ISSN: 0974-5823 Vol. 7 No. 1 January, 2022

International Journal of Mechanical Engineering

PROBABILISTIC ESTIMATION OF FATIGUE DAMAGE IN STRUCTURAL ELEMENTS

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Abstract: One of the topical problems of modern mechanical engineering is to increase the strength safety of structural elements and machine parts exposed to irregular intense loads. Probability theory can sufficiently describe such loads and, therefore, corresponding stresses, mathematical statistics, and random processes theory. Thus, reliable analytical prediction of strength by accumulating fatigue damage and crack occurrence under random loading is a relevant problem. The authors present an analytical estimation of the probability of fatigue damage occurrence in structural elements depending on the action intensity and the time the system is operating. Statistical in formation on the strength of the elements is based on laws of the probability distribution for their fatigue limits considering the cale factor. In contrast, thein formation on the loading is based on correlation functions and energy spectra for the stresses obtained by solving related problems of statisticald ynamics.

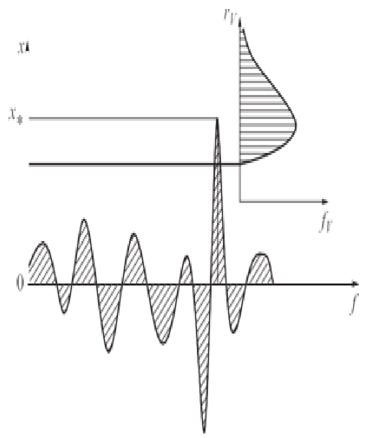
Keywords: statistical mechanics, strength reliability, fatigue, durability, random processes, Gaussian stationary processes, spectral density, three-parameter law, integral function, ac-accumulation off atigue damage, absolute maximum.

The force impact on modern machines details changes over time under deterministic (basically, cyclical) and random laws.

The work objective is to derive probability formulas of fatigue damages in the structural components depending on external loads and the time factor.

Let us consider the situation wherein the structural components appear uniform stresses **x(t)** throughout the volume, which presentsthe Gaussian stationary processes **(1)**with zero mean values and spectral densities**S (ω)**, and endurance strength of **r**structural material is a random variable with the given density of probabilities **f (r)** determined on the metal samples with a standard volume V_0 much less than the volume of V structure.

Let us determine the probability ("danger") that for some time t in the structural component with volume $V>>V_0$ of the stress will exceed at least once the volume-appropriate level of the endurance strength $\mathbf{r_V}$,which will correspond to the start moment of the accumulation process of fatigue damages (pic.1)



Pic.1. Exceeding the stress level of the endurance strength

The sought-for probability is determined with the formula:

$$P\{x(\tau) > r_V, \tau \in (0,t)\} =$$

$$= \int_{r_*}^{\infty} f_V(r_V) \left(\int_{r_V}^{\infty} f_*(x_*) dx_* \right) dr_V, \qquad (1)$$

Where $\mathbf{r_*}$ is a minimum value of the endurance strength; $\mathbf{f_V (r_V)}$ is the distribution density of probabilities of the endurance strength of the structural component with volume $\mathbf{V}$; $\mathbf{f_*}$ $\mathbf{(x^*)}$ is the distribution densityof probabilities for the largest in the interval time 0… t maximum $\mathbf{x_*}$ of the process $\mathbf{x(t)}$.

Let ussolve the given task based on the fact that the distribution of probabilities for the endurance strength of the metal samples with volume $\mathbf{V_0}$is describedby three-parameter Weibull law (2) with an integral function

$$F(r, V_0) = 1 - \exp\left[-\left(\frac{r - r_*}{r_c} \right)^{\alpha} \right] \quad \text{at} \quad \mathbf{r \geq r}$$

and density probabilities

$$f(r, V_0) = \frac{\alpha}{r_c^{\alpha}} (r - r_*)^{\alpha - 1} \exp\left[-\left(\frac{r - r_*}{r_c} \right)^{\alpha} \right], \quad (2)$$

Where α, r_c are parameters of the probabilities distribution.

Parameters α, r_c, $r^*$and the mean value of magnitude ($r - r^*$) are determined with the fatigue test results of the metal samples, since the mean value of the magnitude ($r-r^*$), its variation coefficient δ and quantity r^* become known based on these tests.

The equation solution determines parameter α:

$$\frac{\sqrt{\Gamma(1+2/\alpha) - \Gamma^2(1-1/\alpha)}}{\Gamma(1+1/\alpha)} = \delta, \qquad (3)$$

Where gamma-function

$$\Gamma(x)=\int_0^{\infty} e^{-t}\, t^{x-1} dt.$$

Equation solution results (3) are shown in Pic. 2 and Table 1

Parameter r_c has the following form:

$$r_c=\frac{<r-r_*>}{\Gamma\left(1+\frac{1}{\alpha}\right)},$$

Where <> is the averaging sign.

The probability is that in the volume $V={}_nV_0$ (n=1,2,3…) the endurance strength will exceed some value r, we define under the multiplication theorem on probability and the condition for the uniform distribution of stresses throughout the volumeby the following formula:

$$P\{r,V\}=\left[1-F(r,V_0)\right]^n. \qquad (4)$$

Then, under expressions (1) and (4) the probability distribution function for endurance strength in volume V takes the following form:

$$F(r,V)=1-\left[1-F(r,V_0)\right]^n=$$
$$=1-\exp\left[-\frac{V}{V_0}\left(\frac{r-r_*}{r_c}\right)^{\alpha}\right]. \qquad (5)$$

The relevant probability density

$$f_V(r_V)=\frac{\alpha V}{V_0}\frac{(r-r_*)^{\alpha-1}}{r_c^{\alpha}}\exp\left[-\left(\frac{r-r_*}{r_c}\right)^{\alpha}\right]. \qquad (6)$$

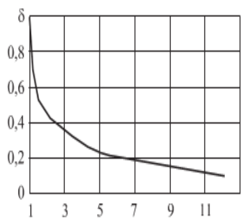
The expression (6) is included in this form's formula (1). Calculation dependence F from r, obtained by formula (5) at different values V and V_0 is shown in Pic.3.

It follows from the concordances that (5), (6) and Pic. 3, in the statistical sense (3) $r_V< r_0$ (r_0 is the endurance strength of the sample with volume V_0).

Let uscalculate the probability distribution for the largest maximum of the process x(t) in the time interval 0…t, based on the following Steinberg functional (4) to determine the number of zeros (zero level crossings) of the function x(t) in time t:

$$n_0(t)=\int\left|\dot{x}(\tau)\right|\delta\left\{x(\tau)\right\}d\tau, \qquad (7)$$

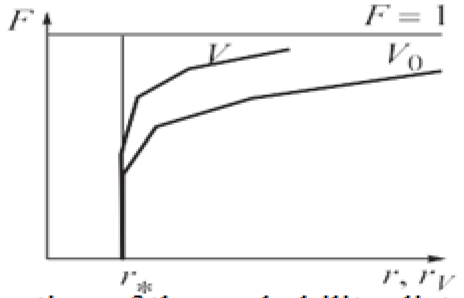
Where δ is the Dirac delta function; $\hat{x}(\tau)$ is the first derivative of the process x(t)



Pic. 2. Calculation dependence of the coefficient of variation δ from parameter α

Table 1

δ	α	δ	α
1,00	1,00	0,50	2,10
0,90	1,11	0,40	2,70
0,80	1,26	0,30	3,71
0,70	1,45	0,22	5,34
0,60	1,72	0,10	12,15



Pic. 3. Calculation function of the probability distribution for the endurance strength at different values V and V_0

Introducing into consideration the joint probability density f (x,x) for the process x(t) and its first derivative x(t), we get the Reis formula (5) from the expression (7) to determine the average number of emissions by the process x(t) of the level x in time t:

$$\overline{n}(x,t) = \frac{t}{2\pi}\frac{S_{\dot{x}}}{S_x}\exp\left(-\frac{x^2}{2S_x^2}\right),$$

where the process dispersion x(t) has the following form

$$S_x^2 = \int_{-\infty}^{\infty} S(x)\, d\omega,$$

and the first derivative dispersion has the following form:

$$S_{\dot{x}}^2 = \int_{-\infty} \omega^2 S(x)\, d\omega.$$

The expected number of maxima for a narrowband random process x(t) in the interval dx is determined by the formula

$$dn_{\max} = \left|\overline{n}(x) - \overline{n}(x+dx)\right| = n_{\max} f_{\max}(x)\, dx, \quad (8)$$

Where n_{max}= n (0,t) is the average number of maxima in time t; f_{max} is the maxima probability density.

The relation (8) shows that the distribution of maxima in narrowband processes correspond to the Rayleigh law [6]with probability density

$$f_{\max}(x) = \frac{1}{n_{\max}}\left|\frac{d\overline{n}(x)}{dx}\right| = \frac{x}{S_{\dot{x}}^2}\exp\left(-\frac{x^2}{2S_{\dot{x}}^2}\right)$$

and integral function

$$F_M(x) = 1 - \exp\left(-\frac{x^2}{2S_{\dot{x}}^2}\right).$$

The probability that in the flow from n_{max} the largest of them will be less than some values x*, they are determinedunder multiplication theorem on probability by the formula

$$F_*(x_*) = \left[F_M(x_*)\right]^{n_{\max}}.$$

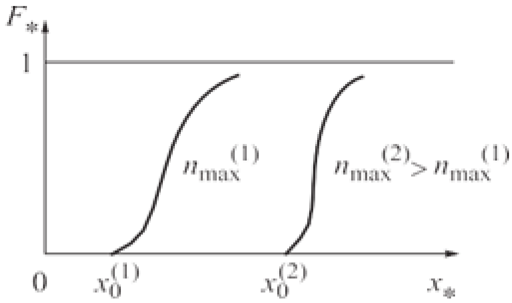
Introducing into consideration the quantity z=n_{max} (1-Fm (x*))at a large number n_{max}, we get the following formula:

$$F_*(x_*)=\left(1-\frac{z}{n_{max}}\right)^{n_{max}}\to\exp\left[-n_{max}\left(1-F_M(x_*)\right)\right]\approx$$
$$\approx 1-n_{max}\left[1-F_M(x_*)\right]=$$
$$=\begin{cases}0 \text{ при } x_*<x_0=S_x\sqrt{2\ln n_{max}};\\ 1-n_{max}\exp\left(-\frac{x_*^2}{2S_x^2}\right) \text{ при } x_*\ge x_0,\end{cases} \quad (9)$$

where →− is the arrow to the large number of maxima n_{max}

The type of function (9) at various values of n_{max} is shown in Pic. 4.

In the statistical sense, the value of the largest maximum increases in time (7)



Pic.4. Calculation function of the distribution of probabilities for the largest maximum at different values n_{max}

The probability distribution density for the largest maximum

$$f_*(x_*)=\begin{cases}0 \text{ при } x_*<x_0;\\ \frac{n_{max}x_*}{S_x^2}\exp\left(-\frac{x_*^2}{2S_x^2}\right) \text{ при } x_*\ge x_0.\end{cases} \quad (10)$$

Substituting relations (6) and (10) into formula (1), we determinethe sought-for probability of the accumulation process start of the fatigue damages in the time interval 0…t.

In approximate estimates, instead of expression (10) it can be assumed that

$$f_*(x_*)=\delta(x_*-x_0).$$

Taking into account that

$$\int_{r_V}^{\infty}\delta\left(x_*-x_0\right)=\begin{cases}0 \text{ при } r_V>x_0;\\ 1 \text{ при } r_V\le x_0,\end{cases}$$

we get

$$P\left\{x(\tau)>r_V,\tau\in(0,t)\right\}=\int_{r_*}^{x_0}f_V(r_V)dr_V=F_V\left(x_0\right)=$$
$$=1-\exp\left[-\frac{V}{V_0}\left(\frac{x_0-r_*}{r_c}\right)^{\alpha}\right] \text{ при } n_{max}>n_*,$$

where the largest maximum of the loading process

$$x_0 = S_x \sqrt{2 \ln n_{\max}},$$

and the critical number of maxima at which the process of fatigue damages accumulation starts,

$$n_* = \exp\left[\frac{1}{2}\left(\frac{r_*}{S_x}\right)^2\right].$$

For instance, we consider a structural component from carbon steel with endurance strength r, given with the test results of standard volume samples V_0 by a polygon of frequencies (Pic. 5).

According to estimate results, we have obtained the following values:

- minimum value of endurance strength r* = 300 MPa;
- mean value

$$\overline{r} = \sum P_i r_i = 413{,}5 \text{ МПа};$$

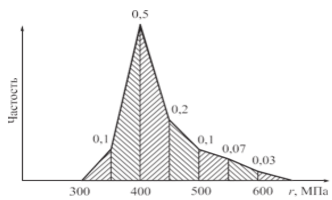
- second-order moment

$$<r^2> = \sum P_i r_i^2 = 1{,}79 \cdot 10^5 \text{ МПа}^2;$$

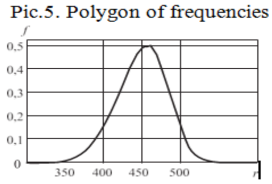
- third-order moment

$$<r^3> = \sum P_i r_i^3 = 7{,}87 \cdot 10^7 \text{ МПа}^3;$$

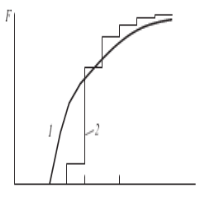
$$S = \sqrt{<r^2> - \overline{r}^2} = 89{,}12 \text{ МПа};$$



Pic.5. Polygon of frequencies



Pic.6. Calculated density of probability distribution of endurance strength



Pic. 7. Theoretical (1) and empirical (2) functions of the probability distribution for endurance strength

Table 2

λ	K	λ	K
0,5	0,036	1,1	0,822
0,6	0,14	1,2	0,89
0,65	0,208	1,4	0,96
0,75	0,373	1,5	0,978
0,8	0,456	1,6	0,99
1	0,73	1,8	0,999

• variation coefficient δ=S/<r>=0,2155. Solving the equation, we get α=5,344. Parameter r_c=162,7 MPa.

Function graph (2) is shown in Pic. 6.

Pic. 7 shows the theoretical and empirical functions of the probability distribution for endurance strength. The most significant difference between them does not exceed the value Δ=0,05, that at n=100 of tests correspond to proximity parameter of two probability distributions $\lambda = \Delta\sqrt{n}$ =0,5.

Some function values of the goodness-of-fit criteria tests by A.N.Kolgomorov K(λ) (8) are shown in Table 2.

In this case, K(λ)= 0,036 shows that the obtained theoretical probability distribution does not contradict the empirical data with high probability.

For instance, we will consider the structural component with characteristics of fatigue resistance (9): r* - 30 MPa, α = 4, r_c = 16,6 MPa. This element is exposed to random stress effects with Sx = 7MPa.

Estimates show that with very low probability,the fatigue damage accumulation process will occur approximately after 104 loading cycles. At 106 loading cycles, this process will occur with a probability of 99%.

Thus, the proposed estimation method is quite effective for starting the fatigue damage accumulation process probabilistic forecasting.

Conclusions

1. When exposed to random loads, the formulas describing the probability of fatigue cracks in a loaded body have been obtained.

2. Comparison of estimated and experimental results shows their good conformity. Obtained expressions may be used to estimatethe strength reliability of the parts.

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