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Filtering of digital images by the convolution method

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In the world, scientific research is being conducted to improve the quality level of digital television images, methods for modeling filtration processes and highly efficient control systems in a number of priority areas, including: on the formation of mathematical models of filtration processes, improving the methods of wavelet, Fourier, Haar, Walsh-Hadamard, Karhunen-Loev in increasing the clarity and brightness of images based on linear and nonlinear differential equations; creation of methods for eliminating additive, pulsed and adaptive-Gaussian types of noise in images using additive and adaptive filtering; methods of algorithms and software for introducing intra-frame and inter-frame image transformations; methods of adaptive brightness system control using the Chebyshev matrix series; methods of gradient, static and Laplace methods for image segmentation and dividing it into contours; formation of criteria and conditions for evaluating image quality.

Conducting scientific research in the above research areas confirms the relevance of the topic of this article.

Introduction

Today, in the world in the field of information and communication technologies, close attention is paid to the control system for processing digital television images in video information systems. In the conditions of intensive improvement of modern information and communication systems to increase the volume and information flow, one of the urgent problems is to improve the quality of television images and control the filtration processes from excess information. In this direction, in the field of information and communication technologies in the leading countries of the world, the demand and need for improving filtering methods and increasing the brightness of digital television images are increasing.

Currently, one of the most important issues is the formation of digital television images, based on them, the improvement of the image processing control system, methods of numerical models and algorithms for solving problems of filtering various digital television images using Fourier and wavelet methods. Purposeful scientific research is carried out in this area, including close attention is paid in the following areas: improved method of classification and selection of criteria for observing and evaluating image quality, methods for controlling image clarity at given values of medium-intensity pixels, creating algorithms for modeling the image processing process, methods for controlling the processes of ensuring the level of clarity of a digital image.

Methodology

Image filtering $L_c(x,y)$ by the convolution method with an impulse response $h(x,y)$ the case of a continuous image is mathematically described as follows [10; 7-8-p.]:

$$L_{c\Omega}(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}L_c(\xi,\eta)h(x-\xi,y-\eta,)d\xi d\eta,$$

where $L_{c\Omega}(x,y)-$brightness distribution in the image after filtering, $\xi,\eta-$integration variables. When implementing this method of filtering digitally, the original image, the image after filtering, as well as the impulse response are represented as arrays of numbers, the elements of which are denoted respectively by $L_c(k,n), L_{c\Omega}(k,n)$ и $h(k,n)$, and the numbers of rows and columns-through k and n . In this case, the brightness of the pixels of the filtered image is calculated as follows:

$$L_{c\Omega}(k,n)=\sum_{k'=-\frac{K-1}{2}}^{\frac{K-1}{2}}\sum_{n'=-\frac{N-1}{2}}^{\frac{N-1}{2}}L_c(k+k',n+n')h(k',n'), \tag{2.1}$$

where K and $N-$the length of the two-dimensional impulse response in both directions. Values K and N they are selected odd in order to avoid shifting the filtered image relative to the original one.

When filtering, the image is scanned by a window (pulse response), the dimensions of which are $K \times N$ pixels.Each window sample represents a weighting factor (the value of the impulse response), by which the image pixel covered by this window sample is multiplied. In this case, the intensity of the pixel of the filtered image, whose coordinates coincide with the coordinates of the center of the window, is found by summing all the products.

Impulse response $h(k,n)$ when developing a digital filter, it is found as follows. First, the frequency transfer function of the analog filter is found $K(\omega_x, \omega_y)$. Then, by applying a two-dimensional integral Fourier transform to it, the corresponding impulse response is found $h(x,y)$:

$$h(x,y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(\omega_x, \omega_y) \exp\left[i(\omega_x x + \omega_y y)\right] d\omega_x d\omega_y. \qquad (2.2)$$

The impulse response found in this way must be converted into a discrete form by means of its spatial sampling, while the step of spatial sampling must be the same as the step of spatial sampling of the filtered image. The next operation to be performed on the sampled impulse response is its truncation, i.e., limiting its size by rows and columns to reasonable limits. The fact is that frequency transfer functions bounded in the frequency space by boundary frequencies $\omega_{x гр}, \omega_{y гр}$, correspond to the impulse characteristics that are unlimited in the coordinate space (x, y). The last and final operation is the normalization of the truncated impulse response, as a result of which the sum of its samples should become equal to one, i.e.

$$\sum_{k=-\frac{K-1}{2}}^{\frac{K-1}{2}} \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} h(k,n) = 1.$$

Due to the normalization of the impulse response after its truncation, the correct reproduction of the average brightness in the filtered image is ensured, which would otherwise be disturbed due to the truncation operation. Turning to the problem of truncation of the impulse response, we note that the greater its length, the greater the amount of calculations must be performed when implementing digital filtering by the method under consideration. In addition, the edge effect will appear on most of the image. Simple truncation of the impulse response by multiplying it by the window function $W(k,n)$, удовлетворяющуюусловию[6; 2-3-p.]

$$W(k,n) = \begin{cases} 1 \; when \; |k|, |n| \le \dfrac{N-1}{2} \\ 0 \; when \; the \; condition \; is \; not \; met \end{cases}$$

it leads to the appearance of undesirable "undulation" of the frequency transfer function, as well as to its expansion in the frequency domain. To achieve a compromise between the length of the impulse response in the image space and the frequency transfer function in the frequency space, a number of windows of a special shape were developed, among which the most famous are: the triangular Bartlett window, the Blackman window, the Hahn window, the Kaiser window, and the Hamming window satisfying the condition

$$W(k,n) = \begin{cases} 0.54 + 0.46 cos \dfrac{2\pi n}{N-1} \; when \; |n| \le \dfrac{N-1}{2} \\ 0 \; when \; the \; condition \; is \; not \; met \end{cases}$$

An important feature of these windows is that when approaching the truncation boundary, the value of $W(k,n)$ it gradually decreases, due to which the effects of "undulation" and the expansion of the frequency transfer function are weakened. After finding the impulse response $h(k,n)$ it is necessary to investigate it for separability with respect to variables k and n. If it turns out that it is separable, i.e. if $h(k,n) = h(k)h(n)$,

where $h(k), h(n) -$ one-dimensional impulse characteristics, then the expression (2.1) it should be converted to the form

$$L_{c\Omega}(k,n) = \sum_{k'=-\frac{K-1}{2}}^{\frac{K-1}{2}} h(k') \sum_{n'=-\frac{N-1}{2}}^{\frac{N-1}{2}} L_c(k+k', n+n') h(n'). \qquad (2.3)$$

Calculating values $L_{c\Omega}(k,n)$ according to the formula (2.3) allows you to significantly reduce the number of necessary mathematical operations compared to the number of mathematical operations when using the formula (2.1). So, for example, if when calculating $L_{c\Omega}(k,n)$ according to the formula (2.2to determine the value of one sample of the filtered image, you need to perform $(K-1)N$ multiplication operations and $(K-1)(N-1)$ addition operations, then in the case of calculation $L_{c\Omega}(k,n)$ according to the formula (2.3) the number of necessary multiplication operations is reduced to $K+N,$ and the number of addition operations is reduced to $K+N-2.$ If you accept $K=7, N=7,$ what is quite a bit for typical filtering problems, even in this case, the gain in the amount of necessary computational costs provided by using the separability property of the impulse response will be 3.5 times for multiplication operations and 3 times for addition operations. In fact, the gains when using the separability property of the impulse response are significantly greater. It should be noted that a number of impulse characteristics, which often have to deal with in practice, are separable. These include: the impulse response described by the Gaussian law, the impulse response having a constant value inside a rectangular window, and some others. Next, you should pay attention to two more significant circumstances that are important to keep in mind when developing a filter. First, it is necessary to set limiters for the brightness value of the filtered image before its presentation with an eight-digit code, preventing it from going beyond the accepted dynamic range. The appearance of such brightness values is possible if there are outliers on the transition characteristic of the filter, due, for example, to a sharp decline in the frequency transfer function. In this case, the absence of limiters will lead to an overflow of the discharge grid, which will lead to the appearance of black dots and spots on the light areas of the filtered image, and white dots and spots, respectively, on the dark areas. The use of limiters of the dynamic range of the signal from the white side and from the black side allows you to avoid these artifacts, although it introduces the so-called restriction noise into the filtered image.

Secondly, the restriction of the discharge grid leads to the appearance of peculiar distortions — noise. The fact is that the convolution filtering algorithm includes a multiplication operation, as a result of which the number of code bits that represent the samples of the filtered image is equal to the sum of the number of code bits used to represent the original image and the impulse response. Since both the original image and the impulse response are usually represented by eight-bit numbers, the result of filtering is an image that requires a sixteen-bit code to accurately represent the intensities. In order to switch to the previous type of recording of the filtered image, it is necessary to bring it into an eight-bit representation, i.e. to round off, which causes rounding noise.

The essence of the convolution method with a pulse characteristic in the spectral region is that at the beginning there is an array of samples that represents the original image $L_c(k,n),$ in this case, the achromatic image is transformed according to some basis into an array of spectral coefficients $M_c(u,v),$ and then each of the spectral coefficients is scalar multiplied by the corresponding sample of the discrete frequency transfer function $K(u,v):$

$$M_{c\Omega}(u,v)=M_c(u,v)K(u,v),$$

where $M_{c\Omega}(u,v)-$ spectral coefficients of the filtered image, $u,v-$ indexes that determine the position of the spectral coefficients, as well as the samples of the frequency transfer function in the column and row of the corresponding arrays. At the final stage of this filtering method, the found array of spectral coefficients is converted into an array of samples of the filtered image $L_{c\Omega}(k,n).$

The described method is completely analogous to filtering an analog image $L_c(x,y)$ in the frequency (spectral) domain. It should be noted that in this case, for the original image, its spectrum is found by a two-dimensional integral Fourier transform $M_c(\omega_x,\omega_y),$ which is then multiplied by the frequency transfer function $K(\omega_x,\omega_y),$ and the spectrum of the filtered image obtained in this way $M_{c\Omega}(\omega_x,\omega_y)$ by the inverse two-dimensional integral transformation the Fourier transform is converted to a filtered image $L_{c\Omega}(x,y).$ This filtering method can be implemented in an optical way.

Result

When filtering an image in the spectral region, you should pay special attention to the definition of the discrete frequency transfer function of the filter $K(u,v).$ Usually, the frequency transfer function of a digital filter is determined based on the previously found frequency transfer function of an analog filter $K\left(\omega_x,\omega_y\right)$. However, you should immediately warn against the temptation to get a frequency transfer function $K\left(u,v\right)$ by simply sampling the function $K\left(\omega_x,\omega_y\right)$. The fact is that the view $K\left(u,v\right)$, like the view $M_c\left(u,v\right)$, it is determined by the basis that is used when calculating the spectral coefficients of a digital image. Figure 1 shows discrete amplitude spectra $\left|M_c\left(u,v\right)\right|$ the same image, but obtained by converting it according to two different bases.

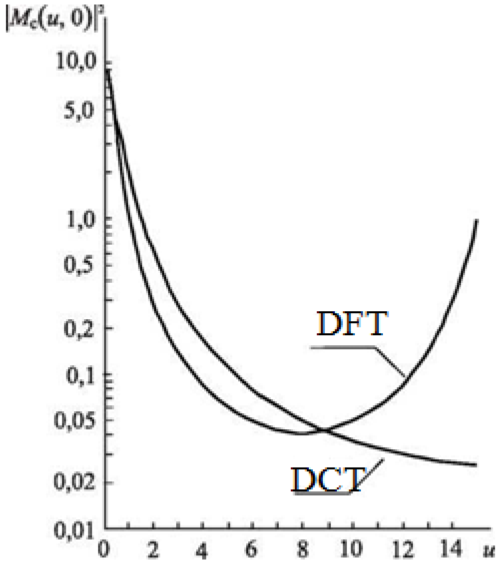

Fig. 1. Dependence of the amplitude of the spectral coefficients on the u index for the DCT and DFT

In one case, when determining the spectral coefficients $\left|M_c\left(u,v\right)\right|$ The discrete cosine transform (DCT) was used, and the discrete Fourier transform (DFT) was used in the other. It can be seen from the figure that for two different bases, the discrete spectra differ greatly from each other.Therefore, the definition $K\left(u,v\right)$ it should be carried out as follows. First, by means of a two-dimensional integral Fourier transform based on a known analog frequency transfer function $K\left(\omega_x,\omega_y\right)$ according to (2.2) the corresponding impulse response is found $h\left(x,y\right)$ which is subjected to spatial sampling at the same interval as the filtered image. After that, the discrete impulse response is normalized so that the sum of all its samples is equal to one. And only then is the discrete frequency transfer function calculated $K\left(u,v\right)$ by applying to $h\left(k,n\right)$ a two-dimensional orthogonal transformation using the same basic functions as when calculating spectral coefficients.

Filtering images distorted by Gaussian noise

A widespread type of interference is random additive noise, statistically independent of the video signal. The additive noise model is used when the signal at the output of the imaging system or at some intermediate stage of the transformation can be considered as the sum of a useful signal and some random signal (noise). The additive noise model well describes the effect of film grain, fluctuation noise in radio engineering systems, quantization noise in analog-to-digital converters, etc.[10; 23-24-p.].

In practice, additive noise is considered as a stationary random field and is characterized by a variance and a correlation function. Additive noise can be uncorrelated or weakly correlated.

The noise sources can be different:

1. Imperfect equipment for capturing images — a video camera, scanner, etc.;

2. Poor shooting conditions — for example, strong noises that occur during night photo/video shooting;

3. Interferenceduringtransmissionviaanalogchannels — interferencefromsourcesofelectromagneticfields, intrinsicnoiseofactivecomponents (amplifiers) ofthetransmissionline.

Let $x(m,n)$ - samples (elements of brightness functions) of the original (ideal) image, and $y(m,n)$ - samples (elements of brightness functions) of the original (ideal) image, and

$$y(m,n)=x(m,n)+v(m,n), \qquad (2.4)$$

where $v(m,n)$ - samples of a noise random field with a zero mean and a correlation function

$$B_{\nu}(k,l) = E\{\nu(m,n)\nu(m+k,n+l)\} = D_{\nu}\sigma(k,l),$$

where $E\{\cdot\}$ - the mathematical expectation operator; D_{ν} - noise dispersion;

$\sigma(k,l)$ - two-dimensional delta pulse.

The intensity of additive noise is characterized by a signal-to-noise ratio

$$d^2 = \frac{D_x}{D_{\nu}},$$

where D_x - the dispersion of the signal (image).

If the effect of noise does not affect the entire length of the image field, but only at randomly located points where the values of the brightness function are replaced by random values, then the noise is called pulsed [6; 2-3-p.]. In the image, such interference looks like isolated contrast points.

The most common methods of noise removal:

1. smoothing filters;
2. Wiener filters;
3. median filters;
4. rankingfilters.

Suppose we assume that the distorted points are evenly distributed throughout the image field, and the brightness of the distorted points has a uniform distribution in a certain range. Pulse noise is characteristic for image transmission systems over radio channels using nonlinear modulation methods, as well as for digital image transmission and storage systems. In particular, pulse

noise is inherent in devices for entering images from a television camera.

The effect of pulse noise can be described by the following mathematical model[4; 7-8-p.]:

$$\square(\square,\square) = \begin{cases} \zeta(\square,\square)\square\square\square h\,\square\square\square\square\square\square\square\square\square\square\square\;\square \\ x(\square,\square)\square\square\square h\,\square\square\square\square\square\square\square\square\square\square\square(1-\square), \end{cases}$$

where $\xi(m,n)$ - noise field samples that are statistically independent of each other and evenly distributed in the range $[\xi_{\min}, \xi_{\max}]$.

Thus, the pulse noise is characterized by the probability ppoint distortion $(0 \le p \le 1)$ and a range of values $[\xi_{\min}, \xi_{\max}]$.

The next noise model is multiplicative. The noise model is used when the samples of the observed image are obtained by multiplying the samples of the original image by a random signal. This model well describes the diffusivity noise in coherent optical and holographic imaging systems. Since the logarithm of the product is equal to the sum of the logarithms of the multipliers, the logarithmic element-by-element transformation of the noisy image leads the multiplicative noise model to an additive model (2.4).

A linear model of image observation under interference conditions is a model that takes into account dynamic spatial distortions along with additive noise. If such distortions can be described by a spatially homogeneous (shift-invariant) linear system with an impulse response $h(k,l)$, then the observation model takes the form:

$$y(m,n) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} h(k,l)x(m-k,n-l) + \nu(m,n). \qquad (2.5)$$

The model (2.5) describes distortions caused by the motion of the registration system relative to the object, atmospheric turbulence, aberrations of the optical system, inaccuracy of focusing, etc.

Let's consider the scheme of distortion and filtering (restoration, restoration) of images, presented in fig. 2.

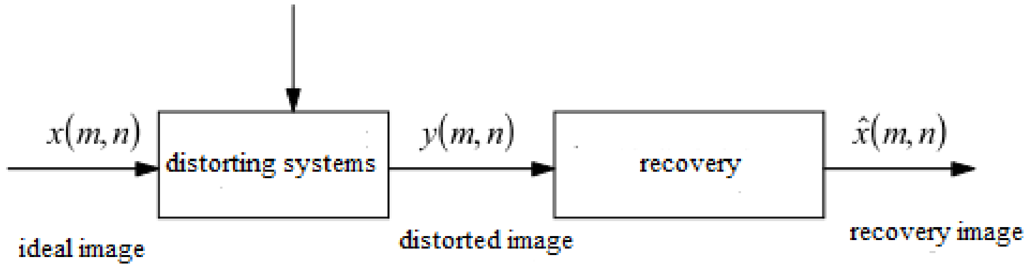

Fig.2. Distortion and filtering scheme

The purpose of restoring a distorted image $y(m,n)$ is getting from it with the help of some image processing $\hat{x}(m,n)$, which is close to the perfect image $x(m,n)$ according to the specified criterion. The resulting image as a result of processing $\hat{x}(m,n)$ we will call it the evaluation of the original (ideal) image $x(m,n)$. Let's determine the estimation error at each point of the image

$$\varepsilon(m,n) = \hat{x}(m,n) - x(m,n), m = \overline{0, M-1}, n = \overline{0, N-1}, \qquad (2.6)$$

and also the average quadratic error (COEX) through its square, i.e. the variance of the error:

$$\bar{\varepsilon}^2 = \frac{1}{MN}\sum\sum\left[\hat{x}(m,n) - x(m,n)\right]^2. \qquad (2.7)$$

Discussion

Minimum square criterion COEX) $\left(\bar{\varepsilon}^2 \to \min\right)$ It is the most universal and common criterion for the quality of recovery when designing image filtering algorithms due to its mathematical simplicity. However, this criterion has the disadvantage that it does not always agree with the subjective (psychovisual) quality criterion, based mainly on the accuracy of the transmission of contours.

This criterion is constructive and allows us to theoretically calculate optimal (giving minima of the square of the COEX system) filtering algorithms for the considered observation models. However, optimal algorithms are very difficult to calculate and implement. For example, an optimal Wiener filter requires performing a two-dimensional discrete Fourier transform over a field of size, which leads to significant machine time costs. In an interactive automated image processing system, preference is given to the so-called quasi-optimal algorithms, which give a minimum of the square of the COEX system in a certain class of algorithms with a given structure and differ slightly from the optimal ones according to this criterion. Let's consider the simplest linear filtering algorithms based on local processing of a noisy image by a small-sized" window".

A linear smoothing filter with a finite impulse response can be described by the two-dimensional convolution equation

$$\hat{x}(m,n) = \sum_{(k,l)\in W}\sum a(k,l)\, y(m-k, n-l) = a(k,l) ** y(m,n). \qquad (2.8)$$

where $a(k,l)$ - coefficients of the linear filter mask (pulse characteristic of the restoring LPP system); W - the "windows" area.

Coefficients $a(k,l)$ you can choose the optimal way from the condition of minimizing the variance of the filtering error, which is the mathematical expectation of the average quadratic error:

$$\bar{\varepsilon}^2 = E\left\{\left[\hat{x}(m,n) - x(m,n)\right]^2\right\} \to \min. \qquad (2.9)$$

Suppose that the ideal and noisy images are stationary two-dimensional random sequences with zero mathematical expectations

$$B_x(k,l) = E\left[x(m,n)\, x(m+k, n+l)\right],$$

and known correlation functions $B_y(k,l) = E\left[y(m,n)\, y(m+k, n+l)\right],$ (2.10)

$$B_{xy}(k,l) = E\left[x(m,n)\, y(m+k, n+l)\right],$$

It follows from (2.8) and (2.9) that

$$\varepsilon^2 = \sum_{(k,l)\in W}\sum \sum_{(p,q)\in W}\sum a(k,l)a(p,q)B_y(k-p,l-q)-$$
$$-2\sum_{(k,l)\in W}\sum a(k,l)B_{xy}(-k,-l)+D_x. \qquad (2.11)$$

Equating the partial derivatives to zero $\frac{\partial\varepsilon^2}{\partial a(k,l)}$. we find the optimal values of the coefficients $a(k,l)$, minimizing the quadratic form (2.11). At the same time, relatively unknown coefficients $a(k,l)$:we obtain a system of linear equations

$$\sum_{(k,l)\in W}\sum a(k,l)B_y(m-k,n-l)=B_{xy}(-m,-n),(m,n)\in W. \qquad (2.12)$$

In this case, the optimal linear mask is made up of the coefficients $a_*(k,l)$,satisfying the given system and delivering a minimum (2.15),

$$A_* = \{a_*(k,l)\},(k,l)\in W.$$

The minimum variance of the filtering error corresponding to the optimal mask is equal to

$$\varepsilon_*^2 = D_x - \sum_{(k,l)\in W}\sum a_*(k,l)B_{xy}(-k,-l). \qquad (2.13)$$

For the special case corresponding to the additive white noise model (2.6), (2.7), the system of equations (2.13) and the error variance (2.12) have the form:

$$\sum_{(k,l)\in W}\sum a(k,l)B_x(m-k,n-l)+a(m,n)D_v = B_x(m,n),(m,n)\in W; \qquad (2.14)$$

$$\varepsilon_*^2 = a_*(0,0)D_v.$$

Next, the filtration efficiency is measured by the noise reduction coefficient, which is equal to the ratio of the filtration error variance to the distortion variance: $K_c = \frac{\varepsilon^2}{\varepsilon_u^2} = \frac{E\left[\hat{x}(m,n)-x(m,n)\right]^2}{E\left[y(m,n)-x(m,n)\right]^2}$.

It can be shown that in the case of additive white noise

$$K_c = a_*(0,0).$$

From the system (2.14), we will find the optimal mask of size 3x3 $A_* = \{a_*(k,l)\}_{k,l=-1}^{1}$ for filtering additive white noise with an isotropic exponential correlation function of an ideal image:

$$B_x(k,l) = D_x\rho^{\sqrt{k^2+l^2}}.$$

With the correlation coefficient of neighboring samples $\rho = 0{,}95$ and the "signal/noise" relationship $d^2 = 10$ and $d^2 = 1$ we get, respectively

$$A_* = \begin{bmatrix} 0.060 & 0.100 & 0.060 \\ 0.100 & 0.360 & 0.100 \\ 0.060 & 0.100 & 0.060 \end{bmatrix}, A_* = \begin{bmatrix} 0.093 & 0.101 & 0.093 \\ 0.101 & 0.137 & 0.101 \\ 0.093 & 0.101 & 0.093 \end{bmatrix}. \qquad (2.22)$$

Noise reduction coefficients K_c^* (maximum permissible) in these cases are equal to 0.360 (для $d^2 = 10$) and 0,137 (для $d^2 = 1$).

In practice, the measurement or theoretical calculation of the correlation function (2.10), as well as the solution of the system (2.12) is not always possible. Therefore, a different approach is often used, based on the construction of a so-called smoothing filter. Since noise (both additive and multiplicative) is usually spatially uncorrelated (white), its spectrum contains higher spatial frequencies than the spectrum of an ideal image. This confirms that simple low-frequency filtering can serve as an effective means of noise suppression. In principle, any FIR filter with non-negative coefficients has smoothing properties. Focusing on the samples of optimal masks (2.15), we can offer the following smoothing masks:

$$A_1 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, A_2 = \frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}, A_3 = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}. \qquad (2.16)$$

The coefficients of the masks (2.16) are normalized $\left(\sum_{(k,l)\in W} \sum a(k,l) = 1 \right)$ so that the interference suppression procedure does not cause the average brightness of the processed image to shift relative to the original one. This property is only approximately possessed by optimal masks (2.22), since the optimal FIR filter is designed for zero average brightness values of ideal and noisy images. As a rule, the absence of pre-centering when processing images with linear masks is equivalent to artificially overestimating the actual signal-to-noise ratio and ultimately leads to an increase in the filtration COEX. If the ratio of the square of the average value to the image variance is large, then the loss of optimality can be significant.

Masks (2.16) differ in the degree of noise smoothing (at the mask A_1 itmax, on A_3 - min). The selection of the mask coefficients should be made experimentally. With an increase in the degree of noise smoothing, the high-frequency component of the useful image is also suppressed, which causes the disappearance of small details and smearing of contours. For example, with a signal-to-noise ratio $d^2 = 10$ using a smoothing mask instead of the optimal mask (2.15) A_1 (2.16) leads to $K_c = 1.28,$ and the masks A_3 - к $K_c = 0.380,$ and with a signal-to-noise ratio $d^2 = 1$ the use of these smoothing masks gives, respectively, the following noise reduction coefficients: $K_c = 0.147$ and $K_c = 0.164$. Thus, a simple smoothing filter can even increase the filtering COE if the mask coefficients are selected incorrectly.

If the required degree of smoothing using a 3x3 mask is not achieved, then large smoothing masks (5x5, 7x7,...) should be used. Another method is to iteratively apply a 3x3 smoothing mask:

$$\hat{x}(m,n) = \underbrace{q(k,l) ** q(k,l) ** \ldots ** q(k,l)}_{R} ** y(m,n),$$

where R - the number of passes.

Note that with an unlimited increase in the number of passes, the image with a constant brightness value:

$$\hat{x}(m,n) \to const = E\left[(m,n)\right].$$

Therefore, for a specific observation model, there is an optimal number of passes that can be determined experimentally.

The simplest method for attenuating additive Gaussian white noise in images is to filter it using a low-frequency linear filter with a rectangular frequency transfer function. Since the spectral intensity of the image at high spatial frequencies decreases in proportion to the square of the spatial frequencies, and the spectral intensity of white noise remains constant, as a result of such filtering, the noise energy is weakened to a greater extent than the image energy, but, unfortunately, the sharpness of the boundaries on the corrected image decreases.

When using the Wiener filter, somewhat better results can be obtained in which the frequency transfer function of which in the case of noise has the form:

$$K_{кор}\left(\omega_x, \omega_y\right) = \frac{S_c\left(\omega_x, \omega_y\right)}{S_c\left(\omega_x, \omega_y\right) + S_ш},$$

where $S_c\left(\omega_x, \omega_y\right) -$ spectral intensity of the corrected image, $S_ш -$ the spectral intensity of the noise, which does not depend on the spatial frequencies.

Another approach to restoring images distorted by noise is to use adaptive filtering. The adaptive filter in question is also a low-frequency filter, and therefore a filter that smooths not only noise, but also brightness and color boundaries in the image. When performing this type of filtering, the neighborhood of each pixel is first analyzed, for which a brightness estimate is made. This neighborhood is a pixel-sized window. As a result of this analysis, the average brightness value is found $\overline{L}_c$ inside the window

$$\overline{L}_c = \frac{1}{KN}\sum_{-K/2}^{K/2}\sum_{-N/2}^{N/2} L_c(k,n),$$

and also the average square of the deviation from the average brightness value $\sigma_{\sum}$ in this window

$$\sigma_{\Sigma}^2 = \frac{1}{KN}\sum_{-K/2}^{K/2}\sum_{-N/2}^{N/2}\left[L_c(k,n) - \overline{L}_c\right]^2,$$

after that, the evaluation itself is performed (filtering). It should be noted that $\sigma_{\sum}$ includes as a component due to noise $\sigma_{ш}$, so is the component caused by a change in brightness in a noiseless image σ_L , in turn, due to its texture and contours. Since these components are not mutually correlated, then $\sigma_{\Sigma}^2 = \sigma_{ш}^2 + \sigma_L^2.$

Adaptive filtering is performed in such a way that in those places of the image for which the brightness variance is large $\sigma_{\Sigma}^2,$ the filter performs weak smoothing, since a significant proportion of the brightness deviation from the average value in these places of the image is due to the presence of light boundaries or texture that must be preserved. In the same places of the image for which it is small, the filter performs smoothing to a greater extent, since the deviation of brightness from the average in these places of the image is due to noise that needs to be attenuated. In this case, the brightness values of the pixels in the filtered image are determined by the formula

$$\hat{L}_c(k,n) = \overline{L}_c + \frac{\sigma_{\Sigma}^2 - \sigma_{ш}^2}{\sigma_{\Sigma}^2}\left[L_c(k,n) - \overline{L}_c\right].$$

It follows that in the absence of noise, when $\sigma_{ш}^2 = 0,$ the filtering result is the same as the original brightness value of the filtered image. When the noise variance is large, the multiplier before the square brackets in the formula becomes very small and the pixel brightness estimate becomes $\hat{L}_c(k,n)$ approaching the average brightness value $\overline{L}_c$ inside the window. As a result of this approach, as a rule, the filtering result is better than with non-adaptive filtering.

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