ISSN: 0974-5823 Vol. 7 No. 1 January, 2022

International Journal of Mechanical Engineering

Data-driven modeling of automotive suspension dynamics using Polynomial chaos

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Abstract

A data-driven method based on parametrical uncertainty analysis of the car suspension system was learned in this study. Four degree-of-freedom (DOF) mathematical models have been established for the car's passive and semi-active suspension systems. The uncertainty of the models has been studied using polynomial chaos methods. Compared to the Monte Carlo simulation approach, the results indicate that Polynomial Chaos (PC) approaches are more potent than the Monte Carlo (MC) approach and are a sufficiently robust tool for the simulation of uncertainty in dynamic systems.

Index Terms – stochastic car model, polynomial chaos, uncertainty, Monte Carlo method

INTRODUCTION

The parameters of car engineering are usually known as determinist, but several unknown parameters exist that the dynamic performance of the car would be significantly influenced. Such uncertainties can be caused by manufacturing defects, such as mechanical wear. In engineering, if the ranges of uncertain parameters are very limited, the traditional deterministic approach has a little error. The deterministic method does not provide the correct response when ranges of uncertain parameters are large or the number of uncertain parameters has risen significantly [1]. Methods that have recently been utilized for different problems are Polynomial Chaos expansion and Monte Carlo techniques in numerical simulation of uncertainties quantifcation. To estimate system dynamics with uncertainty that the polynomial chaos method has been proposed in this paper. With a small number of possible parameters but a large magnitude of uncertainties the polynomial chaos approach offered an effective mathematical solution for the large nonlinear system.
Although the PC method has been successfully extended to several issues, including car dynamics prediction. A half-car model four-degree-of-freedom suspension system is given with these polynomial chaos-based methods in this work [2].
After introduction in the chapter 2 we quickly review PC expansion and its mathematical formulations. Then, we address performance of PC expansion compared to MC and show examples in chapter 3 and chapter 4. Conclusions are provided by the results obtained from the simulations in the last section of the paper.

POLYNOMIAL CHAOS EXPANSION

2.1. Polynomial chaos expansion
The polynomial chaos expansion is used to represent a stochastic process of polynomial functions of uncertainty dynamic system in this section. We also show how to build model of the system using this approach [5].
Let us take the following actual parametric system into consideration:

$$Y = \mathrm{M}(X) \qquad \textbf{(1)}$$

where Y ∈ R is the output of the system and X = { x_i , i = 1, ..., N } is a vector which captures the parameters of the system. The aim is to construct a compact and precise mathematical representation $\tilde{\mathrm{M}}$ is the so called surrogate model of the system M.
Polynomial Chaos expansions of stochastic processes:

$$Y \approx \tilde{M}_{PC}(X) = \sum_{k=0}^{k} a_k \psi_k(X) \qquad \textbf{(2)}$$

- Y is the random response and X is the random variables of a mechanical model.
- $K+1 = \dfrac{(p+n)!}{p!n!}$, n is the number of random variables and p is the maximum order of the polynomial basis.
- a_k are coefficients to be computed (coordinates)
- $\psi_k(X)$ is an orthonormal basis [4]

2.2. Building of a polynomial chaos in basis in simple circuit

Example building of a polynomial chaos in simple resistance-inductance-capacitance (RLC) series circuit and its tranfer function is $|H(x)| = |LCx^2 / (LCx^2 + RCx + 1|$.

As can be seen from the transfer function that we have three variables.

We consider the model depending on three Gaussian random x_1, x_2 and x_3.

Upon applying the linear mapping $x_1 = \mu_1 + \sigma_1\zeta_1$, $x_2 = \mu_2 + \sigma_2\zeta_2$ and $x_3 = \mu_3 + \sigma_3\zeta_3$. In terms of standard Gaussian random variables the response may be recast, that is $Y = \tilde{M}_{PC}(\zeta_1, \zeta_2, \zeta_3)$.

The orthogonal polynomial family with regard to the standard Gaussian probability density function (PDF) $\varphi(x) = 1/\sqrt{2\pi}e^{-x^2/2}$ is the family of Hermite polynomials $\{\mathrm{He}_k(\mathrm{x}),\ \mathrm{k} \in N\}$. The following recurrence relationship describes them as:

$$\begin{aligned} &\mathrm{He}_{-1}(\mathrm{x}) \equiv \mathrm{He}_0(\mathrm{x}) = 1 \\ &\mathrm{He}_{j+1}(\mathrm{x}) \equiv x\mathrm{He}_j(\mathrm{x}) - \mathrm{j}\mathrm{He}_{j-1}(\mathrm{x}) = 1 \end{aligned} \qquad \textbf{(3)}$$

The polynomials that come up are orthogonal, but not in orthonormal form. Specifically, it is known $\langle He_j, He_j \rangle = j!$.

Consequently, the family $\{He_k(x)/\sqrt{j!}, k \in N\}$ is orthonormal. The extension of the random response Y to the maximum degree of the PC basis is p = 2. The preserved polynomials are constructed in ζ_1, ζ_2 and ζ_3 from products of Hermite polynomials. An approximation of the model response is indeed obtained in the form:

$$\begin{aligned} Y &= a_0 + a_1\zeta_3 + a_2\zeta_2 + a_3\zeta_1 + a_4\zeta_1\zeta_2 + a_5\zeta_3\zeta_2 + a_6\zeta_1\zeta_3 \\ &+ a_7(\zeta_1^2 - 1)/\sqrt{2} + a_8(\zeta_2^2 - 1)/\sqrt{2} + a_9(\zeta_3^2 - 1)/\sqrt{2} \end{aligned} \qquad \textbf{(4)}$$

where the coefficients { a_k , k = 0, ..., 9} must be determined [3].

2.2. Coefficient

The notable point of constructing the expansion of the PC since one of the several methods is to find the PC coefficients. Different strategies can be used to approximate the PC coefficients in (2) like sampling-based approaches based on linear regression. The technique of linear regression is applied here. Generating a series of observations $\{(x_i, y_i)\}_{i=1}^{L}$ the original system is tested for L randomly selected samples of parameters x. These results are adapted to the PC expansion (2) in the least square case. The system is provided as

$$Aa = b \qquad \textbf{(5)}$$

where $a \in R^{K+1}$ is a vector which collects the uncertain coefficients of PC expansion, $b \in R^L$ is a vector which collects the observations $\{y_i\}_{i=1}^{L}$, and $A \in R^{L\times(K+1)}$ is a Vandermondelike matrix of basis polynomials measured at random ranges of parameters, and that solving by regression (probably L = 2(K + 1)) is recommended for an appropriate assessment) [8]

$$a = (A^T A)^{-1} A^T b \qquad \textbf{(6)}$$

MATHEMATICAL FORMULATION OF CAR SUSPENSION SYSTEM

To reduce vibrations that are distributed from the ground surface to the car body, the car suspension system has been used. A soft suspension is needed for good riding convenience and a hard suspension is needed to handle higher weight. An active or semiactive suspension system is preferred over a simple passive suspension system to satisfy these two concepts were mentioned in the above. In optimizing both ride comfort and road holding that active and semiactive suspensions are getting more interest.

Semiactive suspension provides a desirable performance improved by an active suspension without costly hardware [6,13]. So, the most frequently used in specific applications are semiactive and active prototypes. However, nonlinear equations of uncertainty are difficult to achieve advanced optimization solutions.
The mathematical representation of the suspension system for cars is discussed in this section. We can be used the passive suspension system of half car model as linear case and the semiactive suspension systsm of half car model as nonlienar case [12].
First, the model of passive suspension was shown in Fig.1.

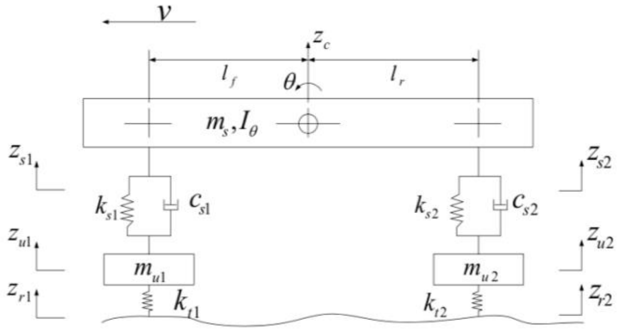

FIGURE 1
HALF CAR REPRESENTATION OF PASSIVE SUSPENSION SYSTEM

Mathematical equations that describe the suspension system should first be formulated in order to simulate the suspension system. For this reason, the basic theory of dynamics was adopted to the suspension system. Few basic assumptions were considered for mathematical relations for sprung and unsrung masses.
There are some assumptions such as car does not leave the ground, velocity of the car is constant and sprung mass is considered to be uniformly distributed and also pitch angle is given very small [11].
In Fig.1, where the spring mass is described by m_s , and the unsprung mass of the front and rear suspension is described by m_{u1} , m_{u2} respectively, and the springs of stiffness of front and rear suspension is described by k_{s1}, k_{s2} respectively and the dampers with damping coefficient of front and rear suspension is described by c_{s1} , c_{s2} respectively.
z_{si} , z_{ui} (i=1,2) are the vertical displacements of the sprung and unsprung mass of front and rear suspension respectively. z_{ri} (i=1,2) is vertical displacement from road surface and z_c is the car (body) displacement. The corresponding stiffness of the front and rear tires is k_{t1} , k_{t2} . l_f , l_r is the length between the center of gravity and the front axle or the rear axle. The moment of inertia is I_0 [10].
It is possible to write z_{ri} (i=1,2) as:

$$z_{s1} = z_c + l_f\theta \quad \textbf{(7)}$$

$$z_{s1} = z_c + l_f\theta \quad \textbf{(8)}$$

The basic form of the State-space model is used to model the suspension system:

$$\dot{q} = Aq + Bu$$

$$y = Cq + Du$$

The model of state space can be given by the following:

$$q = \begin{bmatrix} z_c \\ \theta \\ z_{u1} \\ z_{u2} \\ \dot{z}_c \\ \dot{\theta} \\ \dot{z}_{u1} \\ \dot{z}_{u2} \end{bmatrix} \quad u = \begin{bmatrix} z_{r1} \\ z_{r2} \end{bmatrix} \quad y = \begin{bmatrix} z_c \\ \theta \\ z_{u1} \\ z_{u2} \end{bmatrix}$$

Second, the semiactive suspention system is studyied to build another model and above assumptions stayed the same.

The system of semiactive suspension is identical to that of the passive system, apart from the fact that variable damping coefficients c_{s1} , c_{s2} substituted with the constant damping coefficients for the front and rear wheels. F_{s1}, F_{s2} seem to be the dampers for front and rear suspension damping coefficients in Fig.2.

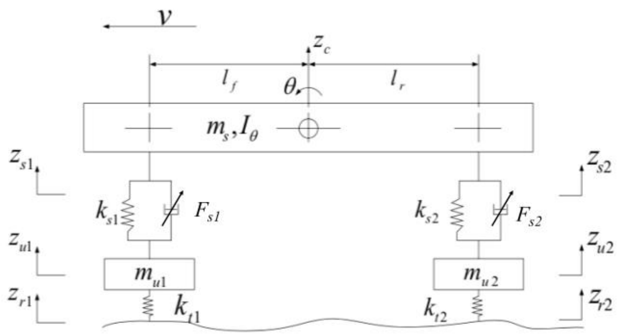

FIGURE 2
HALF CAR REPRESENTATION OF SEMIACTIVE SUSPENSION SYSTEM

It is possible to give the model of state space as follows [10]:

$$q=\begin{bmatrix} z_c \\ \theta \\ z_{u1} \\ z_{u2} \\ \dot{z}_c \\ \dot{\theta} \\ \dot{z}_{u1} \\ \dot{z}_{u2} \end{bmatrix} \quad u=\begin{bmatrix} F_{s1} \\ F_{s2} \\ z_{r1} \\ z_{r2} \end{bmatrix} \quad y=\begin{bmatrix} z_c \\ \theta \\ z_{u1} \\ z_{u2} \end{bmatrix}$$

There is a significant opportunity to add nonlinear parameters and properties of such products and interactions of different elements made of different materials to dampers.
In [14], author gives example of these kind of processes is magneto-rheological (MR) liquids which provide controllable damping force by using MR fluids as semiactive device.
Stiffness component, passive damper and Bouc-Wen hysteresis loop components are formed of the Bouc-Wen model.
Figure 3 shows the schematic representation of the BoucWen model of the MR damper. There is an internal variable y in the hysteresis loop which reflects hysteretic actions and satisfies the next expression (9). The Bouc-Wen model's model equation is expressed as below.

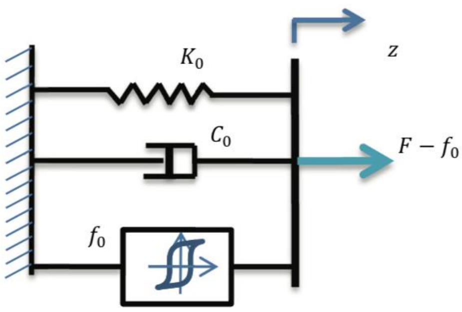

FIGURE 3
SCHEMATIC REPRESENTATION OF BOUC-WEN MODEL OF AN MR DAMPER

$$\dot{y}=\gamma\left|\dot{z}\right|y\left|y\right|^{n-1}-\beta z y^{n}+A\dot{z} \qquad \textbf{(9)}$$

The time evolutionary vector function y depends on the γ , β and A parameters. The force that the MR damper imposed is the function given by

$$F=C_0(u)\dot{z}+K_0 z+\alpha(u)y+f_0 \qquad \textbf{(10)}$$

Where z and $\dot{z}$ are the relative displacement and the velocity respectively, and the α parameter defined by the u control voltage. In the model of the MR damper, K_0 is the stiffness of the MR damper spring component and the $C_0(u)$ and $\alpha(u)$ coefficient values have a linear relationship with the u control voltage and evaluate the model effect on F . The force f_0 needs in order to determine the damper pre-yield stress. The following expressions have been used to assess $C_0(u)$ and $\alpha(u)$ [16]:

$$C_0(u) = C_{0a} + C_{0b}u \quad \alpha_0(u) = \alpha_{0a} + \alpha_{0b}u \qquad \textbf{(11)}$$

SIMULATION RESULTS

The application of PC techniques to the half-car suspension is addressed in this chapter, which are already demonstrated in Fig. 1 and Fig. 2.

The passive suspension system has no external force, and the damping coefficient is not variable. Energy is dissipated while using damper and spring in that system. MATLAB/SIMULINK was used to do all the simulations. The simulink model is a bunch of different simulink blocks which are connected by arrows according to mathematical relations [11]. Passive suspension system of half car model

is used to illustrate our linear case. Values of masses, damping coefficients and spring constants of sprung and unsprung masses on both front and rear wheels are taken into consideration for simulation.

TABLE I PARAMETERS OF CAR

Parameters	Values
sprung mass (m_s)	800 kg
body mass moment of inertia (I_0)	3000 $m \times kg^2$
front and rear unsprung mass (m_{u1} and m_{u2})	35 kg
front and rear suspension stiffness (k_{s1} and k_{s2})	24000 N/m
front and rear suspension damping c_{s1} and c_{s2})	2500 Ns/m
front and rear tyre stiffness (k_{t1} and k_{t2})	180000 N/m
Location of center of gravity from front axle (l_f)	1.5 m
Location of center of gravity from rear axle (l_r)	1 m

The road input is the random road profile of ISO 8608. In accordance with ISO 8608 standard, it is possible to simulate external excitation of the suspension system in the form of a random road profile. z_r excitation from the road surface of the half car system is expressed by means of $z_r = 2\pi\sqrt{G_0 V \omega}$, and where G_0 is the road roughness coefficient; V is the speed of the car; $\omega = [\omega_1, \omega_2]^T$ is Zero mean Gaussian white noise [17].

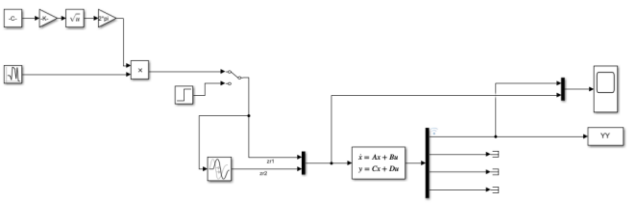

FIGURE 4
SIMULINK MODEL FOR PASSIVE SUSPENSION SYSTEM OF HALF CAR

We refer to the equivalent stiffness of the tire k_{t1}, k_{t2} as uncertainty parameters in order to provide a clear analysis of the problem.

$$k_{t1} = \bar{k}_{t1} + k'_{t1}\zeta_1 \quad \textbf{(12)}$$

$$k_{t2} = \bar{k}_{t2} + k'_{t2}\zeta_2 \quad \textbf{(13)}$$

The $x = [\zeta_1, \zeta_2]$ parameters are uniformly distributed in $X = [-1,1]^2$. A uniform variation of ±50% around its central value is shown each parameter. The model of the system is the following

$$y(f;x) = M(f;x) \quad \textbf{(14)}$$

Figure 5 indicates the significant function distribution (14) resulting from the large uncertainty of the two stochastic parameters and obtained by MC simulation in the $f \in [0-10]Hz$ frequency range [9].

In (14) for the prediction of the probability density function of y, the surrogate model is used. The MC realizations were used in order to train the PC surrogate model, denoted as $\tilde{M}_{PC}(f;x)$ and that model is built by using Legendre polynomials due to the uniform distribution of the parameters.

The maximum total degree is p = 6. The model is estimated from a randomly selected range of $\{(x_i, y_i)\}_{i=1}^{L}$ samples. For a given size L, ten major realizations of the training set are considered.

Figure 6 contrasts the y PDF collected from 10000 MC (14) (solid black line) assessments with those calculated from the surrogate model trained with L = 20, 30 and 40 samples (dashed lines). As a consequence of the ten different realizations of the training set, the two lines represent the minimum and maximum value of the PDF. The model parameters are computed in the case of PC expansion by the solution of a least-square problem, which provides better results for overdetermined systems, as discussed in chapter 2.

According to chapter 2, for a PC expansion of order p = 6 and n = 2 random parameters, the number of PC expansion coefficients is 28. Next, the surrogate models are used to measure the PDF of the transfer function (14) at $f_0 = 0.3Hz$ frequencies, shown by the dashed vertical lines in Fig. 5. [8].

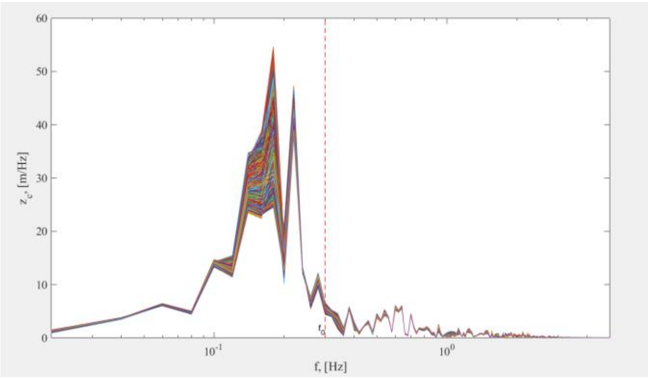

FIGURE 5
CAR BODY DISPLACEMENT AS LINEAR CASE

We can see that linear dynamic systems not to take too much time for simulation. It turns out that the suggested approach is more effective than traditional methods like MC.

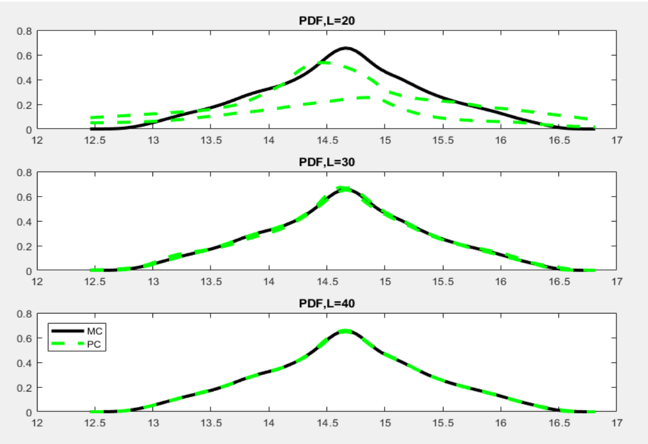

FIGURE 6

PDF OF THE FUNCTION FIG. 5. THE RESULT COMPUTED FROM 10000 MC SAMPLES (SOLID BLACK CURVES) IS COMPARED WITH THE PREDICTIONS ACHIEVED VIA THE PC EXPANSION (DASHED GREEN CURVES). THE THREE PANELS CORRESPOND TO DIFFERENT TRAINING SET SIZES

For the nonlinear case, semiactive suspension system is used to show our PC method application. Then, I attempted to simulate semiactive suspension with the MR damper under random excitation. Before simulate our model that shown Fig. 2, we must consider some parameters stayed as the same passive suspension system. The simulation MR damper model of the system from Bouc-Wen model shown in Figure 7 is built in Simulink by using the equations expressed in (9) and (10). All numerical values for MR damper mode parameters are given in Table 3.

TABLE II
PARAMETERS OF MR DAMPER

Parameter nme	Paramter notation	Values
Parameters of the Hysteresis shape	α, β, A, n	120, 100, 2.7, 2
Stiffness of the spring element	K_0	300 N/m
Damping coefficient	C_0	2500 Ns/m
Other parameters	α	10000
Pre-yield stress	f_0	0 N

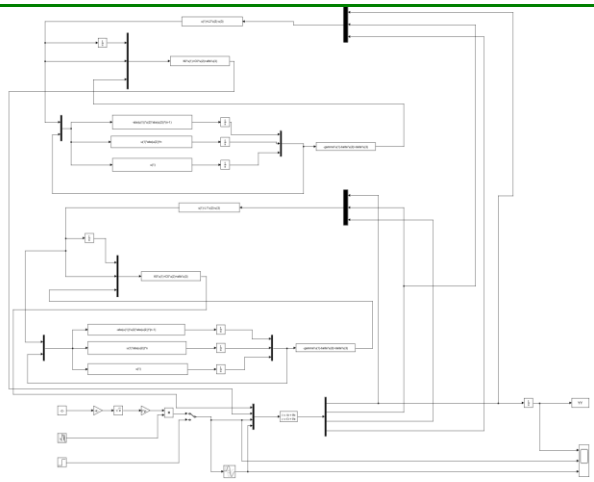

FIGURE 7
SIMULINK MODEL FOR SEMIACTIVE SUSPENSION SYSTEM OF HALF CAR

Like the linear simulation we presented above in Figure 8 the large distribution of the function (14) arising from the large variation of the two stochastic parameters is seen and Figure 9 compares the PDF of y obtained from 10000 MC (solid black line) evaluations of (14) with those calculated from the surrogate model trained with L = 20, 30 and 40 samples (dashed lines).

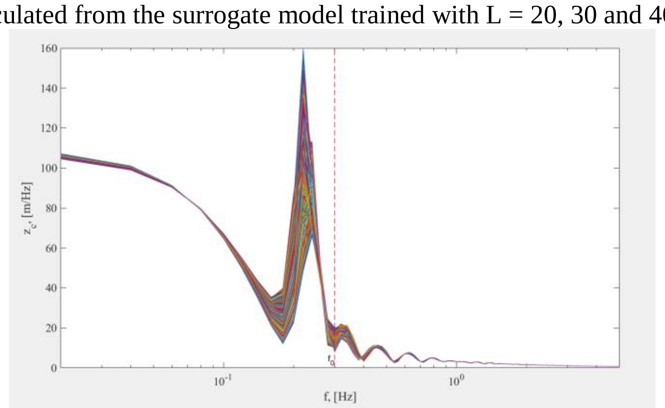

FIGURE 8
CAR BODY DISPLACEMENT AS NONLINEAR CASE

As such, only when the number of training samples is greater than the number of unknowns in the regression problem can the PC model begin to converge to the actual PDF, which is implemented in [7,15].
The model converges on the actual PDF with a significant number of training samples. Computational performance depends only on the number of implementation variables in comparison to the number of training samples [8].
We have seen that nonlinear dynamic systems are having been used large time for simulation. For getting accurate

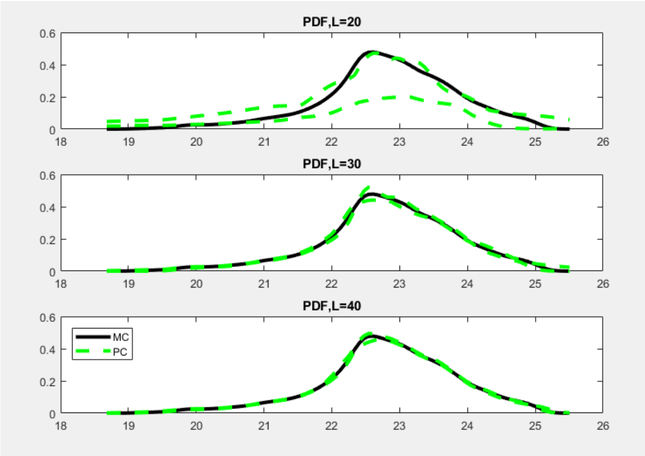

FIGURE 9

PDF OF THE FUNCTION FIG. 8. THE RESULT COMPUTED FROM 10000 MC SAMPLES (SOLID BLACK CURVES) IS COMPARED WITH THE PREDICTIONS ACHIEVED VIA THE PC EXPANSION (DASHED GREEN CURVES). THE THREE PANELS CORRESPOND TO DIFFERENT TRAINING SET SIZES

CONCLUSION

This paper explores an effective technique for generating a system surrogate model that focuses on highly variable parameters, beginning with a limited number of samples of response. We also demonstrated a detailed overview of polynomial chaos methodology for solving car suspension systems of uncertain parameters. Then, compared to the Monte Carlo simulation method and the PC method, the researchers concluded that the PC method is more effective than the Monte Carlo method and is a reasonably powerful approach with uncertainty for the parametric modelling dynamic system simulation. Owing to the high CPU time, Monte-Carlo strategies are often impractical in the industrial context. So, the PC dramatically reduces the cost of computing, and it is fast. The linear passive suspension system of the half-car model takes less than half an hour for simulation, the nonlinear semiactive suspension system of the half-car model takes more than 5 hours for simulation. A small quantity of training set is enough to get the result of MC evaluation. It means that the PC method beneficial nonlinear situation. The important thing is that to model non-linearity, and model differentiability are robust in simulation. The suggested approach can be used in such systems to produce accurate information.

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