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Approximation of Expectation and Variance on $[a, b]$ Interval, with Probability Density Function in $L_p[a, b], 0 < p < 1$

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Abstract :

In this article we use Taylor's formula to approximate expression in terms of expectation and variance simultaneously with probability density function in $L_p, 0 < p < 1$.

1.1.Introduction

If **X** is a random variable, have probability density function

$f: [a, b] \to R$. We know that the expectation of the random variable **X** is

$$E(X) = \int_{\infty} tf(t)dt.$$

Therefore, the variance of the random vaiable X is

$$\sigma^2 = \int_a^b (t - E(X)f(t)dt)^2 = E(X^2) - (E(X))^2$$

In our article we use these inequalities to approximate (estimate) expectation and variance with measurable probability density functions, in the aid of Taylor's formula.

1.2. The Main Results

To prove our main theorem we need the following auxiliary Lemmas

Lemma 1.2.1:[2]

$$\int_a^b (b - t)\,(t - a)f(t)dt = |b - E(X)||E(X) - a| - \sigma^2(X), t \in [a, b]$$

Lemma 1.2.2.[1]

If $p < q$, then

$$\left(\sum_{i=1}^{\infty} |x_i|^q\right)^{\frac{1}{q}} \leq \left(\sum_{i=1}^{\infty} |x_i|^p\right)^{\frac{1}{p}}.$$

Now let us introduce our main Theorem.

Theorem 1.2.3

Let **X** be a random variable defined on $[\mathbf{a}, \mathbf{b}]$ with the probability density function $f: [a, b] \to R$ belongs to $L_p[a, b], p < 1$.Then we have,

$$|b - E(X)||E(X) - a| - \sigma^2(X) \leq c(b - a)^{2+p-\frac{1}{p}}\|f\|_p,$$

$0 < p < 1$,where c is a positive constant.

Proof

Since

$$\int_a^b (b-t)\,(t-a)f(t)dt \le \int_a^b (b-a)^2\,|f(t)|dt$$

$$= (b-a)^2 \int_a^b |f(t)|dt, \qquad (1)$$

Now

Let $t_1 < t_2 < t_3 < \cdots < t_n$ be a partition for $[a,b], a = t_\circ$, $\Delta = \frac{b-a}{n}$,

$$t_1 = a + \frac{b-a}{n}, t_2 = a + \frac{2(b-a)}{n}, \dots, t_n = a + \frac{i(b-a)}{n}.$$

This implies,

$$\int_a^b f(t)dt \cong \sum_{i=1}^{n} f(t_i)\frac{b-a}{n}, [54] \qquad (2)$$

If $p < 1$, by using Lemma 1.2.2 and (2), we get

$$\int_a^b f(t)dt \le \left(\int_a^b |f(t)|^{\frac{1}{p}}\right)^p \le c\left(\sum_{i=1}^{n} |f(t_i)|^{\frac{1}{p}}\frac{b-a}{n}\right)^p$$

By using Holder inequality when $q > 1, k > 1$ and $\frac{1}{q} + \frac{1}{k} = 1$, we get

$$\int_a^b f(t)dt \le c(b-a)^p\left(\left(\sum_{i=1}^{n} |f(t_i)|^{\frac{q}{p}}\right)^{\frac{p}{q}}\left(\sum_{i=1}^{n}\left|\frac{1}{n}\right|^k\right)^{\frac{p}{k}}\right)$$

$$\le c(b-a)^p\left(\left(\sum_{i=1}^{n} |f(t_i)|^{\frac{q}{p}}\right)^{\frac{p}{q}}\sum_{i=1}^{n}\frac{1}{n^k}\right), k > 1$$

$$\le c(b-a)^p\left(\left(\sum_{i=1}^{n} |f(t_i)|^{p}\right)^{\frac{1}{p}}\frac{1}{n^{k-1}}\right)$$

Assume $\frac{1}{p} = k - 1$, then

$$\int_a^b f(t)dt \le \frac{c(b-a)^p}{(b-a)^{\frac{1}{p}}}\left(\left(\sum_{i=1}^{n} |f(t_i)|^{p}\frac{(b-a)}{n}\right)^{\frac{1}{p}}\right)$$

$$\le \frac{c(b-a)^p}{(b-a)^{\frac{1}{p}}}\left(\left(\int_a^b |f(t_i)|^{p}\right)^{\frac{1}{p}}\right)$$

$$\le \frac{c(b-a)^p}{(b-a)^{\frac{1}{p}}}\|f\|_p\,. \qquad (3)$$

Thus

$$\int_a^b (b-t)\,(t-a)f(t)dt \le c(b-a)^{2+p-\frac{1}{p}}\ \|f\|_p$$

Then by repeating of using of (2), we get,

$$\int_a^b (b-t)\,(t-a)f(t)dt \le (b-a)^{2+p-\frac{1}{p}}\|f\|_{p.}$$

Then by using Lemma 1.2.1 , we get,

$$|b-E(X)||E(X)-a|-\sigma^2(X)\le (b-a)^{2+p-\frac{1}{p}}\ \|f\|_p,$$

$\mathbf{0<p<1}$ ■

Theorem 1.2.4

Let $\mathbf{X}$ be a random variable defined in $[\mathbf{a},\mathbf{b}]$ with the probability density function $\mathbf{f}:[\mathbf{a},\mathbf{b}]\to\mathbf{R}$ belongs to $\mathbf{L_p}[\mathbf{a},\mathbf{b}]$, $\mathbf{p<1}$.Then we have,

$$\left||\mathbf{b}-\mathbf{E}(\mathbf{X})||\mathbf{E}(\mathbf{X})-\mathbf{a}|-\boldsymbol{\sigma}^2(\mathbf{X})-\frac{(\mathbf{b}-\mathbf{a})^3}{\mathbf{6}}\right|\le\frac{(\mathbf{b}-\mathbf{a})}{\mathbf{2}}\left((\mathbf{a}+\mathbf{x})^2+(\mathbf{b}+\mathbf{x})^2\|\mathbf{f}(\mathbf{t})\|_{\mathbf{p}}\right)+\frac{(\mathbf{b}-\mathbf{a})^3}{\mathbf{6}}$$

Proof

Recall pre-Gruss inequality when $\mathbf{0<p<1}$

$$\left|\frac{1}{b-a}\int_a^b h(t)g(x,t)dt-\int_a^b h(t)\,dt\frac{1}{b-a}\int_a^b g(x,t)dt\right|$$

$$\le\frac{1}{2}\left\{(a+x)^2+(b+x)^2\|h(t)\|_p\right\}+\left|\int_a^b h(t)dt\frac{1}{b-a}\int_a^b g(x,t)dt\right|\ (4)$$

Put $h(t)=f(t), g(x,t)=(b-t)(t-a)$, in (4), we get.

$$\left|\frac{1}{b-a}\int_a^b f(t)(b-t)(t-a)dt-\int_a^b f(t)\,dt\frac{1}{b-a}\int_a^b (b-t)(t-a)dt\right|$$

$$\le\frac{1}{2}\left((a+x)^2+(b+x)^2\|f(t)\|_p\right)+\left|\int_a^b f(t)dt\frac{1}{b-a}\int_a^b (b-t)(t-a)dt\right|.\qquad (5)$$

Now let us compute $\int_a^b (b-t)\,(t-a)dt$

$$\int_a^b (b-t)\,(t-a)dt$$

$$=\int_a^b (bt-ab-t^2+at)dt=\frac{t^3}{3}-abt-\frac{t^3}{3}+\frac{at^2}{2}\bigg|_a^b$$

$$=\left(\frac{b^3}{2}-ab^2-\frac{b^3}{3}+\frac{ab^2}{2}\right)-\left(\frac{a^2b}{2}-ba^2-\frac{a^3}{3}+\frac{a^3}{2}\right)$$

$$=\left(\frac{3b^3-6ab^2-2b^3+3ab^2}{6}\right)-\left(\frac{3a^2b-6ba^2-2a^3+3a^3}{6}\right)$$

$$=\frac{(b^3-3ab^2)-(a^3-3ba^2)}{6}=\frac{b^3-3ab^2+3ba^2-a^3}{6}=\frac{(b-a)^3}{6}\qquad (6)$$

Then using (6) to complete our estimate in (5). Also we have $\int_a^b f(t)dt=1$, we obtain

$$\left|\frac{1}{b-a}\int_a^b f(t)(b-t)(t-a)dt-\frac{(b-a)^3}{6}\right|$$

$$\le\frac{1}{2}\left((a+x)^2+(b+x)^2\|f(t)\|_p\right)+\frac{(b-a)^2}{6}.$$

Then,

$$\left|\int_a^b f(t)(b-t)(t-a)dt-\frac{(b-a)^3}{6}\right|$$

$$\le\frac{(b-a)}{2}\left((a+x)^2+(b+x)^2\|f(t)\|_p\right)+\frac{(b-a)^3}{6},$$

Now by using Lemma 1.2.1 , we get,

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^3}{6}\right| \le$$

$$\frac{(b-a)}{2}\left((a+x)^2+(b+x)^2\|f(t)\|_p\right)+\frac{(b-a)^3}{6} \qquad \blacksquare$$

Corollary 1.2.4

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^3}{6}\right| \le (b+a)^3\left(1+\|f\|_p\right)$$

Proof:

Since,

By using Theorem 1.2.4, we get,

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^3}{6}\right| \le$$

$$\frac{(b-a)}{2}\left((a+x)^2+(b+x)^2\|f(t)\|_p\right)+\frac{(b-a)^3}{6} \le (b+a)^3\left(1+\|f\|_p\right) \qquad \blacksquare$$

Theorem 1.2.5

Let **X** be a random variable defined in $[\mathbf{a},\mathbf{b}]$ with the probability density function $f:[a,b]\to R$.If $f,\acute{f}\in L_p[a,b], p<1$,then we have,

$$|b-E(X)||E(X)-a|-\sigma^2(X)\le 2^{\frac{1}{p}+3}b^{\frac{4}{p}}\left\|\acute{f}\right\|_p, 0<p<1$$

Proof:

Recall the "Pre-Chebychev inequality"

$$\left|\frac{1}{b-a}\int_a^b f(x)g(x)dx-\frac{1}{b-a}\int_a^b f(x)\,dx\,\frac{1}{b-a}\int_a^b g(x)dx\right| \le$$

$$(b-a)\left\|\acute{f}\right\|_p\left|\frac{1}{(b-a)}\int_a^b |g(x)|^2dx-\left(\frac{1}{(b-a)}\int_a^b |g(x)|\,dx\right)^p\right|^{\frac{1}{p}} \qquad (7)$$

In (7) put $g(x)=(t-a)(b-t)$, we get

$$\left|\frac{1}{b-a}\int_a^b f(t)(t-a)(b-t)dt-\frac{1}{b-a}\int_a^b f(t)\,dt\,\frac{1}{b-a}\int_a^b (t-a)(b-t)dt\right|$$

$$\le (b-a)\left\|\acute{f}\right\|_p\left|\frac{1}{(b-a)}\int_a^b |(t-a)(b-t)|^2dt-\left(\frac{1}{(b-a)}\int_a^b |(t-a)(b-t)|\,dt\right)^p\right|^{\frac{1}{p}} \qquad (8)$$

Since

$$\int_a^b |(t-a)(b-t)|\,dt=\frac{(b-a)^3}{6}, \int_a^b f(t)dt=1.$$

And

$$\int_a^b |(t-a)(b-t)|^2dt=\int_a^b (t-a)^2(b-t)^2dt=\frac{(b-a)^5}{30}.$$

Then (8) become,

$$\left|\frac{1}{b-a}\int_a^b f(t)(t-a)(b-t)dt-\frac{(b-a)}{6}\right|\leq$$

$$(b-a)\,\|\acute{f}\|_p\left|\frac{1}{(b-a)}\int_a^b\frac{(b-a)^5}{30}dt-\left(\frac{(b-a)^2}{6}\right)^p\right|^{\frac{1}{p}}.$$

Then,

$$\left|\int_a^b f(t)(t-a)(b-t)dt-\frac{(b-a)^2}{6}\right|\leq$$

$$(b-a)^2\,\|\acute{f}\|_p\left|\left[\frac{(b-a)^4}{30}-\left(\frac{(b-a)^2}{6}\right)^p\right]\right|^{\frac{1}{p}}$$

By using Lemma 1.2.1, we get

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^2}{6}\right|\leq$$

$$(b-a)^2\,\|\acute{f}\|_p\left|\left[\frac{(b-a)^4}{30}-\left(\frac{(b-a)^2}{6}\right)^p\right]\right|^{\frac{1}{p}},$$

This implies,

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^2}{6}\right|\leq$$

$$(b-a)^2\,\|\acute{f}\|_p 2^{\frac{1}{p}-1}\left(\left(\frac{(b-a)^4}{30}\right)^{\frac{1}{p}}+\frac{(b-a)^2}{6}\right)$$

$$\leq(b-a)^2\,\|\acute{f}\|_p 2^{\frac{1}{p}-1}\left(\frac{2^{\frac{4}{p}-1}\left(b^{\frac{4}{p}}+a^{\frac{4}{p}}\right)}{30^{\frac{1}{p}}}+\frac{2}{6}(b^4+a^4)\right)$$

$$\leq(b-a)^2\,\|\acute{f}\|_p 2^{\frac{1}{p}-1}\left(b^{\frac{4}{p}}+a^{\frac{4}{p}}+b^4+a^4\right)$$

$$\leq(\mathrm{b}-\mathrm{a})^2\,\|\acute{\mathrm{f}}\|_{\mathrm{p}} 2^{\frac{1}{\mathrm{p}}-1}\left(\mathrm{b}^{\frac{4}{\mathrm{p}}}+\mathrm{a}^{\frac{4}{\mathrm{p}}}+\mathrm{b}^{\frac{4}{\mathrm{p}}}+\mathrm{a}^{\frac{4}{\mathrm{p}}}\right)$$

$$\leq(b-a)^2\,\|\acute{f}\|_p 2^{\frac{1}{p}}\left(b^{\frac{4}{p}}+a^{\frac{4}{p}}\right)$$

$$\leq 2(b^2+a^2)\,\|\acute{f}\|_p 2^{\frac{1}{p}}\left(b^{\frac{4}{p}}+a^{\frac{4}{p}}\right)$$

$$\leq\ 2^{\frac{1}{p}+2}\left(b^{\frac{4}{p}}+a^{\frac{4}{p}}\right)\|\acute{f}\|_p$$

$$\leq\ 2^{\frac{1}{p}+3}b^{\frac{4}{p}}\|\acute{f}\|_p,$$

Then ,we get

$$\left||b-E(X)||E(X)-a|-\sigma^2(X)-\frac{(b-a)^2}{6}\right|\leq 2^{\frac{1}{p}+3}b^{\frac{4}{p}}\|\acute{f}\|_p\quad\blacksquare$$

Lemma 5.2.7[4]

If $g, h, \acute{h} \in L_1[a,b]$, then

$$\left|\frac{1}{b-a}\int_a^b h(t)g(t)dt - \frac{1}{b-a}\int_a^b h(t)\,dt\,\frac{1}{b-a}\int_a^b g(t)dt\right|^2 \leq$$

$$\left(\frac{b-a}{\pi}\right)^2 \int_a^b |\acute{h}|^2 \left[\frac{1}{b-a}\int_a^b g(t)^2 dt - \left(\frac{1}{b-a}\int_a^b g(t)\,dt\right)^2\right]$$

Is Pre-Lupas inequality, when g, h and $\acute{h} \in L_2[a,b]$

Now let us generalize Pre-Lupas inequality for $L_p[a,b], p < 1$ spaces.

Theorem 1.2.8

If $g, h, \acute{h} \in L_p[a,b], 0 < p < 1$, then

$$\left|\frac{1}{b-a}\int_a^b h(t)g(t)dt - \frac{1}{b-a}\int_a^b h(t)\,dt\,\frac{1}{b-a}\int_a^b g(t)dt\right| \leq$$

$$\frac{c(b-a)^{p-\frac{1}{p}}}{\pi}\|\acute{h}\|_p \left(\frac{1}{b-a}\int_a^b g(t)^2 dt - \int_a^b g(t)dt\right)$$

Where c is a positive constant.

Proof

By using Lemma 1.2.7 and (2) we get,

$$\left|\frac{1}{b-a}\int_a^b h(t)g(t)dt - \frac{1}{b-a}\int_a^b h(t)\,dt\,\frac{1}{b-a}\int_a^b g(t)dt\right| \leq$$

$$\frac{c(b-a)}{\pi}\left(\sum_{i=1}^{n}\frac{b-a}{n}|\acute{h}(t_i)|^2\right)^{\frac{1}{2}}\left(\frac{1}{b-a}\int_a^b g(t)^2 dt - \int_a^b g(t)dt\right)$$

Then using (3) secondly, we get

$$\left|\frac{1}{b-a}\int_a^b h(t)g(t)dt - \frac{1}{b-a}\int_a^b h(t)\,dt\,\frac{1}{b-a}\int_a^b g(t)dt\right| \leq$$

$$\frac{c(b-a)}{\pi}\|\acute{h}\|_p \left(\frac{1}{b-a}\int_a^b g(t)^2 dt - \int_a^b g(t)dt\right) \qquad \blacksquare$$

Collorally 1.2.9

$$\left||b-E(X)||E(X)-a| - \sigma^2(X) - \frac{(b-a)^3}{6}\right| \leq \frac{c(b-a)^{5+p-\frac{1}{p}}}{6\pi}\left(\frac{b-a}{5} - 1\right)\|\acute{f}\|_p,$$

where c is a positive constant.

Proof

Put $h(t) = f(t), g(x) = (t-a)(b-t)$ in Theorem 1.2.8, we get

$$\left|\frac{1}{b-a}\int_a^b f(t)(t-a)(b-t)dt - \frac{1}{b-a}\int_a^b f(t)\,dt\,\frac{1}{b-a}\int_a^b (t-a)(b-t)dt\right| \leq$$

$$\frac{c(b-a)^{p-\frac{1}{p}}}{\pi}\|\acute{f}\|_p \left(\frac{1}{b-a}\int_a^b [(t-a)(b-t)]^2 dt - \int_a^b (t-a)(b-t)dt\right) \qquad (9)$$

By using Lemma 1.2.1, we get

$$\int_a^b (b-t)(t-a)f(t)dt = |b-E(X)||E(X)-a| - \sigma^2(X),$$

Since

$\int_a^b (t-a)^2(b-t)^2\,dt = \frac{(b-a)^5}{30}$ and $\int_a^b f(t)dt = 1.$

Also,

$$\int_a^b (b-t)(t-a)dt = \frac{(b-a)^3}{6},$$

So ,(9) implies,

$$\left||b-E(X)||E(X)-a| - \sigma^2(X) - \frac{(b-a)^3}{6}\right| \le \frac{c(b-a)^{2+p-\frac{1}{p}}}{\pi}\|\acute{f}\|_p\left(\frac{(b-a)^4}{30} - \frac{(b-a)^3}{6}\right).$$

Then , we get,

$$\left||\mathrm{b}-\mathrm{E(X)}||\mathrm{E(X)}-\mathrm{a}| - \sigma^2(\mathrm{X}) - \frac{(b-a)^3}{6}\right| \le \frac{c(b-a)^{5+p-\frac{1}{p}}}{6\pi}\left(\frac{b-a}{5} - 1\right)\|\acute{f}\|_p\,,$$

where c is a positive constant ■

Theorem 1.2.10

Let **X** be a random variable with the probability density function $f:[a,b]\to R$.If $f\in L_p^n[a,b] = \{f:[a,b]\to R, f, f^{(n)} \in L_p^n[a,b], 0<p<1\}$,then we have

$$\left||b-E(X)||E(X)-a| - \sigma^2(X) - \sum_{i=0}^{n}\frac{(i+1)(b-a)^{i+1}f^{(i)}(a)}{(i+3)!}\right| \le \frac{c}{n!}\|f^{(n+1)}\|_p\frac{(b-a)^{(n+1)p+3}}{(np+2)(np+3)},$$

where c is an absolute constant.

Proof

The Taylor's formula with integral remainder [13] is

$$f(t) = \sum_{i=0}^{n}\frac{(t-a)^n}{i!}f^{(i)}(a) + \frac{1}{n!}\int_a^t (t-s)^n f^{(n+1)}(s)\,ds$$

$t\in[a,b]$ (10)

By using Lemma1.2.1 and (10), we have

$|b-E(X)||E(X)-a| - \sigma^2(X)$

$$= \int_a^b (b-t)(t-a)\left[\sum_{i=0}^{n}\frac{(t-a)^n}{i!}f^{(i)}(a) + \frac{1}{n!}\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right]dt,$$

$$= \sum_{i=0}^{n}\frac{(t-a)^n}{i!}f^{(i)}(a)\int_a^b (b-t)(t-a)dt + \frac{1}{n!}\left[\int_a^b (b-t)(t-a)\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right]dt \quad (11)$$

Using the transformation $t = (1-u)a + ub,\ t\in[a,b]$

If $t=a$, then

$a = (1-u)a + ub$

$a = a - au + ub$

$0 = u(b-a).$

This implies, $u=0$ where $t=a$.

Similarly,

If $t=b$, we obtain $u=1$. Also, $dt = (b-a)du,$

$$\int_a^b (t-a)^{i+1}(b-t)dt = (b-a)^{i+3}\int_0^1 u^{i+1}(1-u)\,du = \frac{1}{(i+2)(i+3)}$$

By using (11), we deduce that,

$|b-E(X)||E(X)-a| - \sigma^2(X)$

$$= \sum_{i=0}^{n} \frac{1}{(i+2)(i+3)} \frac{(b-a)^{i+1} f^{(i)}(a)}{i!} + \frac{1}{n!}\left[\int_a^b (b-t)(t-a)\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right] dt$$

This implies,

$$\left| |b-E(X)||E(X)-a| - \sigma^2(X) - \sum_{i=0}^{n} \frac{(i+1)(b-a)^{i+1} f^{(i)}(a)}{(i+3)!} \right|$$

$$\leq \frac{1}{n!}\int_a^b (b-t)(t-a)\left|\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right| dt \qquad (12)$$

Since

$$\left|\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right| \leq \int_a^t |t-s|^n \left|f^{(n+1)}(s)\right| ds.$$

Then by using (3),this implies,

$$\left|\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right| \leq c(b-a)^{p-\frac{1}{p}}\left(\int_a^t |t-s|^{pn}\left|f^{(n+1)}(s)\right|^p ds\right)^{\frac{1}{p}},$$

Then , we get,

$$\left|\int_a^t (t-s)^n f^{(n+1)}(s)\,ds\right| \leq c(b-a)^{p-\frac{1}{p}}|t-s|^{pn}\left\|f^{(n+1)}\right\|_p,$$

$$0 < p < 1. \qquad (13)$$

Put (13) in (12), we get,

$$\left| |b-E(X)||E(X)-a| - \sigma^2(X) - \sum_{i=0}^{n} \frac{(i+1)(b-a)^{i+1} f^{(i)}(a)}{(i+3)!} \right|$$

$$\leq \frac{c(b-a)^{p-\frac{1}{p}}}{n!}\left\|f^{(n+1)}\right\|_p \int_a^b (b-t)(t-a)^{pn+1} dt. \qquad (14)$$

Assume $t = (1-u)a + ub$.

So

$$\left| |b-E(X)||E(X)-a| - \sigma^2(X) - \sum_{i=0}^{n} \frac{(i+1)(b-a)^{i+1} f^{(i)}(a)}{(i+3)!} \right|$$

$$\leq \frac{c(b-a)^{p-\frac{1}{p}}}{n!}\left\|f^{(n+1)}\right\|_p (b-a)^{np+3}\int_0^1 u^{np+1}(1-u)\,du$$

$$\leq \frac{c(b-a)^{p-\frac{1}{p}}}{n!}\left\|f^{(n+1)}\right\|_p \frac{(b-a)^{np+3}}{(nP+2)(nP+3)}$$

$$= \frac{c}{n!}\left\|f^{(n+1)}\right\|_p \frac{(b-a)^{(n+1)p+3-\frac{1}{p}}}{(nP+2)(nP+3)}$$

This implies,

$$\left| |\boldsymbol{b}-\boldsymbol{E}(\boldsymbol{X})||\boldsymbol{E}(\boldsymbol{X})-\boldsymbol{a}|-\boldsymbol{\sigma}^2(\boldsymbol{X})-\sum_{i=0}^{n}\frac{(\boldsymbol{i}+\mathbf{1})(\boldsymbol{b}-\boldsymbol{a})^{i+1}\boldsymbol{f}^{(i)}(\boldsymbol{a})}{(\boldsymbol{i}+\mathbf{3})!}\right| \le \frac{\boldsymbol{c}}{\boldsymbol{n}!}\left\|\boldsymbol{f}^{(n+1)}\right\|_p \frac{(\boldsymbol{b}-\boldsymbol{a})^{(n+1)p+3-\frac{1}{p}}}{(\boldsymbol{nP}+\mathbf{2})(\boldsymbol{nP}+\mathbf{3})}$$

$$\le \frac{\boldsymbol{c}}{\boldsymbol{n}!}\left\|\boldsymbol{f}^{(n+1)}\right\|_p \frac{(\boldsymbol{b}-\boldsymbol{a})^{(n+1)p+3}}{(\boldsymbol{nP}+\mathbf{2})(\boldsymbol{nP}+\mathbf{3})}$$

where c is absolute constant. ■

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