview, a learner analyzed the quadratic equation and used reduction to get to the result. Learners believe that b2-a2 (*you mean* b^2-a^2*??)* is analyzed into prime factors (- a) (b-a) (b), leaving the problem unaswered or leaving the equation without analysis.

Strategy of finding the greatest common denominator in algebraic equations

It was found that learners have more difficulty finding the greatest common denominator when performing addition and subtraction on algebraic fractions. Instead, they simply subtract the numerator and denominator from each other, and concludethat the numerator and denominator are separate numbers.

Prior knowledge strategy

When learners are exposed during the learning process to difficulty in understanding the concept properly, they carry out the process of transferring this information to suit their knowledge, and this process often leads to incorrect understanding or the so-called wrong concept (Steinle, 2004), and since these concepts are built on the basis on which later concepts are built that depends to a large extent on previous knowledge, they constitute a threat to the learner's subsequent knowledge.

Discussion

This study aimed to uncover patterns of common mistakes in solving algebraic problems among ninth graders. It also aimed to identify thinking strategies associated with these mistakes by learners' describing the method of solving these algebraic problems that led to the mistake. The results showed that there are numerous mistakes common to learners,among which; over-simplifying algebraic expressions. It also showed that the thinking strategies that learners use when making mistakes depend on the use of incorrect properties of the algebraic system, such as the property of distributing multiplication on addition, applying it incorrectly, mixing concepts or using incorrect generalizations.

The first question was answered by monitoring learners' performance in the test. The results showed the diversity of common mistakes in solving algebraic expressions and equations. The highest percentage of common mistakes in algebraic concepts among learners was finding a square expansion at a percentage of 57.1%, while the lowest percentage of common mistakes in inflection or combining two dissimilar terms was 18.92%, and this is explained by the learners' belief that the sign of the brackets should give one result and therefore the learner performs a wrong procedure to produce one term. Thus, $3x + 4$ was incorrectly simplified, and this result is in agreement with many studies that have been reviewed in this area.

The common mistake when solving the first question is the replacement of the process of subtracting two terms that are not similar to each other by multiplying them, may not be explained enough. This matter requires future study and research to verify whether this is a pattern or just a mistake that occurred by chance in this cohort, and to know the types of experiences learners are exposed to, which may help provide appropriate explanations. The second type of mistakes is the subtraction of two algebraic expressions, so that the first term is subtracted from the second term of the first expression.

It has also been observed that ninth graders have common mistakes in decoding brackets, and this is consistent with the results of the study of(Abu Awad, 2006). They also make mistakes in multiplying algebraic expressions by not calculating the middle term when multiplying two binary expressions, and this is explained by the learners'belief that multiplying two algebraic expressions is done by multiplying each term in the first expression by its counterpart in the second expression, aresult consistent with the study of (Muhammad, 2004).

With regard to the mistake in finding the square of an algebraic term, it was consistent with the results of the study of(Abu Awad, 2006; Dawkins, 2006), as well as falling into a mistake in finding the square of the sum of two terms has agreed with the results of other studies in this area (Al Yunus, 2004; Dawkins,2005). The results of the study of (Muhammad, 2004), this mistake may be attributed to learners'weakness in adding and subtracting negative integers, in addition to considering the subtraction process as commutative.

We have the possibility to understand the origins and justifications of these strategies. In general, a diversity of thinking strategies associated with learners'mistakes in simplifying algebraic expressions was observed, and the most important of these strategies is the use of incorrect properties of the algebraic system, such as the property of distributing multiplication to the addition, applying it incorrectly, confusing concepts, or using incorrect algebraic generalizations such as deleting parts of them, resorting to generalizations and using them in inappropriate locations. This is consistent with the theory of modification as one of the theories of generating mistakes based on a group of imperfect strategies by applying the principles of deletion to formal representations of the correct action. Mistakes occur when learner encounters an unfamiliar situation during their performance, which leads them to a dead end as they try to find a known method and apply it to the task in a wrong way (Vanlehn & Brown, 1980).

A weakness has been observed in the conceptual and procedural knowledge in operations with variables, and that the most common mistakes are related to the operations of adding and subtracting algebraic expressions. So, the expression 5x^ 2-3x is incorrectly simplified as 2x2. This result is in agreement with the study of(Akg¨un and Ozdemir, 2006) and the results of this study were consistent with regard to strategies related to the distribution of multiplication on addition, and the strategy of collecting algebraic terms that are not similar to each other.

It was observed that learners' use of strategies related to addition and multiplication, where the learner used either of them to replace the other, and this is in agreement with the study of(Levi & Carpenter 2000). The previous study found that the majority of learners adhere to solving strategies associated with common mistakes in algebraic concepts.It was also observed that learners made the same mistakes in the test as in the interview.It was noted that the percentage of using the same solving strategies associated with common mistakes in algebraic concepts is high among learners in most of the questions. This percentage ranged between 50% to 100%, and this indicates that thinking strategies associated with falling into inherent mistakes are not random and they have a deep cognitive structure.

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